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On the Solution of Rough Goal Bi-Level Multi-Objective Linear Programming Problem

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Abstract

This paper proposes a bi-level linear programming problem with linear constraints, in which the linear objective functions are to be maximized with different rough goals, the suggested approach in this paper is mainly based on the iterative goal programming method of Dauer and Krueger to develop the optimal solution of the bi-level decision- maker, then we uses the concepts of tolerance membership function technique to generate the optimal solution for this problem. An auxiliary problem is discussed as well as an example is presented.

Keywords: *Bi –level programming, rough programming, goal programming.*

1 Introduction

Rough set theory has been proposed by Pawlak in 1982, the aim is to maximize or minimize an objective function over certain set of feasible solutions. But in many practical situations, the decision maker may not be in a position to specify the objective and/or the feasible set precisely but rather can specify them in a rough sense ([12], [13]).

Rough set models based on incomplete systems [23], covering rough sets, rough fuzzy sets and fuzzy rough sets ([9], [10]), Rough set theory has been proven to be an excellent mathematical tool dealing with vague description of objects ([3], [7], [8], [16], [20][25]).

Bi-level programming is a powerful and robust technique for solving hierarchical decision making problem. It has been applied in many real life problems such as agriculture, bio-fuel production, economic systems, finance, engineering, banking, management sciences, and transportation problem ([1], [15], [17], [18], [21]).

Goal programming is one powerful tool that has been proposed for the modeling, analysis and solution of multi-objective optimization problems [6]. Dauer and Krueger in [4] suggested an iterative goal programming approach for solving multi objective nonlinear programming problems [2].

In [5] Emam proposed a bi-level integer non-linear programming problem with linear or non-linear constraints, and in which the non-linear objective function at each level are to maximized. It proposed a two planner integer model and a solution method for solving this problem. In [19] Saraj and Safaei used the global criterion method, to solve the bi-level programming by an interval approach on using Karush-Kuhn-Tucker (KKT) conditions, and global criterion method converts to a single objective.

Since Pawlak proposed the concept of the rough set, it has rapidly developed and been applied in many fields. Pawlak and Slowinski applied the rough set approach to multi-attribute decision problems [14]. Xu and Yao discussed a class of linear multi-objective programming problems with random rough coefficients and gave a crisp equivalent model [22]. Youness applied the rough set to the classification of the feasible area in mathematical programming and called it Rough programming [24].

2 Problem Formulation and Solution Concept

Let $x_i \in R^{n_i}, (i = 1,2)$ be a vector variables indicating the first decision level's choice and the second decision level's choice, $n_i \geq 1, (i = 1,2)$.

Let $F_i: R^{n_i} \rightarrow R^{N_i}, (i = 1,2)$ be the first level objective functions, and the second level objective functions, respectively. Let the first level decisions maker and second level decisions maker have N_1 and N_2 objective function, respectively.

Therefore, the bi-level linear programming problem contains rough parameters may be stated as follows:

First Level Decision Maker

$$\max_{x_1} F_1(x,\xi) = \max_{x_1}\left(f_{11},\ldots,f_{1N_1}\right), \tag{1}$$

Where x_2 solves

Second Level Decision Maker

$$\max_{x_2} F_2(x,\xi) = \max_{x_2}\left(f_{21},\ldots,f_{2N_2}\right), \tag{2}$$

Subject to

$$x \in G, \tag{3}$$

$$G = \{(x_1,x_2)|g_i(x_1,x_2) \le y, i = 1,2,\ldots m,$$

$$x_1,x_2 \ge 0\}.$$

Where

$$f_{kj} = \sum_{j=1}^{n} \xi_{kj}x_j \quad , i = 1,2,\ldots,n_k.$$

G is the bi-level linear constraint set. F_1 and F_2 are linear functions contains rough parameters with definite goals.

Now, going back to the bi-level multi-objective linear programming problem contains rough parameters. We can write an associated goal programming for this problem with $(N_1 + N_2)$ goals as follows:

[First Level Decision Maker]

Achieve $f_{11}(x,\xi) = k_{11}$,

Achieve $f_{12}(x,\xi) = k_{12}$, (4)

.

.

.

Achieve $f_{1N_1}(x,\xi) = k_{1N_1}$,

Where x_2 solves

[Second Level Decision Maker]

Achieve $f_{21}(x,\xi) = k_{21}$,

Achieve $f_{22}(x, \xi) = k_{22}$, (5)

.

