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Periodic Solutions for A Predator-Prey Model with Beddington-DeAngelis Type Functional Response on Time Scales

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Abstract

TIn this paper, the existence of periodic solutions of a predator-prey model with Beddington-DeAngelis type functional response is investigated by using the Gaines and Mawhin's continuation theorem of coincidence degree theory on time scales. Some conditions are obtained for the existence of periodic solutions. The approach is unified to provide the existence of the desired solutions for the continuous differential equations and discrete difference equations.

Keywords: *Beddington-DeAngelis type, coincidence degree, periodic solution, predator-prey system, time scales.*

1 Introduction

From the population data of Canadian Lynx and Snowshoe hare from 1840s, we know that interaction between a pair of predator-prey influences the population growth of both species. The first predator-prey model was formulated by Alfred

James Lotka in 1925[12], and Vito Volterra in 1926[15]. After that, a lot of more complicated but realistic continuous and discrete predator-prey models have been proposed and investigated by many authors[5,6,10,14,16]. In 2010, Baek [2] studied the stability and periodic solutions of the following predator-prey model:

$$\begin{cases} \dot{x}_1 = ax_1(t)\left[1 - \frac{x(t)}{K}\right] - \frac{ex_1(t)x_2(t)}{bx_2(t)+x_1(t)+c}, \\ \dot{x}_2 = -Dx_2(t) + \frac{ex_1(t)x_2(t)}{bx_2(t)+x_1(t)+c}, \end{cases} \tag{1}$$

where $x_1(t), x_2(t)$ are the population densities of prey and the predator at time t respectively, the positive constants K, a, m, D are the carrying capacity of the prey, intrinsic growth rate of the prey, the conversion rate, the death rate of the predator, respectively. The term *by* denotes the mutual interference between predators. In details, one can see Ref.[2].

Since biological and environmental parameters are naturally subjected to fluctuation in time, the effects of a periodically varying environment are considered as important selective forces on systems in periodically changing environment, we incorporate the varying property of the parameters into the model and reconstruct it as follows:

$$\begin{cases} \dot{x}_1 = a(t)x_1(t)\left[1 - \frac{x(t)}{K(t)}\right] - \frac{e(t)x_1(t)x_2(t)}{b(t)x_2(t)+x_1(t)+c(t)}, \\ \dot{x}_2 = -D(t)x_2(t) + \frac{e(t)x_1(t)x_2(t)}{b(t)x_2(t)+x_1(t)+c(t)}. \end{cases} \tag{2}$$

It must be pointed out that although there are numerous papers investigating the existence of positive periodic solutions of differential or difference equations by using the coincidence degree theory in mathematical ecology, one often deal with these types of equations in a different way to prove the existence results. This motivated us to think wether we can explore such an existence problem in an unified way. In order to unify continuous and discrete analysis, the theory of time scales(measure chain), which has recently received a great many attention, was introduced by Stefan Hilger in his PhD thesis in 1998. After that, people have done a great many research about dynamic equations on time scales. Moreover, many results on the existence of periodic solutions of dynamic equations have been reported[1,3,4,8,9]. Motivated by Refs.[1,3,4,8,9], in this paper, we will devote our attention to investigating the existence of periodic solutions of the following predator-prey model with Beddington-DeAngelis type on time scales:

$$\begin{cases} x_1^{\Delta}(t) = a(t)\left[1 - \frac{\exp(x(t))}{K(t)}\right] - \frac{e(t)\exp(x_2(t))}{b(t)\exp(x_2(t))+\exp(x_1(t))+c(t)}, \\ x_2^{\Delta}(t) = -D(t) + \frac{e(t)\exp(x_1(t))}{b(t)\exp(x_2(t))+\exp(x_1(t))+c(t)}, \end{cases} \tag{3}$$

where $x_1(t), x_2(t)$ are the population densities of prey and the predator at time t respectively, $K(t), a(t), m(t), D(t)$ are the carrying capacity of the prey, intrinsic growth rate of the prey, the conversion rate, the death rate of the predator at time t respectively and they are rd-continuous positive ω-periodic functions.

We believed that it is the first time to deal with the existence problem of periodic solution for system (3) on time scales.

