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Intuitionistic Fuzzy ω-Extremally Disconnected Spaces

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Abstract

In this paper, a new class of intuitionistic fuzzy topological spaces called intuitionistic fuzzy ω extremally disconnected spaces is introduced and several other properties are discussed.

Keywords: *Intuitionistic fuzzy ω extremally disconnected spaces, lower (resp.upper) intuitionistic fuzzy ω continuous functions.*

1 Introduction

After the introduction of the concept of fuzzy sets by Zadeh [13], several researches were conducted on the generalizations of the notion of fuzzy set. The concept of "Intuitionistic fuzzy sets" was first published by Atanassov [2] and many works by the same author and his colleagues appeared in the literature [3-5]. Later this concept was generalized to "Intuitionistic L-fuzzy sets" by Atanassov and stoeva [6]. An introduction to intuitionistic fuzzy topological space was introduced by Dogan Coker [8]. Several types of fuzzy connectedness in intuitionistic fuzzy topological spaces defined by Coker (1997). The construction is based on the idea of intuitionistic fuzzy set developed by Atanassov (1983, 1986; Atanassov and Stoeva, 1983). The concept of fuzzy extremally disconnected spaces was studied in [7]. In this paper a new class of intuitionistic fuzzy topological spaces namely, intuitionistic fuzzy ω extremally disconnected spaces is introduced by using the notions introduced in [7, 11, 12], The concept of fuzzy ω-open set was studied in [10]. Tietze extension theorem for intuitionistic fuzzy ω-extremally disconnected spaces has been discussed as in [1]. Some interesting properties and characterizations are studied.

2 Preliminaries

Definition 2.1[4]: *Let* X *be a non empty fixed set. An intuitionistic fuzzy set (IFS for short)* A *is an object having the form* $A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle : x\in X\}$ *where the functions* $\mu_A : X\to I$ *and* $\gamma_A : X\to I$ *denote the degree of membership (namely* $\mu_A(x)$*) and the degree of non membership (namely* $\gamma_A(x)$*) of each element* $x\in X$ *to the set* A*, respectively, and* $0\le\mu_A(x)+\gamma_A(x)\le 1$ *for each* $x\in X$.

Remark 2.1[8]: For the sake of simplicity, we shall use the symbol $A=\langle x,\mu_A,\gamma_A\rangle$.

Definition 2.2[4]: *Let* X *be a non empty set and the IFSs* A *and* B *be in the form* $A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle : x\in X\}$, $B=\{\langle x,\mu_B(x),\gamma_B(x)\rangle : x\in X\}$. *Then*

(a) $A\subseteq B$ *iff* $\mu_A(x)\le\mu_B(x)$ *and* $\gamma_A(x)\ge\gamma_B(x)$ *for all* $x\in X$;
(b) $A=B$ *iff* $A\subseteq B$ *and* $B\subseteq A$*;*
(c) $\overline{A}=\{\langle x,\gamma_A(x),\mu_A(x)\rangle : x\in X\}$*;*
(d) $A\cap B=\{\langle x,\mu_A(x)\wedge\mu_B(x),\gamma_A(x)\vee\gamma_B(x)\rangle : x\in X\}$*;*
(e) $A\cup B=\{\langle x,\mu_A(x)\vee\mu_B(x),\gamma_A(x)\wedge\gamma_B(x)\rangle : x\in X\}$*;*
(f) $[\ ]A=\{\langle x,\mu_A(x),1-\mu_A(x)\rangle : x\in X\}$*;*
(g) $\langle\ \rangle A=\{\langle x,1-\gamma_A(x),\gamma_A(x)\rangle : x\in X\}$.

Definition 2.3[8]: *Let* X *be a non empty set and let* $\{A_i : i\in J\}$ *be an arbitrary family of IFSs in* X *. Then*
(a) $\cap A_i=\{\langle x,\wedge\mu_{A_i}(x),\vee\gamma_{A_i}(x)\rangle : x\in X\}$*;*
(b) $\cup A_i=\{\langle x,\vee\mu_{A_i}(x),\wedge\gamma_{A_i}(x)\rangle : x\in X\}$.