.

.

Achieve $f_{2N_2}(x, \xi) = k_{2N_2}$,

Subject to

$$x \in G. \tag{6}$$

Where k_{1N_1}, k_{2N_2} are scalars and represent the aspiration levels associated with the objectives of the First level decision maker and Second level decision maker, respectively.

3 The Transformation of Random Rough Coefficient [22]

To convert the bi-level multi-objective linear programming problem with random rough coefficient in the objective functions into the respective crisp equivalents for solving a trust probability constrains, this process is usually hard work for many cases but the transformation process is introduced in the following theorem.

Theorem 1: *Assume that random rough variable* $\tilde{\bar{c}}_{ij}$ *is characterized by* $\tilde{\bar{c}}_{ij}(\lambda) \sim \mathcal{N}(c_{ij}(\lambda), V_i^c)$, *where* $c_{ij}(\lambda)\left(\left(c_{ij}(\lambda)\right)_{nx1} = \left(c_{i1}(\lambda), c_{i2}(\lambda), \dots, c_{in}(\lambda)\right)^T\right)$ *is a rough variable and* V_i^c *is a positive definite covariance matrix. It follows that* $c_i(\lambda)^T x = ([a, b], [c, d])$ *(where* $c \le a \le b \le d$*) is a rough variable and characterized by the following trust measure function:*

$$\mathrm{Tr}\{c_i(\lambda)^T x \ge t\} = \begin{cases} 0 & \text{if } d \le t, \\ \dfrac{d-t}{2(d-c)} & \text{if } b \le t \le d, \\ \dfrac{1}{2}\left(\dfrac{d-t}{d-c} + \dfrac{b-t}{b-a}\right) & \text{if } a \le t \le b, \\ \dfrac{1}{2}\left(\dfrac{d-t}{d-c} + 1\right) & \text{if } c \le t \le a, \\ 1 & \text{if } t \le c. \end{cases}$$

Then, we have $\mathrm{Tr}\{\lambda | \Pr\{c_i(\lambda)^T x \ge f_i(x)\} \ge \delta_i\} \ge \gamma_i$ if and only if

$$\begin{cases} b + R \le f_i \le d - 2\gamma_i(d - c) + R & \text{if } b \le M \le d, \\ a + R \le f_i \le \dfrac{d(b-a) + b(d-c) - 2\gamma_i(d-c)(b-a)}{d - c + b - a} + R & \text{if } a \le M \le b, \\ c + R \le f_i \le d - (d - c)\left(2\gamma_i - 1\right) + R & \text{if } c \le M \le a, \\ \quad f_i \le c + R & \text{if } M \le C. \end{cases}$$

Where $M = f_i - \emptyset^{-1}(1 - \delta_i)\sqrt{x^T V_i^c x}$ and $R = \emptyset^{-1}(1 - \delta_i)\sqrt{x^T V_i^c x}$, and Φ is the standardized normal distribution and $\delta_i, \gamma_i \in [0,1]$ are predetermined confidence levels.

To proof theorem 1 above, the reader is referred to [22].

3.1 The Equivalent Crisp Problem of Bi-Level Rough Linear Problem

The equivalent bi-level multi-objective linear programming problem equivalent to the bi-level multi-objective linear programming problem contains rough parameters with definite goals in objective functions may be stated as follows:

[First Level Decision Maker]

$$\max_{x_1} h_1(x) = \max_{x_1} \left(h_{11}, \dots, h_{1N_1}\right),$$

[Second Level Decision Maker]

$$\max_{x_2} h_2(x) = \max_{x_2} \left(h_{21}, \dots, h_{2N_2}\right),$$

Subject to

$$x \in G,$$

$$G = \{(x_1, x_2) | g_i(x_1, x_2) \le y, i = 1,2, \dots m,$$

$$x_1, x_2 \ge 0\}.$$

Where h_1, h_2 are the objective functions of the first level decision maker (FLDM), and second level decision maker (SLDM).