The remainder of the paper is organized as follows: in Section 2, we present some preliminary results including some basic knowledge for dynamic systems on time scales. In Section 3, we prove our main result.

2 Preliminary Results on Time Scales

In order to make an easy and convenient reading of this paper, we present some foundational definitions and notations on time scales so that the paper self-contained. For more detail, one can see[4,9].

Definition 2.1 *A time scale is an arbitrary nonempty closed subset $\mathbb{T}$ of $\mathbb{R}$, the real numbers. The set $\mathbb{T}$ inherits the standard topology of $\mathbb{R}$.*

Definition 2.2 *The forward jump operator $\sigma : \mathbb{T} \to \mathbb{T}$, the backward jump operator $\sigma : \mathbb{T} \to \mathbb{T}$, and the graininess $\mu : \mathbb{T} \to \mathbb{R}^+ = [0, \infty)$ are defined, respectively, by*

$$\sigma(t) := \inf\{s \in \mathbb{T} : s > t\},\ \rho(t) := \sup\{s \in \mathbb{T} : s < t\},\ \mu(t) = \sigma(t) - t \ for\, t \in \mathbb{T}.$$

If $\sigma(t) = t$, then t is called right-dense (otherwise: right-scattered), and if $\rho(t) = t$, then t is called left-dense (otherwise: left-scattered).

Definition 2.3 *A function $f : \mathbb{T} \to \mathbb{R}$ is said to be rd-continuous if it is continuous at right-dense points in $\mathbb{T}$ and its left-sides limits exists(finite) at left-dense points in $\mathbb{T}$. The set rd-continuous functions is shown by $C^1_{rd} = C_{rd}(\mathbb{T}) = C_{rd}(\mathbb{T}, \mathbb{R})$.*

Definition 2.4 *For $f : \mathbb{T} \to \mathbb{R}$ and $t \in \mathbb{R}$, we define $f^{\Delta}(t)$, the delta-derivative of f at t, to be the number(provided it exists) with the property that, given any $\varepsilon > 0$, there is a neighborhood U of t in $\mathbb{T}$ such that*

$$|[f(\sigma(t)) - f(s)] - f^{\Delta}(t)[\sigma(t) - s]| \le \varepsilon|\sigma(t) - s| \ for \ all \ s \in U.$$

Thus f is said to be delta-differentiable if its delta-derivative exists. The set of functions $f : \mathbb{T} \to \mathbb{R}$ that are delta-differentiable and whose delta-derivative are rd-continuous functions is denoted by $C_{rd} = C^1_{rd}(\mathbb{T}) = C^1_{rd}(\mathbb{T}, \mathbb{R})$.

Definition 2.5 *A function $F : \mathbb{T} \to \mathbb{R}$ is called a delta-antiderivative of $f : \mathbb{T} \to \mathbb{R}$ provided $F^{\Delta}(t) = f(t)$, for all $t \in \mathbb{T}$. Then we write $\int_r^s f(t)\Delta t := F(s) - F(r)$ for all $s, t \in \mathbb{T}$.*

For the usual time scales $\mathbb{T} = \mathbb{R}$, rd-continuous coincides with the usual continuity in calculus. Moreover, every rd-continuous function on $\mathbb{T}$ has a delta-antiderivative[11]. For more information about the above definitions and their related concepts, one can see Refs.[1,3,4,8,9].

3 Existence of Periodic Solutions

For convenience and simplicity in the following discussion, we always use the notations below throughout the paper. The time scale $\mathbb{T}$ is assumed to be ω-periodic, i.e., $t \in \mathbb{T}$ implies $t + \omega \in \mathbb{T}$, $\kappa = \min\{\mathbb{R}^+ \cap \mathbb{T}\}$, $I_\omega = [\kappa, \kappa + \omega] \cap \mathbb{T}$, $g^l = \inf_{t\in\mathbb{T}} g(t)$, $g^u = \max_{t\in\mathbb{T}} g(t)$, $\bar{g} = \frac{1}{\omega}\int_{I_\omega} g(s)\Delta s = \frac{1}{\omega}\int_\kappa^{\kappa+\omega} g(s)\Delta s$, where $g \in C_{rd}(\mathbb{T})$ is an ω-periodic real function, i.e., $g(t + \omega) = g(t)$ for all $t \in \mathbb{T}$.

