Gen. Math. Notes, Vol. 4, No. 1, May 2011, pp. 70-84
ISSN 2219-7184; Copyright © ICSRS Publication, 2011
www.i-csrs.org
Available free online at http://www.geman.in

Properties of Contra Sg-Continuous Maps

O. Ravi[1], M. Lellis Thivagar[2] and R. Latha[3]

[1]Department of Mathematics, P.M.T. College, Usilampatti, Madurai,
Tamil Nadu, India.
E-mail: siingam@yahoo.com
[2]Department of Mathematics, Arul Anandar College, Karumathur, Madurai,
Tamil Nadu, India
E-mail: mlthivagar@yahoo.co.in
[3]Department of Mathematics, Prince SVP Engineering College, Ponmar,
Chennai-48, Tamil Nadu, India.
E-mail: ar.latha@gmail.com

(Received: 30-12-10 /Accepted: 11-4-11)

Abstract

In [Dontchev J. Contra-continuous functions and strongly S-closed spaces. Int. J. Math. Math. Sci. 1996; 19(2) : 303 – 310], Dontchev introduced and investigated a new notion of continuity called contra-continuity. Following this, many authors introduced various types of new generalizations of contra-continuity called contra-α-continuity [22], contra-semi-continuity [7], contra precontinuity [21], contra-super-continuity [23], contra-β-continuity [3], almost-contra-super-continuity [14], contra-δ-precontinuity [11], almost-contra-precontinuity [12] and contra sg-continuity [7] and so on [13 and 33]. In this paper, we investigate a generalization of contra-continuity by utilizing semi-generalized closed sets [1].

Keywords: *sg-closed set, contra sg-continuous map, contra sg-graph, sg- $T_{1/2}$ space, sg-normal space, sg-closed-compact space.*

1 Introduction

General Topology has shown its fruitfulness in both the pure and applied directions. In data mining [38], computational topology for geometric design and molecular design [31], computer – aided geometric design and engineering design (briefly CAGD), digital topology, information systems, non-commutative geometry and its application to particle physics [25], one can observe the influence made in these realms of applied research by general topological spaces, properties and structures. Rosen and Peters [39] have used topology as a body of mathematics that could unify diverse areas of CAGD and engineering design research. They have presented several examples of the application of topology to CAGD and design.

2 Preliminaries

The concept of closedness is fundamental with respect to the investigation of general topological spaces. Levine [28] initiated the study of the so-called g-closed sets and by doing this; he generalized the concept of closedness. Following this, in 1987, Bhattacharyya and Lahiri [1] introduced the notion of semi-generalized closed sets in topological spaces by means of semi-open sets of Levine [27]. In continuation of this work, in 1991, Sundaram et al [43] studied and investigated semi-generalized continuous maps and semi-$\mathbf{T}_{½}$-spaces. Recently, Dontchev and Noiri [7] have defined the concept of contra-sg-continuity between the topological spaces. In this paper, we investigate the properties of contra-sg-continuous maps.
In this paper, spaces (X, τ), (Y, σ) and (Z, ρ) (shortly X, Y and Z) mean topological spaces. Let A be a subset of a space X. For a subset A of (X, τ), cl(A) and int(A) represent the closure of A and the interior of A respectively.

Definition 1. *A subset A of a space X is said to be*

(i) semi-open [27] if $A \subseteq cl(int(A))$.
The complement of a semi-open set is called semi-closed.
(ii) preopen [30] if $A \subseteq int(cl(A))$.
The complement of a preopen set is called preclosed.
(iii) regular open [41] if $A = int(cl(A))$.
The complement of a regular open is called regular closed.
(iv) δ-open [44] if it is the union of regular open sets.
The complement of a δ-open set is called δ-closed.

The intersection of all semi-closed sets containing A is called the semi-closure [4] of A and is denoted by scl(A).

Definition 2. *Let A be a subset of X. Then A is called sg-closed [1] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is a semi-open set.*
The complement of a sg-closed set is called an sg-open set.
The intersection of all sg-closed sets containing a set A is called the semi-generalized closure of A and is denoted by sgcl(A) [2]. If a subset A is sg-closed in a space X, then A = sgcl(A). The converse of this implication is not true in general as shown in [2].
The family of all sg-open (resp. sg-closed, closed) sets of X is denoted by SGO(X) (resp. SGC(X), C(X)). The family of all sg-open (resp. sg-closed, closed) sets of X containing a point $x \in X$ is denoted by SGO(X, x) (resp. SGC(X, x), C(X, x)).

