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On Totally sg-Continuity, Strongly sg-Continuity and Contra sg-Continuity

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Abstract

In this paper, sg-closed sets and sg-open sets are used to define and investigate a new class of functions. Relationships between this new class and other classes of functions are established.

Keywords: *Topological spaces, sg-closed set, sg-open set, totally sg-continuity, strongly sg-continuity, contra sg-continuity.*

1 Introduction

Jain [9], Levine [12] and Dontchev [5] introduced totally continuous functions, strongly continuous functions and contra continuous functions, respectively.

Levine [10] also introduced and studied the concepts of generalized closed sets. The notion has been studied extensively in recent years by many topologists. As generalization of closed sets, sg-closed sets were introduced and studied by Bhattacharya and Lahiri [2]. This notion was further studied by Navalagi [14, 15]. In this paper, we will continue the study of some related functions by using sg-open sets and sg-closed sets. We introduce and characterize the concepts of totally sg-continuous, strongly sg-continuous and contra sg-continuous functions.

2 Preliminaries

Throughout this paper (X, τ), (Y, σ) and (Z, η). (or X, Y and Z) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), cl(A), int(A) and A^c denote the closure of A, the interior of A and the complement of A in X, respectively. We set $C(X, x) = \{V \in C(X) \mid x \in V\}$ for $x \in X$, where C(X) denotes the collection of all closed subsets of (X, τ). The set of all clopen subsets of (X, τ) is denoted by CO(X, τ).

We recall the following definitions, which are useful in the sequel.

Definition 2.1 *A subset A of a space (X, τ) is called:*

(i) semi-open [11] if A $\subseteq$ cl(int(A)).
(ii) α-open [16] if A $\subseteq$ int(cl(int(A))).

The complements of the above mentioned open sets are called their respective closed sets.
The intersection of all semi-closed sets of X containing a subset A is called the semi-closure of A and is denoted by scl(A).

Defintion 2.2 *A subset A of a space (X, τ) is called:*

(i) a $\hat{g}$-closed set [23] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi-open in (X, τ). The complement of $\hat{g}$-closed set is called $\hat{g}$-open.
*(ii) a *g-closed set [22] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $\hat{g}$-open in (X, τ). The complement of *g-closed set is called *g-open.*
*(iii) a $^{\#}$g-semi-closed(briefly $^{\#}$gs-closed) set [24] if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is *g-open in (X, τ). The complement of $^{\#}$gs-closed set is called $^{\#}$gs-open.*
(iv) a $\tilde{g}$-semi-closed (briefly $\tilde{g}$s-closed) set [20] if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $^{\#}$gs-open in (X, τ). The complement of $\tilde{g}$s-closed set is called $\tilde{g}$s-open

(v) *a generalized semi-closed (briefly gs-closed) set [1] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ). The complement of gs-closed set is called gs-open*

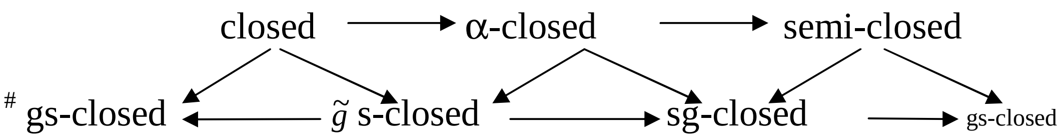
(vi) *a semi-generalized closed (briefly sg-closed) set [2] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X, τ). The complement of sg-closed set is called sg-open. The class of all sg-open sets of (X, τ) is denoted by $SG(X, \tau)$.*

(vii) *a sg-clopen if it is both sg-open and sg-closed.*

We set $SG(X, x) = \{V \in SG(X, \tau) \mid x \in V\}$ for $x \in X$.

Remark 2.1

From the Definitions 2.1 and 2.2, we have the following implications.



None of the above implications is reversible as the following example shows

Example 2.1

(i) Let $X = \{a, b, c\}$, $\tau = \{\phi, \{a\}, X\}$. The set $\{b\}$ is α-closed, #gs- closed and $\tilde{g}$ s-closed but not closed.

(ii) Let $X = \{a, b, c\}$, $\tau = \{\phi, \{a, b\}, X\}$. The set $\{a, c\}$ is $\tilde{g}$ s-closed but not α-closed.

(iii) Let $X = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{b, c\}, X\}$. The set $\{a, b\}$ is sg-closed, #gs-closed but not $\tilde{g}$ s-closed.