In order to explore the existence of positive periodic solutions of (3) and for the reader's convenience, we shall first summarize below a few concepts and results without proof, borrowing from Ref.[7].

Let X, Y be normed vector spaces, $L : \mathrm{Dom}L \subset X \to Y$ is a linear mapping, $N : X \to Y$ is a continuous mapping. The mapping L will be called a Fredholm mapping of index zero if $\mathrm{dimKer}L = \mathrm{codimIm}L < +\infty$ and $\mathrm{Im}L$ is closed in Y. If L is a Fredholm mapping of index zero and there exist continuous projectors $P : X \to X$ and $Q : Y \to Y$ such that $\mathrm{Im}P = \mathrm{Ker}L, \mathrm{Im}L = \mathrm{Ker}Q = \mathrm{Im}(I - Q)$, It follows that $L \mid \mathrm{Dom}L \cap \mathrm{Ker}P : (I - P)X \to \mathrm{Im}L$ is invertible. We denote the inverse of that map by K_P. If Ω is an open bounded subset of X, the mapping N will be called $L-$compact on $\bar{\Omega}$ if $QN(\bar{\Omega})$ is bounded and $K_P(I - Q)N : \bar{\Omega} \to X$ is compact. Since $\mathrm{Im}Q$ is isomorphic to $\mathrm{Ker}L$, there exist isomorphisms $J : \mathrm{Im}Q \to \mathrm{Ker}L$.

Lemma 3.1 ([7]Continuation Theorem) *Let L be a Fredholm mapping of index zero and let N be $L-$compact on $\bar{\Omega}$. Suppose*
(a) For each $\lambda \in (0, 1)$, every solution x of $Lx = \lambda Nx$ is such that $x \notin \partial\Omega$;
(b) $QNx \neq 0$ for each $x \in KerL \bigcap \partial\Omega$, and $\deg\{JQN, \Omega \bigcap \partial KerL, 0\} \neq 0$;
Then the equation $Lx = Nx$ has at least one solution lying in $DomL \bigcap \bar{\Omega}$.

Lemma 3.2 [3] *Let $t_1, t_2 \in I_\omega$ and $t \in \mathbb{T}$. If $g : \mathbb{T} \to \mathbb{R}$ is ω-periodic, then*

$$g(t) \leq g(t_1) + \int_\kappa^{\kappa+\omega} |g^\Delta(s)|\Delta s$$

and

$$g(t) \leq g(t_2) - \int_\kappa^{\kappa+\omega} |g^\Delta(s)|\Delta s.$$

Lemma 3.3 *The following equation:*

$$\begin{cases} \bar{a} - \overline{\left(\frac{a}{K}\right)} \exp{(x_1)} = 0, \\ -\bar{D} + \frac{1}{\omega}\int_{\kappa}^{\kappa+\omega} \frac{m(t)\exp(x_1)}{b(t)\exp(x_2)+\exp(x_1)+c(t)}\Delta t = 0 \end{cases} \tag{4}$$

has a unique positive solution $(x_1^*, x_2^*)^T$.

The proofs of Lemma 3.3 are easy, so we omitted it here.

Theorem 3.1 *Let* S_1 *be defined by (12). Suppose that*
$(H1)$ $\bar{a} > \overline{\left(\frac{a}{K}\right)} + \overline{\left(\frac{e}{b}\right)}$;
$(H2)$ $\overline{\left(\frac{m}{b}\right)} \exp(S_1) > \bar{D}$;
$(H3)$ $m^l \exp(-S_1) - \bar{D}[\exp(S_1) + c^u] > \bar{D}b^u$
hold, then (3) has at least one ω*-periodic solution.*

Proof. Define

$$X = Z = \{(x_1, x_2)^T \in C(\mathbb{T}, \mathbb{R}^2) | x_i \in C_{rd},\ x_i(t+\omega) = x_i(t), i = 1, 2\},$$

$$||(x_1, x_2)^T|| = \sum_{i=1}^{2} \max_{t \in I_\omega} |x_i(t)|, (x_1, x_2)^T \in X(\text{or } Z).$$

$$\text{Dom}L = \{x = (x_1, x_2)^T \in X | x_i \in C_{rd}, i = 1, 2\}.$$

It is easy to see that X and Z are both Banach spaces if they are endowed with the above norm $||.||$.