Definition 3. *A map $f : X \rightarrow Y$ is called :*

(i) *contra-continuous [6] if $f^{-1}(V)$ is closed in X for each open set V in Y;*
(ii) *contra semi-continuous [7] if $f^{-1}(V)$ is semi-closed in X for each open set V in Y;*
(iii) *contra sg-continous [7] if $f^{-1}(V)$ is sg-closed in X for each open set V in Y;*
(iv) *sg-continous [2] if $f^{-1}(V)$ is sg-closed in X for each closed set V in Y;*
(v) *sg-irresolute [2] if $f^{-1}(V)$ is sg-closed in X for each sg-closed set V in Y;*
(vi) *preclosed [16] if f(V) is preclosed in Y for each closed set V in X;*
(vii) *irresolute [4] if $f^{-1}(V)$ is semi-closed in X for each semi-closed set V in Y.*

Definition 4. *A space X is called :*

(i) *a locally indiscrete [35] if each open subset of X is closed in X;*
(ii) *semi-$T_{½}$-space [1] if each sg-closed subset of X is semi-closed in X;*
(iii) *sg-connected [2] if X cannot be written as a disjoint union of two non-empty sg-open sets;*
(iv) *ultra normal [42] if each pair of non-empty disjoint closed sets can be separated by disjoint clopen sets;*
(v) *weakly Hausdorff [40] if each element of X is an intersection of regular closed sets;*
(vi) *ultra Hausdorff [42] if for each pair of distinct points x and y in X, there exist clopen sets A and B containing x and y, respectively, such that $A \cap B = \phi$.*

Result 5. *Let X be a topological space. Then*

(i) *Every semi-closed set of X is sg-closed in X, but not conversely. [2]*
(ii) *Every closed set of X is sg-closed in X, but not conversely. [2]*

Let S be a subset of a space X. The set $\cap\{U \in \tau : S \subset U\}$ is called the kernel of S and is denoted by ker(S). [32]

Lemma 6 [10]. *The following properties hold for the subsets U, V of a space X.*

(1) $x \in ker(U)$ if and only if $U \cap F \neq \phi$ for any closed set F containing x.
(2) $U \subset ker(U)$ and $U = ker(U)$ if U is open in X.
(3) $U \subset V$, then $ker(U) \subset ker(V)$.

3 Characterizations of Contra Sg-Continuous Maps

Remark 7. *From the definitions we stated above, we observe that*

(i) Every contra-continuous map is contra sg-continuous.
(ii) Every contra semi-continuous map is contra sg-continuous.

However the separate converses of the above relations are not true from the following examples.

Example 8. Let X = Y = {a, b, c}, $\tau = \{\phi, X, \{a\}, \{a, b\}\}$ and $\sigma = \{\phi, Y, \{a\}\}$. Define $f : X \to Y$ as f(a) = c; f(b) = a; f(c) = b. Clearly f is contra sg-continuous map but it is not contra-continuous.

Example 9. Let X = Y = {a, b, c}, $\tau = \{\phi, X, \{a\}, \{b, c\}\}$ and $\sigma = \{\phi, Y, \{a\}\}$. Define $f : X \to Y$ as f(a) = b; f(b) = c; f(c) = a. Clearly f is contra sg-continuous map but it is not contra semi-continuous.

Theorem 10. *Let $f : X \to Y$ be a map. The following statements are equivalent.*

(i) f is contra sg-continuous.
(ii) The inverse image of each closed set in Y is sg-open in X.

Proof. Let G be a closed set in Y. Then Y\G is an open set in Y. By the assumption of (i), $f^{-1}(Y\backslash G) = X\backslash f^{-1}(G)$ is sg-closed in X. It implies that $f^{-1}(G)$ is sg-open in X. Converse is similar.

Theorem 11. *Suppose that SGC(X) is closed under arbitrary intersections. Then the following are equivalent for a map $f : X \to Y$.*

(i) f is contra sg-continuous.
(ii) the inverse image of every closed set of Y is sg-open in X.
(iii) For each $x \in X$ and each closed set B in Y with $f(x) \in B$, there exists an sg-open set A in X such that $x \in A$ and $f(A) \subset B$.
(vi) $f(sgcl(A)) \subset ker(f(A))$ for every subset A of X.
(v) $sgcl(f^{-1}(B)) \subset f^{-1}(ker\ B)$ for every subset B of Y.