Definition 1: *For any $x_1 (x_1 \in G_1 = \{x_1 | (x_1, x_2) \in G\})$ achieves the first level decision maker goals with under attainment or over attainment, if the decision-making variable $x_2 (x_2 \in G_2 = \{x_2 | (x_1, x_2) \in G_1\})$ achieves is the second level decision maker goals with under attainment or over attainment, then (x_1, x_2) is a feasible solution of the rough goal bi-Level multi-objective linear programming problem.*

Definition 2: *If* (x_1^*, x_2^*) *is a feasible solution of the rough goal bi-Level multi-objective linear programming problem, such that the first level decision maker achieves all goals; so* (x_1^*, x_2^*) *is the Pareto optimal solution of the rough goal bi-Level multi-objective linear programming problem.*

4 A Goal Approach for the Bi-Level Multi-Objective Linear Programming Problem

To solve the bi-level multi-objective linear programming problem with definite goals, one first get the optimal solution of the first level decision maker with definite goals, and the second level decision maker should get his optimal solution with definite goals, as follows:

4.1 The First Level Decision Maker

First, the first level decision maker solves the following problem:

$$\text{Achieve } (h_{11}(x), \dots, h_{1N_1}(x)) = (k_{11}, \dots\ k_{1N_1}), \tag{7}$$

Subject to

$$x \in G.$$

Where $k_{11}, \dots, k_{1N_1}$ are scalars, and represent the aspiration levels associated with the objectives, $h_{11}, \dots, h_{1N_1}$, respectively.

We consider the following bi-level multi-objective linear programming problem associated to the first goal as:

$$\mathrm{P}_{11}: \quad \mathrm{Minimize} \mathrm{D}_{11} = \mathrm{d}_{11}^- + \mathrm{d}_{11}^+, \tag{8}$$

Subject to

$$h_{11}(x) + d_{11}^- - d_{11}^+ = \mathrm{k}_{11},$$

$$x \in G,$$

$$d_{11}^-, d_{11}^+ \geq 0.$$

Where d_{11}^- and d_{11}^+ are the under attainment and over attainment, respectively, of the first goal and $\mathrm{d}_{11}^- \mathrm{X}\, \mathrm{d}_{11}^+ = 0$.

Then the attainment problem associated with the second goal is equivalent to the optimization problem P_{12} , where:

$$P_{12}: \quad \mathrm{Minimize}\ \mathrm{D}_{12} = \mathrm{d}_{12}^- + \mathrm{d}_{12}^+, \tag{9}$$

Subject to

$$h_{12}(x) + d_{12}^{-} - d_{12}^{+} = k_{12,}$$

$$h_{11}(x) + d_{11}^{-} - d_{11}^{+} = k_{11},$$

$$d_{11}^{-} + d_{11}^{+} = D_{11}^{*},$$

$$x \in G,$$

$$d_{1t}^{-}, d_{1t}^{+} \geq 0.$$

The optimal solution of the first level decision maker $x^{*} = (x_1^F, x_1^F)$.

4.2 The Second Level Decision Maker

Second, in the same way, the second level decision maker independently solves:

$$\text{Achieve}(h_{21}(x), \dots, h_{2N_2}(x)) = (k_{21}, \dots\ k_{2N_2}), \qquad (10)$$

Subject to

$$x \in G.$$

Where $k_{21}, \dots, k_{2N_2}$ are scalars, and represent the aspiration levels associated with the objectives, $h_{21}, \dots, h_{2N_2}$, respectively.

The second level decision maker will do the same action as the first level decision maker till he obtain his optimal solution $x^{*} = (x_1^S, x_1^S)$.

5 Fuzzy Approach of Bi-Level Linear Programming with Rough Parameters Problem

Now the solution of the first level decision maker and second level decision maker are disclosed. However, two solutions are usually different because of nature between two levels goals. The first level decision maker knows that using the optimal decisions x_1^F as a control factors for the second level decision maker are not practical. It is more reasonable to have some tolerance that gives the second level decision maker an extent feasible region to search for his/her optimal solution, and reduce searching time or interactions.

In this way, the range of decision variable x_1 should be around x_1^F with maximum tolerance t_1 and the following membership function specify $x_1$as:

$$\mu(x_1) = \begin{cases} \frac{x_1-(x_1^F-t_1)}{t_1} & x_1^F - t_1 \le x_1 \le x_1^F, \\ \frac{(x_1^F+t_1)-x_1}{t_1} & x_1^F \le x_1 \le x_1^F - t_1. \end{cases} \tag{11}$$

Where X_1^F is the most preferred solution; the $(X_1^F - t_1)$ and $(X_1^F + t_1)$ are the worst acceptable decision; and that satisfaction is linearly increasing with the interval of $\lfloor X_1^F - t_1, X_1 \rfloor$ and linearly decreasing with$\lfloor X_1, X_1^F - t_1 \rfloor$, and other decision are not acceptable.