For $(x_1, x_2)^T \in X$, we define

$$N \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} a(t)\left[1 - \frac{\exp(x(t))}{K(t)}\right] - \frac{e(t)\exp(x_2(t))}{b(t)\exp(x_2(t))+\exp(x_1(t))+c(t)} \\ -D(t) + \frac{e(t)\exp(x_1(t))}{b(t)\exp(x_2(t))+\exp(x_1(t))+c(t)} \end{bmatrix},$$

$$L \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}(t) = \begin{bmatrix} x_1^\Delta \\ x_2^\Delta \end{bmatrix}(t),\ P \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}(t) = Q \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}(t) = \begin{bmatrix} \frac{1}{\omega}\int_\kappa^{\kappa+\omega} x_1(t)\Delta t \\ \frac{1}{\omega}\int_\kappa^{\kappa+\omega} x_2(t)\Delta t \end{bmatrix},$$

where $(x_1, x_2)^T$. Then

$$\begin{aligned} \text{Ker}L &= \{(x_1, x_2)^T \in X | (x_1(t), x_2(t))^T = (h_1, h_2)^T \in \mathbb{R}^2 \text{ for } t \in \mathbb{T}\}, \\ \text{Im}L &= \{(x_1, x_2)^T \in X | \int_\kappa^{\kappa+\omega} x_1(t)\Delta t = 0,\ \int_\kappa^{\kappa+\omega} x_2(t)\Delta t = 0,\ \text{for } t \in \mathbb{T}\}. \end{aligned}$$

Then dim Ker$L$ = 2 = codim ImL. Since ImL is closed in Z, L is a Fredholm mapping of index zero, it is easy to show that P and Q are continuous projections and ImP = Ker$L$, ImL = Ker$Q$ = Im$(I - Q)$. Clearly, QN and $K_p(I - Q)N$ are continuous. It can be shown that N is L-compact on $\bar{\Omega}$ for every open bounded set, $\Omega \subset X$.

Now we are at the point to search for an appropriate open, bounded subset Ω for the application of the continuation theorem. Corresponding to the operator equation $L(x_1, x_2)^T = \lambda N(x_1, x_2)^T, \lambda \in (0, 1)$, we have

$$\begin{cases} x_1^{\Delta}(t) = \lambda \left[a(t) \left(1 - \frac{\exp(x(t))}{K(t)} \right) - \frac{e(t)\exp(x_2(t))}{b(t)\exp(x_2(t)) + \exp(x_1(t)) + c(t)} \right], \\ x_2^{\Delta}(t) = \lambda \left[-D(t) + \frac{e(t)\exp(x_1(t))}{b(t)\exp(x_2(t)) + \exp(x_1(t)) + c(t)} \right]. \end{cases} \tag{5}$$

Suppose that $x(t) = (x_1(t), x_2(t))^T \in X$ is an arbitrary solution of system (5) for a certain $\lambda \in (0, 1)$, Integrating (5) over the set I_ω, we obtain

$$\begin{cases} \bar{a}\omega = \int_\kappa^{\kappa+\omega} \frac{a(t)\exp(x_1)}{K(t)} \Delta t + \int_\kappa^{\kappa+\omega} \frac{e(t)\exp(x_2(t))}{b(t)\exp(x_2(t)) + \exp(x_1(t)) + c(t)} \Delta t, \\ \bar{D}\omega = \int_\kappa^{\kappa+\omega} \frac{e(t)\exp(x_1(t))}{b(t)\exp(x_2(t)) + \exp(x_1(t)) + c(t)} \Delta t. \end{cases} \tag{6}$$

Since $(x_1, x_2)^T \in X$, there exists $\xi_i, \eta_i \in [\kappa, \kappa + \omega], i = 1, 2, 3$ such that

$$x_i(\xi_i) = \min_{t \in [\kappa, \kappa+\omega]} \{x_i(t)\}, x_i(\eta_i) = \max_{t \in [\kappa, \kappa+\omega]} \{x_i(t)\}.$$