Proof. (i) $\Rightarrow$ (iii): Let $x \in X$ and B be a closed set in Y with $f(x) \in B$. By (i), it follows that $f^{-1}(Y\backslash B) = X\backslash f^{-1}(B)$ is sg-closed and so $f^{-1}(B)$ is sg-open. Take $A = f^{-1}(B)$. We obtain that $x \in A$ and $f(A) \subset B$.
(iii) $\Rightarrow$ (ii): Let B be a closed set in Y with $x \in f^{-1}(B)$. Since $f(x) \in B$, by (iii) there exist an sg-open set A in X containing x such that $f(A) \subset B$. It follows that $x \in A \subset f^{-1}(B)$. Hence $f^{-1}(B)$ is sg-open.
(ii) $\Rightarrow$ (i): Follows from the previous theorem.
(ii) $\Rightarrow$ (iv): Let A be any subset of X. Let $y \notin ker(f(A))$. Then there exists a closed set F containing y such that $f(A) \cap F = \phi$. Hence, we have $A \cap f^{-1}(F) = \phi$ and $sgcl(A) \cap f^{-1}(F) = \phi$. Hence we obtain $f(sgcl(A)) \cap F = \phi$ and $y \notin f(sgcl(A))$. Thus, $f(sgcl(A)) \subset ker(f(A))$.
(iv) $\Rightarrow$ (v): Let B be any subset of Y. By (iv), $f(sgcl(f^{-1}(B))) \subset ker(B)$ and $sgcl(f^{-1}(B)) \subset f^{-1}(ker(B))$.
(v) $\Rightarrow$ (i): Let B be any open set of Y. By (v), $sgcl(f^{-1}(B)) \subset f^{-1}(ker(B)) = f^{-1}(B)$ and $sgcl(f^{-1}(B)) = f^{-1}(B)$. We obtain that $f^{-1}(B)$ is sg-closed in X.

Theorem 12. *Let $f : X \rightarrow Y$ be a map and $g : X \rightarrow X \times Y$ the graph function of f, defined by $g(x) = (x, f(x))$ for every $x \in X$. If g is contra sg-continuous, then f is contra sg-continuous.*

Proof. Let U be an open set in Y. Then $X \times U$ is an open set in $X \times Y$. It follows that $f^{-1}(U) = g^{-1}(X \times U)$ is sg-closed in X. Thus, f is contra sg-continuous.

For a map $f : X \rightarrow Y$, the subset $\{(x, f(x)) : x \in X\} \subset X \times Y$ is called the graph of f and is denoted by G(f).

Definition 13. *The graph G(f) of a map $f : X \rightarrow Y$ is said to be contra sg-graph if for each $(x, y) \in (X \times Y)\backslash G(f)$, there exist an sg-open set U in X containing x and a closed set V in Y containing y such that $(U \times V) \cap G(f) = \phi$.*

Proposition 14. *The following properties are equivalent for the graph G(f) of a map f:*

(i) *G(f) is contra sg-graph.*
(ii) *For each $(x, y) \in (X \times Y)\backslash G(f)$, there exist an sg-open set U in X containing x and a closed V in Y containing y such that $f(U) \cap V = \phi$*

Theorem 15. *If $f : X \rightarrow Y$ is contra sg-continuous and Y is Urysohn, G(f) is contra sg-graph in $X \times Y$.*

Proof. Let $(x, y) \in (X \times Y)\backslash G(f)$. It follows that $f(x) \neq y$. Since Y is Urysohn, there exist open sets B and C such that $f(x) \in B$, $y \in C$ and $cl(B) \cap cl(C) = \phi$. Since f is contra sg-continuous, there exists an sg-open set A in X containing x such that $f(A) \subset cl(B)$. Therefore $f(A) \cap cl(C) = \phi$ and G(f) is contra sg-graph in $X \times Y$.

Theorem 16. *Let $\{Xi \,/\, i \in I\}$ be any family of topological spaces. If $f : X \to \Pi Xi$ is a contra sg-continuous map, then $Pri \circ f : X \to Xi$ is contra sg-continuous for each $i \in I$, where Pri is the projection of ΠX_i onto X_i.*

Proof. We shall consider a fixed $i \in I$. Suppose U_i is an arbitrary open set of X_i. Since Pr_i is continuous, $Pr_i^{-1}(U_i)$ is open in ΠX_i. Since f is contra sg-continuous, we have by definition, $f^{-1}(Pr_i^{-1}(U_i)) = (Pr_i \circ f)^{-1}(U_i)$ is sg-closed in X. Therefore $Pr_i \circ f$ is contra sg-continuous.

Definition 17. *A space X is said to be sg-$T_{½}$ space if every sg-closed set of X is closed in X.*

Lemma 18. *Let (X, τ) be a topological space. Then $sg\text{-}\tau = \{U \subset X : sgcl(X \backslash U) = X \backslash U\}$ is a topology for X.*

Theorem 19. *Let (X, τ) be a topological space. Then every sg-closed set is closed if and only if $sg\text{-}\tau = \tau$.*

Proof. Let $A \in sg\text{-}\tau$. Then $sgcl(X\backslash A) = X\backslash A$. By hypothesis, $cl(X\backslash A) = sgcl(X\backslash A) = X\backslash A$ and $A \in \tau$. Conversely, let A be a sg-closed set. Then $sgcl(A) = A$ and hence $X\backslash A \in sg\text{-}\tau = \tau$. Hence, A is closed.