First, the first level decision maker goals may reasonably consider $h_1 \ge h_1^F$ is absolutely acceptable and $h_1 < h_1' = h_1(x_1^S, x_2^S)$ is absolutely unacceptable, and that the preference with $[h_1', h_1^F]$ is linearly increasing. This due to the fact that the second level decision maker obtained the optimum at(x_1^S, x_2^S), which in turn provides the first level decision maker the objective function values h_1', makes any $h_1 \ge h_1' = h_1(x_1^S, x_2^S)$ unattractive in practice.

The following membership functions of the first level decision maker can be stated as:

$$\mu[h_1(x)] = \begin{cases} 1 & \text{if } h_1(x) > h_1^F, \\ \frac{h_1(x)-h_1'}{h_1^F-h_1'} & \text{if } h_1' \le h_1(x) \le h_1^F, \\ 0 & \text{if } h_1' \ge h_1(x). \end{cases} \tag{12}$$

Second, the second level decision maker goals may reasonably consider the $h_2 \ge h_2^S$ is absolutely acceptable and $h_2 < h_2' = h_2(x_1^F, x_2^F)$ is absolutely unacceptable, and that the preference with $[h_2', h_2^S]$ is linearly increasing. In this way, the second level decision maker has the following membership functions for his/her goal:

$$\mu[h_2(x)] = \begin{cases} 1 & \text{if } h_2(x) > h_2^S, \\ \frac{h_2(x)-h_2'}{h_2^S-h_2'} & \text{if } h_2' \le h_2(x) \le h_2^S, \\ 0 & \text{if } h_2' \ge h_2(x). \end{cases} \tag{13}$$

Finally, in order to generate the satisfactory solution, which is also a Pareto optimal solution with overall satisfaction for all decision-makers, we can solve the following Tchebycheff problem.

$$max\ \delta, \tag{14}$$

Subject to

$$\frac{(x_1^F + t_1) - x_1}{t_1} \ge \delta,$$

$$\frac{x_1 - (x_1^{\mathrm{F}} - t_1)}{t_1} \geq \delta,$$

$$\grave{\mu}[\mathrm{h}_1(\mathrm{x})] \geq \delta,$$

$$\grave{\mu}[\mathrm{h}_2(\mathrm{x})] \geq \delta,$$

$$(x_1, x_2) \in G,$$

$$t_1 > 0\ , \delta \in [0,1].$$

Where δ is the over all satisfaction.

If the first level decision maker is satisfied with solution then satisfactory solution is reached. Otherwise, he/she should provide new membership function for the control variable and objectives to the second level decision maker, until a satisfactory solution is reached.

6 Numerical Example

To demonstrate the solution method for bi-level multi-objective linear programming problem under random rough coefficient in objective functions can be written as:

[1st Level]

$$\max_{x_1} f_{11}(x) \;=\; x_1 + x_2 + x_3,$$

$$\max_{x_1} f_{12}(x) \;=\; c_1 x_1 + c_2 x_2 + c_3 x_3,$$

Where x_2 solves

[2nd Level]

$$\max_{x_2} f_{21}(x) \;=\; x_1 + x_2 + x_3,$$

$$\max_{x_2} f_{22}(x) \;=\; c_1 x_1 + c_2 x_2 + c_3 x_3,$$

Subject to

$$\mathrm{Tr}\{\lambda |\, \Pr\{\xi_1 x_1 + \xi_2 x_2 + \xi_3 x_3 \geq h_1\} \geq \delta_1\} = k_{11}$$

$$\mathrm{Tr}\{\lambda |\, \Pr\{c_1 \xi_4 x_1 + c_2 \xi_5 x_2 + c_3 \xi_6 x_3 \geq h_2\} \geq \delta_2\} = k_{12}$$

$$\mathrm{Tr}\{\lambda |\, \Pr\{\xi_7 x_1 + \xi_8 x_2 + \xi_9 x_3 \geq h_3\} \geq \delta_1\} = k_{21}$$

$$\text{Tr}\{\lambda|\ \Pr\{c_1\xi_{10}x_1 + c_2\xi_{11}x_2 + c_3\xi_{12}x_3 \geq h_4\} \geq \delta_2\} = k_{22}$$

$$x_1 + x_2 + x_3 \leq 1000,$$

$$2x_1 + x_2 + x_3 \leq 2000,$$

$$4x_1 + 2x_2 + x_3 \leq 9000,$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$$