It follows from (6) that

$$\begin{aligned} \int_\kappa^{\kappa+\omega} |x_1^{\Delta}(t)| \Delta t &\leq \lambda \left[\bar{a}\omega + \int_\kappa^{\kappa+\omega} \frac{a(t)\exp(x_1)}{K(t)} \Delta t \right. \\ &\quad \left. + \int_\kappa^{\kappa+\omega} \frac{e(t)\exp(x_2(t))}{b(t)\exp(x_2(t)) + \exp(x_1(t)) + c(t)} \Delta t \right] \\ &< 2\bar{a}\omega. \end{aligned} \tag{7}$$

$$\begin{aligned} \int_\kappa^{\kappa+\omega} |x_2^{\Delta}(t)| \Delta t &\leq \lambda \left[\bar{D}\omega + \int_\kappa^{\kappa+\omega} \frac{e(t)\exp(x_1(t))}{b(t)\exp(x_2(t)) + \exp(x_1(t)) + c(t)} \Delta t \right] \\ &< 2\bar{D}\omega. \end{aligned} \tag{8}$$

From the first equation of (6), it follows that

$$\bar{a}\omega > \overline{\left(\frac{a}{K}\right)} \omega \exp(x_1(\xi_1)), \bar{a}\omega < \overline{\left(\frac{a}{K}\right)} \omega \exp(x_1(\eta_1)) + \overline{\left(\frac{e}{b}\right)} \omega.$$

Then

$$x_1(\xi_1) < \ln \frac{\bar{a}}{\overline{\left(\frac{a}{K}\right)}} := m_1, x_1(\eta_1) > \ln \frac{\bar{a} - \overline{\left(\frac{e}{b}\right)}}{\overline{\left(\frac{a}{K}\right)}} := M_1. \tag{9}$$

From (9), using the Lemma 3.2, we get

$$x_1(t) \leq x_1(\xi_1) + \int_\kappa^{\kappa+\omega} |x_1^{\Delta}(t)| \Delta t \leq m_1 + 2\bar{a}\omega =: A_1 \tag{10}$$

and

$$x_1(t) \geq x_1(\eta_1) - \int_\kappa^{\kappa+\omega} |x_1^{\Delta}(t)| \Delta t \geq M_1 - 2\bar{D}\omega =: A_2. \tag{11}$$

Thus
$$\max_{t\in I_\omega}|x_1(t)| \le \max\{|A_1|,|A_2|\} := S_1. \tag{12}$$
From the second equation of (6), it follows that
$$\bar{D}\omega \; < \; \int_\kappa^{\kappa+\omega} \frac{m(t)\exp(x_1)}{b(t)\exp(x_2)}\Delta t < \overline{\left(\frac{m}{b}\right)}\omega\exp(S_1)\frac{1}{\exp(x_2(\xi_2))}, \tag{13}$$
$$\begin{aligned} \bar{D}\omega \; &> \; \int_\kappa^{\kappa+\omega} \frac{m(t)\exp(-S_1)}{b(t)\exp(x_2(\eta_2)) + \exp(S_1) + c(t)}\Delta t \\ &\ge \; \frac{m^l\exp(-S_1)}{b^u\exp(x_2(\eta_2)) + \exp(S_1) + c^u}\omega. \end{aligned} \tag{14}$$

Then
$$x_2(\xi_2) \; < \; \ln\frac{\overline{\left(\frac{m}{b}\right)}\exp(S_1)}{\bar{D}} := m_2, \tag{15}$$
$$x_2(\eta_2) \; > \; \ln\frac{m^l\exp(-S_1) - \bar{D}[\exp(S_1) + c^u]}{\bar{D}b^u} := M_2. \tag{16}$$

From (15),(16) and using the Lemma 3.2, we obtain
$$x_2(t) \le x_2(\xi_2) + \int_\kappa^{\kappa+\omega}|x_2^\Delta(t)|\Delta t \le m_2 + 2\bar{a}\omega =: A_3 \tag{17}$$
and
$$x_2(t) \ge x_2(\eta_2) - \int_\kappa^{\kappa+\omega}|x_2^\Delta(t)|\Delta t \ge M_2 - 2\bar{D}\omega =: A_4. \tag{18}$$
It follows from (17) and (18) that
$$\max_{t\in I_\omega}|x_2(t)| \le \max\{|A_3|,|A_4|\} := S_2. \tag{19}$$

Obviously, S_i $(i = 1, 2)$ are independent of the choice of $\lambda \in (0, 1)$. Take $M = S_1 + S_2 + S_0$, where S_0 is taken sufficiently large such that $S_0 \ge |m_1| + |M_1| + |m_2| + |M_2|$.