Theorem 20. *Let $f : X \to Y$ be a map. Suppose that X is a sg-$T_{½}$ space. Then the following are equivalent.*

(i) f is contra sg-continuous .
(ii) f is contra semi-continuous.
(iii) *f is contra-continuous.*

Proof. The proof is obvious.

Definition 21. *A space X is said to be locally sg-indiscrete if every sg-open set of X is closed in X.*

Theorem 22. *If $f : X \to Y$ is contra sg-continuous with X as locally sg-indiscrete, then f is continuous.*

Proof. Omitted.

Theorem 23. *If $f : X \to Y$ is contra sg-continuous and X is sg-$T_{½}$ space, then f is contra-continuous.*

Proof. Omitted.

Theorem 24. *If f : X → Y is a surjective preclosed contra sg-continuous with X as sg-$T_{½}$ space, then Y is locally indiscrete.*

Proof. Suppose that V is open in Y. Since f is contra sg-continuous, $f^{-1}(V)$ is sg-closed in X. Since X is a sg-$\mathbf{T}_{½}$ space, $f^{-1}(V)$ is closed in X. Since f is preclosed, then V is preclosed in Y. Now we have cl(V) = cl(int(V)) ⊆ V. This means V is closed in X and hence Y is locally indiscrete.

Theorem 25. *Suppose that X and Y are spaces and SGO(X) is closed under arbitrary unions. If a map f : X → Y is contra sg-continuous and Y is regular, then f is sg-continuous*.

Proof. Let x be an arbitrary point of X and V be an open set of Y containing f(x). Since Y is regular, there exists an open set G in Y containing f(x) such that cl(G) ⊂ V. Since f is contra sg-continuous, there exists U ∈ SGO(X) containing x such that f(U) ⊂ cl(G). Then f(U) ⊂ cl(G) ⊂ V. Hence f is sg-continuous.

Theorem 26. *A contra sg-continuous image of a sg-connected space is connected.*

Proof. Let f : X → Y be a contra sg-continuous map of a sg-connected space X onto a topological space Y. If possible, let Y be disconnected. Let A and B form a disconnection of Y. Then A and B are clopen and Y = A∪B where A∩B = ϕ. Since f is a contra sg-continuous map, X = $f^{-1}(Y) = f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are non-empty sg-open sets in X. Also $f^{-1}(A) \cap f^{-1}(B) = \phi$. Hence X is not sg-connected. This is a contradiction. Therefore Y is connected.

Theorem 27. *Let X be sg-connected. Then each contra sg-continuous maps of X into a discrete space Y with atleast two points is a constant map*.

Proof. Let f : X → Y be a contra sg-continuous map. Then X is covered by sg-open and sg-closed covering $\{f^{-1}(\{y\}) : y \in Y\}$. By assumption $f^{-1}(\{y\}) = \phi$ or X for each y ∈ Y. If $f^{-1}(\{y\}) = \phi$ for all y ∈ Y, then f fails to be a map. Then there exists only one point y ∈ Y such that $f^{-1}(\{y\}) \neq \phi$ and hence $f^{-1}(\{y\}) = X$ which shows that f is a constant map.

Theorem 28. *If f is a contra sg-continuous map from a sg-connected space X onto any space Y, then Y is not a discrete space.*

Proof. Suppose that Y is discrete. Let A be a proper nonempty open and closed subset of Y. Then $f^{-1}(A)$ is a proper nonempty sg-open and sg-closed subset of X, which is a contradiction to the fact that X is sg-connected.

Definition 29. *A space X is said to be sg-normal if each pair of non-empty disjoint closed sets can be separated by disjoint sg-open sets.*

Theorem 30. *If $f : X \to Y$ is a contra sg-continuous, closed, injection and Y is ultra normal, then X is sg-normal.*

Proof. Let F_1 and F_2 be disjoint closed subsets of X. Since f is closed and injective, $f(F_1)$ and $f(F_2)$ are disjoint closed subsets of Y. Since Y is ultra normal, $f(F_1)$ and $f(F_2)$ are separated by disjoint clopen sets V_1 and V_2 respectively. Hence $F_i \subset f^{-1}(V_i)$, $f^{-1}(V_i)$ is sg-open in X for i = 1, 2 and $f^{-1}(V_1) \cap f^{-1}(V_2) = \phi$. Thus, X is sg-normal.

Theorem 31. *If $f : X \to Y$ is contra sg-continuous map and X is a semi-$T_{½}$ space, then f is contra-semi-continuous.*

Proof. Omitted.

4 Composition of Two Maps

Theorem 32. *The composition of two contra sg-continuous maps need not be contra sg-continuous.*
The following example supports the above theorem.