(iv) Let $X = \{a, b, c\}$, $\tau = \{\phi, \{a, b\}, X\}$. The set $\{b, c\}$ is sg-closed but not α-closed.

(v) Let $X = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$. The set $\{a\}$ is semi-closed but not α-closed.

(vi) Let $X = \{a, b, c\}$, $\tau = \{\phi, \{a, b\}, X\}$. The set $\{b, c\}$ is sg-closed, gs-closed but not semi-closed.

(vii) Let $X = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{a, c\}, X\}$. The set $\{a, b\}$ is gs-closed but not sg-closed.

Definition 2.3 *A function $f : (X, \tau) \to (Y, \sigma)$ is called:*

(i) *totally continuous [9] if the inverse image of every open subset of (Y, σ) is a clopen subset of (X, τ).*

(ii) *strongly continuous [12] if the inverse image of every subset of (Y, σ) is a clopen subset of (X, τ).*

(iii) *contra-continuous [5] (resp. contra-semi-continuous [6], contra-α-continuous [7]) if the inverse image of every open subset of (Y, σ) is a closed (resp. semi-closed, α-closed) subset of (X, τ).*

(iv) *sg-continuous [21] if the inverse image of every open subset of (Y, σ) is a sg-open subset of (X, τ).*

Definition 2.4 *A function f : (X, τ) $\rightarrow$ (Y, σ) is called:*

(i) *sg-irresolute [21] if the inverse image of every sg-closed set of (Y, σ) is a sg-closed of (X, τ).*

(ii) *sg-open [4] if for each open set U of (X, τ), f(U) is sg-open set of (Y, σ).*

Definition 2.5 [14] *Let (X, τ) be a topological space and A $\subseteq$ X. We define the sg-closure of A (briefly sg-cl(A)) to be the intersection of all sg-closed sets containing A.*

3 Two Classes of Functions via sg-Clopen Sets

We introduce the following definitions:

Definition 3.1 *A function f : (X, τ) $\rightarrow$ (Y, σ) is said to be totally semi-generalized-continuous (briefly totally sg-continuous) if the inverse image of every open subset of (Y, σ) is a sg-clopen (i.e. sg-open and sg-closed) subset of (X, τ).*

It is evident that every totally continuous function is totally sg-continuous. But the converse need not be true as shown in the following example.

Example 3.1 Let X = {a, b, c }, Y = {p, q}, τ = { ϕ, {a}, {b}, {a, b}, X} and σ = {ϕ, {p}, Y}. Define a function f : (X, τ) $\rightarrow$ (Y, σ) such that f(a) = p, f(b) = f(c) = q. Then clearly f is totally sg-continuous, but not totally continuous.

Definition 3.2 *A function f : (X, τ) $\rightarrow$ (Y, σ) is said to be strongly semi-generalized-continuous (briefly strongly sg-continuous) if the inverse image of every subset of (Y, σ) is a sg-clopen subset of (X, τ).*

It is clear that strongly sg-continuous function is totally sg-continuous. But the reverse implication is not always true as shown in the following example.

Example 3.2 Let X = {a, b, c } = Y , τ = { ϕ, {a}, {b}, {a, b}, X} and σ = {ϕ, {a}, Y}. Then the identity function f : (X, τ) $\rightarrow$ (Y, σ) is totally sg-continuous, but not strongly sg-continuous.

Theorem 3.1 *Every totally sg-continuous function into T_1-space is strongly sg-continuous.*

Proof. In a T_1-space, singletons are closed. Hence $f^{-1}(A)$ is sg-clopen in (X, τ) for every subset A of Y.

Remark 3.1 It is clear from the Theorem 3.1 that the classes of strongly sg-continuous functions and totally sg-continuous functions coincide when the range is a T_1-space.
Recall that a space (X, τ) is said to be sg-connected [3] if X cannot be expressed as the union of two non-empty disjoint sg-open sets.

Theorem 3.2 *If f is a totally sg-continuous function from a sg-connected space X onto any space Y, then Y is an indiscrete space.*

Proof. Suppose that Y is not indiscrete. Let A be a proper non-empty open subset of Y. Then $f^{-1}(A)$ is a proper non-empty sg-clopen subset of (X, τ), which is a contradiction to the fact that X is sg-connected.