Definition 2.4[8]: *Let* X *be a non empty fixed set. Then,* $0_{\sim}=\left\{\left\langle x,0,1\right\rangle : x\in X\right\}$ *and* $1_{\sim}=\left\{\left\langle x,1,0\right\rangle : x\in X\right\}$.

Definition 2.5[8]: *Let* X *and* Y *be two non empty fixed sets and* $f: X\rightarrow Y$ *be a function. Then*

(a) If $B=\left\{\left\langle y,\mu_B(y),\gamma_B(y)\right\rangle : y\in Y\right\}$ *is an IFS in Y, then the pre image of* B *under* f*, denoted by* $f^{-1}(B)$*, is the IFS in* X *defined by* $f^{-1}(B)=\left\{\left\langle x, f^{-1}(\mu_B)(x), f^{-1}(\gamma_B)(x)\right\rangle : x\in X\right\}$.

(b) If $A=\left\{\left\langle x,\lambda_A(x),\nu_A(x)\right\rangle : x\in X\right\}$ *is an IFS in* X*, then the image of* A *under* f*, denoted by* $f(A)$*, is the IFS in* Y *defined by*

$f(A)=\left\{\left\langle y, f(\lambda_A)(y),(1-f(1-\nu_A))(y)\right\rangle : y\in Y\right\}$ *where,*

$$f(\lambda_A)(y)=\begin{cases}\sup\limits_{x\in f^{-1}(y)} \lambda_A(x) & \text{if } f^{-1}(y)\neq 0\\ 0, & \text{otherwise,}\end{cases}$$

$$(1-f(1-\nu_A))(y)=\begin{cases}\inf\limits_{x\in f^{-1}(y)} \nu_A(x) & \text{if } f^{-1}(y)\neq 0\\ 1, & \text{otherwise.}\end{cases}$$

for the IFS $A=\left\{\left\langle x,\mu_A(x),\gamma_A(x)\right\rangle : x\in X\right\}$.

Definition 2.6[8]: *Let* X *be a non empty set. An intuitionistic fuzzy topology (IFT for short) on a non empty set* X *is a family* τ *of intuitionistic fuzzy sets (IFSs for short) in* X *satisfying the following axioms: (T1)* $0_{\sim},1_{\sim}\in\tau$*, (T2)* $G_1\cap G_2\in\tau$ *for any* $G_1,G_2\in\tau$,

(T3) $\bigcup\limits_{\subseteq\tau} G_i\in\tau$ *for any arbitrary family* $\{G_i : i\in J\}$.

In this case the pair (X,τ) is called an intuitionistic fuzzy topological space (IFTS for short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS for short) in X.

Definition 2.7[8]: *Let* X *be a non empty set. The complement* $\overline{A}$ *of an IFOS* A *in an IFTS* (X,τ) *is called an intuitionistic fuzzy closed set (IFCS for short) in* X.

Definition 2.8[8]: *Let* (X,τ) *be an IFTS and* $A=\langle x,\mu_A,\gamma_A\rangle$ *be an IFS in* X *. Then the fuzzy interior and fuzzy closure of* A *are defined by*

$$cl(A)=\bigcap\{K:K \text{ is an } IFCS \text{ in } X \text{ and } A\subseteq K\},$$
$$\text{int}(A)=\bigcup\{G:G \text{ is an } IFOS \text{ in } X \text{ and } G\subseteq A\}.$$

Remark 2.2[8]: Let (X,τ) be an *IFTS.* $cl(A)$ is an *IFCS* and $\text{int}(A)$ is an *IFOS* in X , and (a) A is an *IFCS* in X iff $cl(A)=A$; (b) A is an *IFOS* in X iff $\text{int}(A)=A$

Proposition 2.1[8]: *Let* (X,τ) *be an IFTS. For any IFS* A *in* (X,τ)*, we have (a)* $cl(\overline{A})=\overline{\text{int}(A)}$*, (b)* $\text{int}(\overline{A})=\overline{cl(A)}$.

Definition 2.9[8]: *Let* (X,τ) *and* (Y,φ) *be two IFTSs and let* $f:X\to Y$ *be a function. Then* f *is said to be fuzzy continuous iff the pre image of each IFS in* ϕ *is an IFS in* τ.