Example 33. Let X = Y = Z = {a, b, c}, τ = {ϕ, X, {a}, {b}, {a, b}}, σ = {ϕ, Y, {a}, {b, c}} and ρ = {ϕ, Z, {a, b}}. Then the identity map $f : X \to Y$ is contra sg-continuous and the identity map $g : Y \to Z$ is contra sg-continuous. But their composition $g \circ f : X \to Z$ is not contra sg-continuous.

Theorem 34. *Let X and Z be any topological spaces and Y be a semi-$T_{½}$ space. Let $f : X \to Y$ be an irresolute map and $g : Y \to Z$ be a contra sg-continuous map. Then $g \circ f : X \to Z$ is contra semi-continuous map.*

Proof. Let F be any open set in Z. Since g is contra sg-continuous, $g^{-1}(F)$ is sg-closed in Y. But Y is semi-$\mathbf{T}_{½}$ space. Therefore $g^{-1}(F)$ is semi-closed in Y. Since f is irresolute,$f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$ is semi-closed in X. Thus, g o f is contra semi-continuous.

Theorem 35. *Let $f : X \to Y$ be sg-irresolute map and $g : Y \to Z$ be contra sg-continuous map. Then $g \circ f : X \to Z$ is contra sg-continuous.*

Proof. Let F be an open set in Z. Then $g^{-1}(F)$ is sg-closed in Y because g is contra sg-continuous. Since f is sg-irresolute, $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$ is sg-closed in X. Therefore g o f is contra sg-continuous.

Corollary 36. *Let $f : X \to Y$ be sg-irresolute map and $g : Y \to Z$ be contra-continuous map. Then $g \circ f : X \to Z$ is contra sg-continuous.*

Definition 37. *A map $f : X \to Y$ is said to be pre sg-open if the image of every sg-open subset of X is sg-open in Y.*

Theorem 38. *Let $f : X \to Y$ be surjective sg-irresolute pre sg-open and $g : Y \to Z$ be any map. Then $g \circ f : X \to Z$ is contra sg-continuous if and only if g is contra sg-continuous.*

Proof. The 'if' part is easy to prove. To prove the 'only if' part, let $g \circ f : X \to Z$ be contra sg-continuous and let F be a closed subset of Z. Then $(g \circ f)^{-1}(F)$ is a sg-open subset of X. That is $f^{-1}(g^{-1}(F))$ is sg-open. Since f is pre sg-open, $f(f^{-1}(g^{-1}(F)))$ is a sg-open subset of Y. So, $g^{-1}(F)$ is sg-open in Y. Hence g is contra sg-continuous.

Theorem 39. *If $f : X \to Y$ is sg-irresolute map with Y as locally sg-indiscrete space and $g : Y \to Z$ is contra sg-continuous map, then $g \circ f : X \to Z$ is sg-continuous.*

Proof. Let F be any closed set in Z. Since g is contra sg-continuous, $g^{-1}(F)$ is sg-open set in Y. But Y is locally sg-indiscrete, $g^{-1}(F)$ is closed in Y. Hence $g^{-1}(F)$ is sg-closed set in Y. Since f is sg-irresolute, $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$ is sg-closed in X. Therefore $g \circ f$ is sg-continuous.

5 Some New Separation Axioms

Definition 40. *A space X is said to be :*

(i) *sg-compact [2] (strongly S-closed [6]) if every sg-open (respectively closed) cover of X has a finite subcover;*

(ii) *countably sg-compact (strongly countably S-closed) if every countable cover of X by sg-open (resp. closed) sets has a finite subcover;*

(iii) *sg-Lindelöf (strongly S-Lindelöf) if every sg-open (resp. closed) cover of X has a countable subcover.*

Theorem 41. *The surjective contra sg-continuous images of sg-compact [2] (resp. sg-Lindelöf, countably sg-compact) spaces are strongly S-closed [6] (respectively strongly S-Lindelöf, strongly countably S-closed).*

Proof. Suppose that $f : X \to Y$ is a contra sg-continuous surjection. Let $\{V_\alpha : \alpha \in I\}$ be any closed cover of Y. Since f is contra sg-continuous, then $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is an sg-open cover of X and hence there exists a finite subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Therefore, we have $Y = \cup\{V_\alpha : \alpha \in I_0\}$ and Y is strongly S-closed.

The other proofs can be obtained similarly.