Next let us consider the algebraic equations
$$\begin{cases} \bar{a} - \overline{\left(\frac{a}{K}\right)}\exp(x_1) - \frac{1}{\omega}\int_\kappa^{\kappa+\omega}\frac{\mu e(t)\exp(x_2(t))}{b(t)\exp(x_2(t))+\exp(x_1(t))+c(t)}\Delta t = 0, \\ -\bar{D} + \frac{1}{\omega}\int_\kappa^{\kappa+\omega}\frac{m(t)\exp(x_1)}{b(t)\exp(x_2)+\exp(x_1)+c(t)}\Delta t = 0 \end{cases} \tag{20}$$

for $(x, y) \in \mathbb{R}^2$,where $\mu \in [0, 1]$ is a parameter. By carrying out similar arguments as above, it is not difficult to show that any solution (x_1^*, x_2^*) of (20) with $\mu \in [0, 1]$ satisfies
$$m_1 < x_1^* < M_1, m_2 < x_2^* < M_2.$$

Now we define $\Omega := \{(x_1, x_2)^T \in X : ||x|| < M\}$. It is clear that Ω verifies the requirement (a) of Lemma 3.1. If $(x_1, x_2)^T \in \partial\Omega \bigcap KerL = \partial\Omega \bigcap \mathbb{R}^2$, then $(x_1, x_2)^T$ is a constant vector in $\mathbb{R}^2$ with $||(x_1, x_2)^T|| = |x_1| + |x_2| = M$. Then

$$QN \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \bar{a} - \bar{\left(\frac{a}{K}\right)} \exp(x_1) - \frac{1}{\omega}\int_\kappa^{\kappa+\omega} \frac{e(t)\exp(x_2(t))}{b(t)\exp(x_2(t))+\exp(x_1(t))+c(t)}\Delta t \\ -\bar{D} + \frac{1}{\omega}\int_\kappa^{\kappa+\omega} \frac{m(t)\exp(x_1)}{b(t)\exp(x_2)+\exp(x_1)+c(t)}\Delta t \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Now let us consider homotopy $H_\mu(x_1, x_2) = \mu QNx + (1-\mu)Gx, \mu \in [0, 1], x = (x_1, x_2)^T$, where

$$Gx = \begin{bmatrix} \bar{a} - \overline{\left(\frac{a}{K}\right)} \exp(x_1) \\ -\bar{D} + \frac{1}{\omega}\int_\kappa^{\kappa+\omega} \frac{m(t)\exp(x_1)}{b(t)\exp(x_2)+\exp(x_1)+c(t)}\Delta t \end{bmatrix}.$$

Letting J be the identity mapping, according to Lemma 3.3 and by direct calculation, we get

$$\begin{aligned} & \deg\left\{JQN(x_1, x_2)^T; \partial\Omega \bigcap \mathrm{Ker}L; 0\right\} \\ = \; & \deg\left\{QN(x_1, x_2)^T; \partial\Omega \bigcap \mathrm{Ker}L; 0\right\} \\ = \; & \deg\left\{H_1(x_1, x_2); \partial\Omega \bigcap \mathrm{Ker}L; 0\right\} \\ = \; & \deg\left\{H_0(x_1, x_2); \partial\Omega \bigcap \mathrm{Ker}L; 0\right\} \\ = \; & \mathrm{sign}\left\{ \left[\overline{\left(\frac{a}{K}\right)} \exp(x_1)\right] \left[\frac{1}{\omega}\int_\kappa^{\kappa+\omega} \frac{m(t)b(t)\exp(x_1)\exp(x_2)}{(b(t)\exp(x_2)+\exp(x_1)+c(t))^2}\Delta t\right] \right\} \\ = \; & 1 \neq 0, \end{aligned}$$

where $\deg(.,.,.)$ is the Brower degree. Thus we have proved that Ω verifies all requirements of Lemma 3.1, then it follows that $Lx = Nx$ has at least one solution in $\mathrm{Dom}L \cap \overline{\Omega}$. The proof is complete.

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