Definition 2.10[8]: *Let* (X,τ) *and* (Y,φ) *be two IFTSs and let* $f:X\to Y$ *be a function. Then* f *is said to be fuzzy open(resp.closed) iff the image of each IFS in* τ *(resp.(1-* τ *)) is an IFS in* ϕ *(resp.(1-*ϕ*)).*

Definition 2.11[9]: *A subset A of an IFTS* (X,τ) *is called an IF semi-open set if* $A\subseteq IFcl(IF\,\text{int}(A))$ *and an IF semi-closed set if* $IF\,\text{int}(IFcl(A))\subseteq A$.

Definition 2.12[10]: *A subset of a topological space* (X,T) *is called ω-closed in* (X,T) *if* $cl(A)\subseteq U$ *whenever* $A\subseteq U$ *and U is semi-open in* (X,T)*. A subset A is called ω-open if* A^C *is ω-closed.*

An *IFTS* (X,T) represent intuitionistic fuzzy topological spaces and for a subset A of a space (X,T), *IFcl(A), IF*int(A) and $\overline{A}$ denote an intuitionistic fuzzy closure of A, an intuitionistic fuzzy interior of A and the complement of A in X respectively.

Notation 2.1[1]: *Let* X *be any non-empty set and* $A\in\zeta^X$*. Then for* $x\in X$*,* $\langle\mu_A(x),\gamma_A(x)\rangle$ *is denoted by* $A^{\sim}$.

Definition 2.13[1]: *An intuitionistic fuzzy real line* $\mathbb{R}_{\mathbb{I}}(I)$ *is the set of all monotone decreasing intuitionistic fuzzy set* $A\in\zeta^R$ *satisfying* $\bigcup\{A(t):t\in R\}=1^{\sim}$ *and* $\bigcap\{A(t):t\in R\}=0^{\sim}$ *after the identification of an intuitionistic fuzzy sets*

$A, B \in \mathbb{R}_{\mathbb{I}}(I)$ *if and only if* $A(t-) = B(t-)$ *and* $A(t+) = B(t+)$ *for all* $t \in R$ *where* $A(t-) = \bigcap\left\{ A(s) : s < t \right\}$ *and* $A(t+) = \bigcup\left\{ A(s) : s > t \right\}$.

The intuitionistic fuzzy unit interval $\mathbb{I}_{\mathbb{I}}(I)$ is a subset of $\mathbb{R}_{\mathbb{I}}(I)$ such that $[A] \in \mathbb{I}_{\mathbb{I}}(I)$ if the membership and non-membership of *A* are defined by

$$\mu_A(t) = \begin{cases} 1 & t < 0 \\ 0 & t > 1 \end{cases} \text{ and } \gamma_A(t) = \begin{cases} 1 & t < 0 \\ 0 & t > 1 \end{cases} \text{ respectively.}$$

The intuitionistic fuzzy topology on $\mathbb{R}_{\mathbb{I}}(I)$ is generated from the subbasis $\left\{ L_t^{\mathbb{I}}, R_t^{\mathbb{I}}, t \in \mathbb{R} \right\}$ where $L_t^{\mathbb{I}}$ $R_t^{\mathbb{I}}$: $\mathbb{R}_{\mathbb{I}}(I) \to \mathbb{I}_{\mathbb{I}}(I)$ are given by $L_t^{\mathbb{I}}(A) = \overline{A(t-)}$ and $R_t^{\mathbb{I}}(A) = A(t+)$ respectively.

Definition 2.14[1]: *Let* (X, T) *be an intuitionistic fuzzy topological space. The characteristic function of intuitionistic fuzzy set A in X is the function* $\psi_A : X \to \mathbb{I}_{\mathbb{I}}(I)$ *defined by* $\psi_A(x) = A^{\sim}$, *for each* $x \in X$.

Notation 2.2[1]: *Let* (X, T) *be intuitionistic fuzzy topological space and let* $A \subset X$. *Then an intuitionistic fuzzy* χ_A^* *is of the form* $\langle x, \chi_A(x), 1 - \chi_A(x) \rangle$.

3 Intuitionistic Fuzzy ω-Extremally Disconnected Spaces

In this section, the concept of ω extremally disconnectedness in intuitionistic fuzzy topological space is introduced besides proving several other propositions.