Definition 42. *A space X is said to be :*

- *(i) sg-closed-compact if every sg-closed cover of X has a finite sub cover;*
- *(ii) countably sg-closed-compact if every countable cover of X by sg-closed sets has a finite sub cover;*
- *(iii) sg-closed-Lindelöf if every sg-closed cover of X has a countable sub cover.*

Theorem 43. *The surjective contra sg-continuous images of sg-closed-compact (resp. sg-closed-Lindelöf, countably sg-closed-compact) spaces are compact (resp. Lindelöf, countably compact).*

Proof. Suppose that $f : X \to Y$ is a contra sg-continuous surjection. Let $\{V_\alpha : \alpha \in I\}$ be any open cover of Y. Since f is contra sg-continuous, then $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is a sg-closed cover of X. Since X is sg-closed-compact, there exists a finite subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Therefore, we have $Y = \cup\{V_\alpha : \alpha \in I_0\}$ and Y is compact.
The other proofs can be obtained similarly.

Definition 44. *A space X is said to be sg-T_1 [30] iff for each pair of distinct points x and y in X, these exist sg-open sets U and V containing x and y, respectively, such that $y \notin U$ and $x \notin V$.*

Definition 45. *A space X is said to be sg-T_2 [30] iff for each pair of distinct points x and y in X, there exist $U \in SGO(X, x)$ and $V \in SGO(X,y)$ such that $U \cap V = \phi$.*

Theorem 46. *Let X and Y be topological spaces. If*

- *(i) for each pair of distinct points x and y in X, there exists a map f of X into Y such that $f(x) \neq f(y)$,*
- *(ii) Y is an Urysohn space and*
- *(iii) f is contra sg-continuous at x and y,*

then X is sg-T_2.

Proof. Let x and y be any distinct points in X. Then, there exists a Urysohn space Y and a map $f : X \to Y$ such that $f(x) \neq f(y)$ and f is contra sg-continuous at x and y. Let $z = f(x)$ and $v = f(y)$. Then $z \neq v$. Since Y is Urysohn, there exist open sets V and W containing z and v, respectively, such that $cl(V) \cap cl(W) = \phi$. Since f is contra sg-continuous at x and y, then there exist sg-open sets A and B containing x and y, respectively, such that $f(A) \subset cl(V)$ and $f(B) \subset cl(W)$. We have $A \cap B = \phi$ since $cl(V) \cap cl(W) = \phi$. Hence, X is sg-T_2.

Corollary 47. *Let $f : X \to Y$ is a contra sg-continuous injection. If Y is an Urysohn space, then sg-T_2.*

Theorem 48. *If $f : X \to Y$ is a contra sg-continuous injection and Y is weakly Hausdorff, then X is sg-T_1.*

Proof. Suppose that Y is weakly Hausdorff. For any distinct point x and y in X, there exist regular closed sets A, B in Y such that $f(x) \in A$, $f(y) \notin A$, $f(x) \notin B$ and $f(y) \in B$. Since f is contra sg-continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are sg-open subsets of X such that $x \in f^{-1}(A)$, $y \notin f^{-1}(A)$, $x \in f^{-1}(B)$ and $y \in f^{-1}(B)$.
This shows that X is sg-T_1.

Theorem 49. *Let $f : X \to Y$ have a contra sg-graph. If f is injective, then X is sg-T_1.*

Proof. Let x and y be any two distinct points in X. Then, we have $(x, f(y)) \in (X \times Y)\backslash G(f)$. Then, there exist an sg-open set U in X containing x and $F \in C(Y, f(y))$ such that $f(U) \cap F = \phi$; hence $U \cap f^{-1}(F) = \phi$. Therefore, we have $y \notin U$. This implies that X is sg-T_1.

Theorem 50. *Let $f : X \to Y$ be a contra sg-continuous injection. If Y is an ultra Hausdorff space, then X is sg-T_2.*

Proof. Let x and y be any two distinct points in X. Then, $f(x) \neq f(y)$ and there exist clopen sets A and B containing f(x) and f(y), respectively, such that $A \cap B = \phi$. Since f is contra sg-continuous, then $f^{-1}(A) \in SGO(X)$ and $f^{-1}(B) \in SGO(X)$ such that $f^{-1}(A) \cap f^{-1}(B) = \phi$. Hence, X is sg-$T_2$.

Definition 51. *A map $f : X \to Y$ is said to be :*

- *(i) perfectly continuous [36] if $f^{-1}(V)$ is clopen in X for every open set V of Y;*
- *(ii) RC-continuous [7] if $f^{-1}(V)$ is regular closed in X for each open set V of Y;*
- *(iii) Strongly continuous [26] if the inverse image of every set in Y is clopen in X;*
- *(iv) Contra R-map [15] if $f^{-1}(V)$ is regular closed in X for every regular open set V of Y;*
- *(v) Contra super-continuous [23] if for each $x \in X$ and each $F \in C(Y, f(x))$, there exists a regular open set U in X containing x such that $f(U) \subset F$;*
- *(vi) Almost contra-super-continuous [14] if $f^{-1}(V)$ is δ-closed in X for every regular open set V of Y;*
- *(vii) Regular set-connected [9, 24] if $f^{-1}(V)$ is clopen in X for every regular open set V in Y;*