Where $c = (c_1, c_2, c_3) = (1.5, 0.5, 1.5)$, and assume that the rough parameters are defines as:

$\xi_1 \sim \mathcal{N}(\rho_1, 1)$, with $\rho_1 = ([2,3],[1,4])$, $\xi_2 \sim \mathcal{N}(\rho_2, 4)$, with $\rho_2 = ([1,2],[0,3])$,

$\xi_3 \sim \mathcal{N}(\rho_3, 1)$, with $\rho_3 = ([4,5],[3,6])$, $\xi_4 \sim \mathcal{N}(\rho_4, 2)$, with $\rho_4 = ([3,4],[2,5])$,

$\xi_5 \sim \mathcal{N}(\rho_5, 1)$, with $\rho_5 = ([2,3],[0,3])$, $\xi_6 \sim \mathcal{N}(\rho_6, 4)$, with $\rho_6 = ([1,2],[0,3])$,

$\xi_7 \sim \mathcal{N}(\rho_7, 4)$, with $\rho_7 = ([1,2],[0,3])$, $\xi_8 \sim \mathcal{N}(\rho_8, 1)$, with $\rho_8 = ([2,3],[0,3])$,

$\xi_9 \sim \mathcal{N}(\rho_9, 2)$, with $\rho_9 = ([2,3],[1,4])$, $\xi_{10} \sim \mathcal{N}(\rho_{10}, 1)$, with $\rho_{10} = ([0,1],[0,2])$,

$\xi_{11} \sim \mathcal{N}(\rho_{11}, 1)$, with $\rho_{11} = ([3,4],[2,5])$, $\xi_{12} \sim \mathcal{N}(\rho_{12}, 1)$, with $\rho_{12} = ([0,1],[0,3])$,

Let $\delta_i = \gamma_i = 0.4$, then $\Phi^{-1}(1 - \delta_i) = 0.26$.

Now by using theorem 1, the equivalent crisp problem which equivalent to bi-Level multi-objective linear programming problem under rough parameters in objective functions with definite goals, as follows:-

[FLDM]

Achieve $h_{11} = \left(1.6x_1 + 0.6x_2 + 3.6x_3 + 0.26\sqrt{x_1^2 + 4x_2^2 + x_3^2}\right) = k_{11}$,
Achieve $h_{12} = \left(3.9x_1 + 0.3x_2 + 0.9\,x_3 + 0.26\sqrt{2x_1^2 + x_2^2 + 4x_3^2}\right) = k_{12}$,

Where x_2 solves

[SLDM]

Achieve $h_{21} = \left(0.6x_1 + 0.6x_2 + 1.6x_3 + 0.26\sqrt{4x_1^2 + x_2^2 + 2x_3^2}\right) = k_{21}$,
Achieve$h_{22} = \left(0.6x_1 + 1.3\,x_2 + 0.9\,x_3 + 0.26\sqrt{x_1^2 + x_2^2 + x_3^2}\right) = k_{22}$.

Subject to

$$x \in G = \{\ x_1 + \ x_2 + \ x_3 \le 1000,\ 2x_1 + \ x_2 + \ x_3 \le 2000,\ 4x_1 + \ 2x_2 + x_3 \le 9000,\ x_1 \ge 0, x_2 \ge 0, x_3 \ge 0\}.$$

Then, calculating trust for every rough coefficients using trust measure function in theorem 1:

$\mathrm{Tr}\ \{\xi_1\} = 0.9, \mathrm{Tr}\ \{\xi_2\} = 0.9\ , \mathrm{Tr}\ \{\xi_3\} = 0.9, \mathrm{Tr}\ \{\xi_4\} = 0.9, \mathrm{Tr}\ \{\xi_5\} = 0.9, \mathrm{Tr}\ \{\xi_6\} = 0.9, \mathrm{Tr}\ \{\xi_7\} = 0.9, \mathrm{Tr}\ \{\xi_8\} = 0.9\ , \mathrm{Tr}\ \{\xi_9\} = 0.9, \mathrm{Tr}\ \{\xi_{10}\} = 0.7\ , \mathrm{Tr}\ \{\xi_{11}\} = 0.9, \mathrm{Tr}\ \{\xi_{12}\} = 0.6$

So, with trust more than or equal $\propto$ is 0.6 the equivalent crisp problem which equivalent to bi-Level multi-objective linear programming problem under rough parameters in objective functions.