Definition 3.1: *A subset A of an IFTS* (X, T) *is called intuitionistic fuzzy ω closed(IF ω closed for short) if* $IFcl(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is IF semi-open in* (X, T).

Definition 3.2: *A subset A of an IFTS* (X, T) *is called intuitionistic fuzzy ω open (IF ω open for short) if* $\overline{A}$ *is IF ω closed.*

Definition 3.3: *Let* (X, T) *be an intuitionistic fuzzy topological space and A be an intuitionistic fuzzy set in* X. *Then the intuitionistic fuzzy ω closure of A (*$IF\omega\, cl(A)$ *for short) and intuitionistic fuzzy ω interior of A* $(IF\omega\ int(A) for\ short)$ *are defined by*

$IF\omega\ cl(A) = \bigcap\{K: K$ is an intuitionistic fuzzy ω closed set in *X* and $A \subseteq K\}$,

$IF\omega\ int(A) = \cup\{G: G$ is an intuitionistic fuzzy ω open set in X and $G \subseteq A\}$.

Definition 3.4: *A function $f:(X,T) \rightarrow (Y,S)$ is called intuitionistic fuzzy ω continuous if $f^{-1}(V)$ is an intuitionistic fuzzy ω closed set of (X,T) for every intuitionistic fuzzy closed set V of (Y,S).*

Proposition 3.1: *For any intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X,T), the following statements hold:*

(a) $\overline{IF\omega\ cl(A)} = IF\omega\ int(\bar{A})$

(b) $\overline{IF\omega\ int(A)} = IF\omega\ cl(\bar{A})$

Definition 3.5: *Let (X,T) be an intuitionistic fuzzy topological space. Let A be any intuitionistic fuzzy ω open set in (X,T). If an intuitionistic fuzzy ω closure of A is intuitionistic fuzzy ω open, then (X,T) is said to be an intuitionistic fuzzy ω extremally disconnected space.*

Proposition 3.2: *For an intuitionistic fuzzy topological space (X,T) the following statements are equivalent:*

(a) (X,T)*is intuitionistic fuzzy ω extremally disconnected.*

(b) *For each intuitionistic fuzzy ω closed set A, $IF\omega\ int(A)$ is intuitionistic fuzzy ω closed.*

(c) *For each intuitionistic fuzzy ω open set A,* $IF\omega\ cl\big(IF\omega\ int(\bar{A})\big) = \overline{IF\omega\ cl(A)}$

(d) *For each pair of intuitionistic fuzzy ω open sets A and B in (X,T) with* $\overline{IF\omega\ cl(A)} = B$, $IF\omega\ cl(B) = \overline{IF\omega\ cl(A)}$.

Proposition 3.3: *Let (X,T) be an intuitionistic fuzzy topological space. Then (X,T) is an intuitionistic fuzzy ω extremally disconnected space if and only if for any intuitionistic fuzzy ω open set A and intuitionistic fuzzy ω closed set B such that $A \subseteq B$, $IF\omega\ cl(A) \subseteq IF\omega\ int(B)$.*

Notation 3.1: *An intuitionistic fuzzy set which is both intuitionistic fuzzy ω open set and intuitionistic fuzzy ω closed set is called intuitionistic fuzzy ω clopen set.*

Remark 3.1: Let (X,T) be an intuitionistic fuzzy ω extremally disconnected space. Let $\{A_i, \overline{B_i}/i \in N\}$ be a collection such that A_i's are intuitionistic fuzzy ω open sets, B_i's are intuitionistic fuzzy ω closed sets and let A, $\bar{B}$ be intuitionistic fuzzy ω clopen sets respectively. If $A_i \subseteq A \subseteq B_j$ and $A_i \subseteq B \subseteq B_j$ for all $i,j \in N$ then there exists an intuitionistic fuzzy ω clopen set C such that $IF\omega\ cl(A_i) \subseteq C \subseteq IF\omega\ int\big(B_j\big)$ for all $i,j \in N$.