(viii) *Almost s-continuous [37] if for each $x \in X$ and each $V \in SO(Y, f(x))$, there exists an open set U in X containing x such that $f(U) \subset scl(V)$;*

(ix) *(Θ, s)-continuous [24] if for each $x \in X$ and each $V \in SO(Y, f(x))$, there exists an open set U in X containing x such that $f(U) \subset cl(V)$.*

Remark 52. *The following diagram holds for a map $f : X \to Y$:*

Strongly continuous	⇒	*almost s-continuous*
Perfectly continuous	⇒	*regular set-connected*
RC-continuous	⇒	*contra R-map*
Contra super-continuous	⇒	*almost contra-super-continuous*
Contra-continuous	⇒	*(Θ, s)-continuous*
Contra-semi-continuous	⇒	*contra sg-continuous*

Remark 53. *None of these implications is reversible as shown in [7, Ex.3.1 and 13, Remark 9].*

Remark 54. *(Θ, s)-continuity and contra sg-continuity are independent of each other. It may be seen by the following examples.*

Example 55. Let X = {a, b, c}, τ = {ϕ, X, {a}, {a, b}, {a, c}} and σ = {ϕ, X, {a}, {a, b}}. Then the identity map f : (X, τ) → (X, σ) is (Θ, s)-continuous map which is not contra sg-continuous.

Example 56. Let X = {a, b, c}, τ = {ϕ, X, {b}, {c}, {b, c}} and σ = {ϕ, X, {a}, {b}, {a, b}, {a, c}}. Then the identity map f : (X, τ) → (X, σ) is contra sg-continuous map which is not (Θ, s)-continuous.

Definition 57. *A map $f : X \to Y$ is called ß-continuous if $f^{-1}(V)$ is ß-open in X for every open set V of Y.*

Theorem 58. *If X is sg-$T_{½}$, then the following equivalent for a map $f : X \to Y$:*

(i) *f is RC-continuous.*
(ii) *f is ß-continuous and contra sg-continuous.*
(iii) *f is ß-continuous and contra g-continuous.*
(iv) *f is ß-continuous and contra-continuous.*

Proof. Follows easily from the proof of [7, Thm 3.11].

References

[1] P. Bhattacharyya and B.K. Lahiri, Semi-generalized closed sets in topology, *Indian J. Math.*, 29(3) (1987), 375-382.

[2] M.C. Cueva, Semi-generalized continuous maps in topological spaces, *Portugaliae Mathematica*, 52(4) (1995), 399-407.

[3] M. Caldas and S. Jafari, Some properties of contra-β-continuous functions, *Mem. Fac. Sci. Kochi. Univ. (Math).*, 22(2001), 19-28.

[4] S.G. Crossley and S.K. Hildebrand, Semi-topological properties, *Fund. Math.*, 74(1972), 233-254.

[5] R. Devi, H. Maki and K. Balachandran, Semi-generalized closed maps and generalized semi-closed maps, *Mem. Fac. Sci. Kochi. Univ. Ser. A (Math.)*, 14(1993), 41-54.

[6] J. Dontchev, Contra-continuous functions and strongly S-closed spaces, *Internat. J. Math. Math. Sci.*, 19(2) (1996), 303-310.

[7] J. Dontchev and T. Noiri, Contra semi-continuous functions, *Math. Pannon*, 10(2) (1999), 159-168.

[8] J. Dontchev and H. Maki, On the behaviour of sg-closed sets, *Topology Atlas*.

[9] J. Dontchev, M. Ganster and I.L. Reilly, More on almost s-continuity, *Indian J. Math.*, 41(2) (1999), 139-146.

[10] E. Ekici, On a weaker form of RC-continuity, *Analele Univ Vest din Timisoara. Seria Matematica – Informatica*, XLII(fasc. 1) (2004), 79-91.

[11] E. Ekici and T. Noiri, Contra δ-precontinuous functions, *Bull. Cal. Math. Soc*., 98(3) (2006), 275-284.

[12] E. Ekici, Almost contra-precontinuous functions, *Bull. Malays. Math. Sci. Soc. (Second Series)*, 27(2004), 53-65.

[13] E. Ekici, On contra πg-continuous functions, *Chaos, Solitons and Fractals*, 35(2008), 71-81.

[14] E. Ekici, Almost contra super continuous functions, *Studii Si Cercetari Stiintifice Seria: Matematica Univ din Bacau*, 14(2004), 31-42.

[15] E. Ekici, Another form of contra-continuity, *Kochi J. Math.*, 1(2006), 21-29.

[16] S.N. El-Deeb, I.A. Hasanein, A.S. Mashhour and T. Noiri, On p-regular spaces, *Bull. Math. Soc. Sci. Math. R. S. Roumanie*, 27(1983), 311-315.