Now, we can write an associated goal programming for this problem with (N_1, N_2) goals as follows:-

[First Level Decision Maker]

$$\max_{x_1} \mathrm{h}_1(x_1, x_2, x_3) = \max_{x_1} \left[1.6x_1 + 0.6x_2 + 3.6x_3 + 0.26\sqrt{x_1^2 + 4x_2^2 + x_3^2}, 3.9x_1 + 0.3x_2 + 0.9\ x_3 + 0.26\sqrt{2x_1^2 + x_2^2 + 4x_3^2} \right],$$

Where x_2 solves

[Second Level Decision Maker]

$$\max_{x_2} \mathrm{h}_2(x_1, x_2, x_3) = \max \left[0.6x_1 + 0.6x_2 + 1.6x_3 + 0.26\sqrt{4x_1^2 + x_2^2 + 2x_3^2}, 0.6x_1 + 1.3\ x_2 + 0.9\ x_3 + 0.26\sqrt{x_1^2 + x_2^2 + x_3^2} \right],$$

Subject to

$$x \in G = \{x_1 + \ x_2 + \ x_3 \le 1000,\ 2x_1 + \ x_2 + \ x_3 \le 2000,\ 4x_1 + \ 2x_2 + x_3 \le 9000,\ x_1 \ge 0, x_2 \ge 0, x_3 \ge 0.\}$$

1- First, the first level decision maker solves his/her Problem as following:

Achieve $1.6x_1 + 0.6x_2 + 3.6x_3 + 0.26\sqrt{x_1^2 + 4x_2^2 + x_3^2} = k_{11}$,
Achieve $3.9x_1 + 0.3x_2 + 0.9\, x_3 + 0.26\sqrt{2x_1^2 + x_2^2 + 4x_3^2} = k_{12}$,

Subject to
$x \in G$.

The aspiration levels of the goals are assumed to be $k_{11} = 220$, $k_{12,} = 240$, respectively. Then, the optimization problem associated with the first goal is formulated as follows:

P_{11}: $Minimize\ D_{11} = d_{11}^{-} + d_{11}^{+}$,
Subject to
$$1.6x_1 + 0.6x_2 + 3.6x_3 + 0.26\sqrt{x_1^2 + 4x_2^2 + x_3^2} + d_{11}^{-} - d_{11}^{+} = k_{11},$$
$$x \in G,$$
$$d_{11}^{-}, d_{11}^{+} \geq 0.$$

The maximum degree of attainment of problem P_{11} is $D_{11}^{*} = 0.0001$ with the optimal solution $x_1 = (45.5717,30,30)$ and $d_{11}^{-} = 0, d_{11}^{+} = 0.0001$.

The attainment problem for goal 2 of the first level decision maker is equivalent to problemP_{12}, where:

P_{12}: $Minimize\ D_{12} = d_{12}^{-} + d_{12}^{+}$
Subject to
$$3.9x_1 + 0.3x_2 + 0.9\, x_3 + 0.26\sqrt{2x_1^2 + x_2^2 + 4x_3^2} + d_{12}^{-} - d_{12}^{+} = k_{12,}$$
$$1.6x_1 + 0.6x_2 + 3.6x_3 + 0.26\sqrt{x_1^2 + 4x_2^2 + x_3^2} + d_{11}^{-} - d_{11}^{+} = 220,$$
$$d_{11}^{-} + d_{11}^{+} = 0.0001,$$
$$x \in G,$$
$$d_{1t}^{-}, d_{1t}^{+} \geq 0.$$

Therefore, the optimal solution of the model P_{12} is $x_2 = (45.7385,49.9999,24.3159), d_{11}^{-} = 0, d_{11}^{+} = 0.0001,\ d_{12}^{-} = 0.0027\ , d_{12}^{+} = 0$, so the optimal solution of the bi-level multi-objective linear goal programming model is given by x^{*} which will be the optimal solution of the first level decision maker$x^{*} = (x_1, x_2, x_3) = (45.7385,49.9999,24.3159)$.