Proposition 3.4: *Let (X,T) be an intuitionistic fuzzy ω extremally disconnected space. Let $\left(A_q\right)_{q\in Q}$ and $\left(B_q\right)_{q\in Q}$ be the monotone increasing collections of* *intuitionistic fuzzy ω open sets and intuitionistic fuzzy ω closed sets of* (X,T) *respectively and suppose that* $A_{q_1} \subseteq B_{q_2}$ *whenever* $q_1 < q_2$ *(Q is the set of rational numbers). Then there exists a monotone increasing collection* $\{C_q\}_{q\in Q}$ *of intuitionistic fuzzy ω clopen sets of* (X,T) *such that* $IF\omega\ cl(A_{q_1}) \subseteq C_{q_2}$ *and* $C_{q_2} \subseteq IF\omega\ int(B_{q_2})$ *whenever* $q_1 < q_2$.

4 Properties and Characterizations of Intuitionistic Fuzzy ω Extremally Disconnected Spaces

In this section, various properties and characterizations of intuitionistic fuzzy ω extremally disconnected spaces are discussed.

Definition 4.1: *Let* (X,T) *be an intuitionistic fuzzy topological space. A function* $f: X \to R_I(I)$ *is called lower (resp., upper) intuitionistic fuzzy ω continuous, if* $f^{-1}(R_t^I)$ *(resp.,* $f^{-1}(\overline{L_t^I})$*) is an intuitionistic fuzzy ω open set (resp., intuitionistic fuzzy ω clopen) for each* $t \in R$.

Lemma 4.1: *Let* (X,T) *be an intuitionistic fuzzy topological space. Let* $A \in \zeta^X$ *and let* $f: X \to R_I(I)$ *be such that*

$$f(x)(t) = \begin{cases} 1^{\sim} if & t < 0, \\ A^{\sim}\ if & 0 \le t \le 1, \\ 0^{\sim}\ if & t > 1, \end{cases}$$

for all $x \in X$ *and* $t \in R$. *Then* f *is lower (resp., upper) intuitionistic fuzzy ω continuous iff A is intuitionistic fuzzy ω open (resp., intuitionistic fuzzy ω clopen) set.*

Proposition 4.1: *Let* (X,T) *be an intuitionistic fuzzy topological space and let* $A \in \zeta^X$. *Then* ψ_A *is lower (resp., upper) intuitionistic fuzzy ω continuous iff A is intuitionistic fuzzy ω open (resp., intuitionistic fuzzy ω clopen).*

Definition 4.2: *Let* (X,T) *and* (Y,S) *be two intuitionistic fuzzy topological spaces. A function* $f: (X,T) \to (Y,S)$ *is called intuitionistic fuzzy strongly ω continuous if* $f^{-1}(A)$ *is intuitionistic fuzzy ω clopen in* (X,T) *for every intuitionistic fuzzy ω open set in* (Y,S).

Proposition 4.2: *Let* (X,T) *be an intuitionistic fuzzy topological space. Then the following statements are equivalent:*

(a) (X,T) *is intuitionistic fuzzy ω extremally disconnected,*

(b) *If* $g,h: X \to R_I(I)$, *g is lower intuitionistic fuzzy ω continuous, h is upper intuitionistic fuzzy ω continuous and* $g \subseteq h$, *then there exists an* *intuitionistic fuzzy strongly ω continuous function,* $f:(X,T)\rightarrow R_I(I)$ *such that* $g\subseteq f\subseteq h$.

(c) *If* $\overline{A}$ *and* B *are intuitionistic fuzzy ω open sets such that* $B\subseteq A$ *then there exists an intuitionistic fuzzy strongly ω continuous function* $f:(X,T,\leq)\rightarrow I_I(I)$ *such that* $B\subseteq\overline{L_1^I f}\subseteq R_0^I f\subseteq A$.

5 Tietze Extension Theorem for Intuitionistic Fuzzy ω Extremally Disconnected Spaces

In this section, Tietze extension theorem for intuitionistic fuzzy ω extremally disconnected space is studied.

Proposition 5.1: *Let* (X,T) *be an upper intuitionistic fuzzy ω extremally disconnected space and let* $A\subset X$ *be such that* χ_A^* *is an intuitionistic fuzzy ω open set in* (X,T). *Let* $f:(A,T/A)\rightarrow I_I(I)$ *be an intuitionistic fuzzy strongly ω continuous function. Then,* f *has an intuitionistic fuzzy strongly ω continuous extension over* (X,T).

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