[17] M.S. El-Naschie, On the uncertainty of Cantorian geometry and the two-slit experiment, *Chaos, Solitons and Fractals*, 9(3) (1998), 517-529.

[18] M.S. El-Naschie, Quantum gravity from descriptive set theory, *Chaos, Solitons and Fractals*, 19(2004), 1339-1344.

[19] M.S. El-Naschie, On the certification of heterotic strings, M theory and e^{∞} theory, *Chaos, Solitons and Fractals*, 2(2000), 2397-2408.

[20] M.S. El-Naschie, Superstrings, Knots and non commutative geometry in $e^{(\infty)}$ space, *Int. J. Theor. Phys.*, 37(12) (1998), 2935-2951.

[21] S. Jafari and T. Noiri, On contra-precontinuous functions, *Bull. Malays. Math. Sci. Soc. (Second Series)*, 25(2002), 115-128.

[22] S. Jafari and T. Noiri, Contra α-continuous functions between topological spaces, *Iranian Int. J. Sci.*, 2(2001), 153-167.

[23] S. Jafari and T. Noiri, Contra Super-continuous functions, *Annales. Univ. Sci. Budapest.*, 42(1999), 27-34.

[24] J.E. Joseph and M.H. Kwack, On S-closed spaces**,** *Proc. Amer. Math. Soc.*, 80 (1980), 341-348.
[25] G. Landi, An introduction to non commutative spaces and their geometris, *Lecture Notes in Physics*, New York, Springer–Verlag, (1997).
[26] N. Levine, Strong continuity in topological spaces, *Amer. Math. Monthly.*, 67 (1960), 269.
[27] N. Levine, Semi-open sets and semi-continuity in topological spaces, *Amer. Math. Monthly*, 70(1963), 36-41.
[28] N. Levine, Generalized closed sets in topology, *Rend. Circ. Mat. Palermo*, 19(2) (1970), 89-96.
[29] H. Maki, K. Balachandran and R. Devi, Remarks on semi-generalized closed sets and generalized semi-closed sets, *Kyungpook Math. J.*, 36(1) (1996), 155-163.
[30] A.S. Mashhour, M.E. Abd El-Monsef and S.N. El-Deeb, On precontinuous and weak precontinuous mappings, *Proc. Math. Phys. Soc.*, Egypt, 53(1982), 47-53.
[31] E.L.F. Moore and T.J. Peters, Computational topology for geometric design and molecular design, In: DR Ferguson, TJ Peters, editors, *Mathematics in Industry Challenges and Frontiers 2003, SIAM,* (2005).
[32] M. Mrsevic, On pairwise R_0 and pairwise R_1 bitopological spaces, *Bull. Math. Soc. Sci. Math.*, R. S. Roumanie, 30(1986), 141-148.
[33] A.A. Nasef, Some properties of contra-γ-continuous functions, *Chaos, Solitons and Fractals,* 24(2005), 471-477.
[34] G.B. Navalagi, Semi generalized separation axioms in topology**,** *Topology Atlas*.
[35] T. Nieminen, On ultra pseudo compact and related topics, *Ann. Acad. Sci. Fenn. Ser. A. I. Math.*, 3(1977), 185-205.
[36] T. Noiri, Super continuity and some strong forms of continuity**,** *Indian J. Pure Appl. Math.*, 15(1984), 241-250.
[37] T. Noiri, B. Ahmad and M. Khan, Almost s-continuous functions**,** *Kyungpook Math. J.*, 35(1995), 311-322.
[38] Z. Pawlak, Rough sets: theoretical aspects of reasoning about data, *System Theory, Knowledge Engineering and Problem Solving*, Dordrecht: Kluwer, 9(1991).
[39] D.W. Rosen and T.J. Peters, The role of topology in engineering design research, *Res. Eng. Des*, 2(1996), 81-98.
[40] T. Soundararajan, Weakly Housdorff spaces and the cardinality of topological spaces, In : General topology and its relation to modern analysis and algebra, *III Proc. Conf. Kanpur, 1968, Academia*, Prague, (1971), 301-306.
[41] M.H. Stone, Applications of the theory of boolean rings to general topology, *Trans. Amer. Math. Soc.*, 41(1937), 375-381.
[42] R. Staum, The algebra of bounded continuous functions into a non Archimedean field, *Pacific J. Math.*, 50(1974), 169-185.

[43] P. Sundaram, H. Maki and K. Balachandran, Semi-generalized continuous maps and semi-$T_{1/2}$ spaces, *Bull. Fukuoka Univ. Edu. Part–III*, 40(1991), 33-40.

[44] N.V. Veličko, H-closed topological spaces, *Amer. Math. Soc. Transl.*, 78(1968), 103-118.