2- Second, the second level decision maker solves his/her Problem as following:

Achieve $0.6x_1 + 0.6x_2 + 1.6x_3 + 0.26\sqrt{4x_1^2 + x_2^2 + 2x_3^2} = k_{21}$,
Achieve $0.6x_1 + 1.3\, x_2 + 0.9\, x_3 + 0.26\sqrt{x_1^2 + x_2^2 + x_3^2} = k_{22}$,

Subject to

$x \in G.$

The aspiration levels of the goals are assumed to be $k_{21} = 135, k_{22} = 125$ respectively. Then, the optimization problem associated with the first goal is formulated as follows:

P_{21}: $Minimize\ D_{21} = d_{21}^{-} + d_{21}^{+},$

Subject to

$$0.6x_1 + 0.6x_2 + 1.6x_3 + 0.26\sqrt{4x_1^2 + x_2^2 + 2x_3^2} + d_{21}^{-} - d_{21}^{+} = k_{21},$$
$$x \in G,$$
$$d_{21}^{-}, d_{21}^{+} \geq 0.$$

The maximum degree of attainment problem P_{21} is $D_{21}^{*} = 0$ with the optimal solution $x = (54.9969, 44.9952, 26.6205)$ and $d_{21}^{-} = 0, d_{21}^{+} = 0$.

The attainment problem for goal 2 of the second level decision maker is equivalent to problemP_{21} , where:

P_{22}: $Minimize\ D_{22} = d_{22}^{-} + d_{22}^{+},$

Subject to

$$0.6x_1 + 1.3\, x_2 + 0.9\, x_3 + 0.26\sqrt{x_1^2 + x_2^2 + x_3^2} + d_{22}^{-} - d_{22}^{+} = k_{22},$$
$$0.6x_1 + 0.6x_2 + 1.6x_3 + 0.26\sqrt{4x_1^2 + x_2^2 + 2x_3^2} + d_{21}^{-} - d_{21}^{+} = 135,$$
$$d_{21}^{-} + d_{21}^{+} = 0,$$
$$x \in G,$$
$$d_{2t}^{-}, d_{2t}^{+} \geq 0.$$

Therefore, the optimal solution of the model P_{22} is $x = (50.3748, 36.0144, 33.0098)$, $d_{21}^{-} = 0, d_{21}^{+} = 0, d_{22}^{-} = 0.0026, d_{22}^{+} = 0$, so the optimal solution of the bi-level multi-objective linear goal programming model is given byx^{*}which will be the optimal solution of the second level decision maker $x^{*} = (50.3748, 36.0144, 33.0098)$.

3- Finally, we assume the first level decision maker control decision $x_1^F = 45.7385$ with the tolerance 5, the second level decision maker solves the following Tchebycheff problem as follows:

$Max\ \delta,$

Subject to

$x \in G,$

$-x_1 - 5\,\delta \geq -50.7385,$

$x_1 - 5\,\delta \geq 40.7385,$

$\left(1.6x_1 + 0.6x_2 + 3.6x_3 + 0.26\sqrt{x_1^2 + 4x_2^2 + x_3^2}\right) + 25.45518\,\delta \geq 245.45512,$

$$\left(3.9x_1 + 0.3x_2 + 0.9\,x_3 + 0.26\sqrt{2x_1^2 + x_2^2 + 4x_3^2}\right) + 23.91089556\,\delta \geq 263.9082316,$$

$$\left(0.6x_1 + 0.6x_2 + 1.6x_3 + 0.26\sqrt{4x_1^2 + x_2^2 + 2x_3^2}\right) - 10.1100363\,\delta \geq 124.889988,$$

$$\left(0.6x_1 + 1.3\,x_2 + 0.9\,x_3 + 0.26\sqrt{x_1^2 + x_2^2 + x_3^2}\right) + 8.0484499\,\delta \geq 133.0459499,\ \delta \in [0,1].$$

Whose, optimal solution is: $(x_1, x_2, x_3) = (45.9016, 49.6429, 30.000), \delta = 0.9622$, $h_1 = (240.71811, 247.26846), and\ h_2 = (134.61826, 138.3086)$

Overall satisfaction for both decisions makers.

7 Summary and Concluding Remarks:

This paper proposed a bi- level linear programming problem with linear constraints, in which the linear objective functions are to be maximized with different rough goals, the suggested approach in this paper was mainly based on the iterative goal programming method of Dauer and Krueger to develop the optimal solution of the bi-level decision- maker, then we used the concepts of tolerance membership function technique to generate the optimal solution for this problem.

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