Definition 3.3 *A space X is said to be sg-T_2 [21] if for any pair of distinct points x, y of X, there exist disjoint sg-open sets U and V such that $x \in U$ and $y \in V$.*

Lemma 3.1 *The sg-closure of every sg-open set is sg-open.*

Proof. Every regular open set is open and every open set is sg-open. Thus, every regular closed set is sg-closed. Now let A be any sg-open set. There exists an open set U such that $U \subset A \subset cl(U)$. Hence, we have $U \subset$ sg-cl(U) $\subset$ sg-cl(A) $\subset$ sg-cl(cl(U)) = cl(U) since cl(U) is regular closed. Therefore, sg-cl(A) is sg-open.

Theorem 3.3 *A space X is sg-T_2 if and only if for any pair of distinct points x, y of X there exist sg-open sets U and V such that $x \in U$, and $y \in V$ and $sgcl(U) \cap sgcl(V) = \phi$.*

Proof. Necessity. Suppose that X is sg- T_2. Let x and y be distinct points of x. There exist sg-open sets U and V such that $x \in U$, $y \in V$ and $U \cap V = \phi$. Hence $sgcl(U) \cap sgcl(V) = \phi$ and by Lemma 3.1, sgcl(U) is sg-open. Therefore, we obtain $sgcl(U) \cap sgcl(V) = \phi$.

Sufficiency. This is obvious.

Theorem 3.4 *If $f : (X, \tau) \to (Y, \sigma)$ is a totally sg-continuous injection and Y is T_0 then X is sg-T_2.*

Proof. Let x and y be any pair of distinct points of X. Then $f(x) \neq f(y)$. Since Y is T_0, there exists an open set U containing say, f(x) but not f(y). Then $x \in f^{-1}(U)$ and $y \notin f^{-1}(U)$. Since f is totally sg-continuous, $f^{-1}(U)$ is a sg-clopen subset of X. Also, $x \in f^{-1}(U)$ and $y \in X - f^{-1}(U)$. By Theorem 3.3, it follows that X is sg-T_2.

Theorem 3.5 *A topological space (X, τ) is sg-connected if and only if every totally sg-continuous function from a space (X, τ) into any T_0-space (Y, σ) is constant.*

Proof. Suppose that X is not sg-connected and every totally sg-continuous function from (X, τ) to (Y, σ) is constant. Since (X, τ) is not sg-connected, there exists a proper non-empty sg-clopen subset A of X. Let Y = {a, b} and σ = {ϕ, {a}, {b}, Y} be a topology for Y. Let f : (X, τ) → (Y, σ) be a function such that f(A) = {a} and f(Y − A) = {b}. Then f is non-constant and totally sg-continuous such that Y is T_0 which is a contradiction. Hence X must be sg-connected.
Converse is similar.

Theorem 3.6 *Let f : (X, τ) → (Y, σ) be a totally sg-continuous function and Y is a T_1-space. If A is a non-empty sg-connected subset of X, then f(A) is a single point.*

Definition 3.4 *Let (X, τ) be a topological space. Then the set of all points y in X such that x and y cannot be separated by a sg-separation of X is said to be the quasi sg-component of X.*

Theorem 3.7 *Let f : (X, τ) → (Y, σ) be a totally sg-continuous function from a topological space (X, τ) into a T_1-space Y. Then f is constant on each quasi sg-component of X.*

Proof. Let x and y be two points of X that lie in the same quasi-sg-component of X. Assume that f(x) = $\alpha \neq \beta$ = f(y). Since Y is T_1, {α} is closed in Y and so Y – {α} is an open set. Since f is totally sg-continuous, therefore $f^{-1}(\{\alpha\})$ and $f^{-1}(Y-\{\alpha\})$ are disjoint sg-clopen subsets of X. Further, $x \in f^{-1}(\{\alpha\})$ and $y \in f^{-1}(Y-\{\alpha\})$, which is a contradiction in view of the fact that y belongs to the quasi sg-component of x and hence y must belong to every sg-open set containing x.

4 Contra-sg-Continuous Functions

Definition 4.1[17] *A function f : (X, τ) → (Y, σ) is called contra-sg-continuous (briefly csg-continuous) if $f^{-1}(V)$ is sg-open in (X, τ) for every closed set V in (Y, σ).*
It is clear that every strongly sg-continuous function is csg-continuous. But the reverse implication is not always true as shown in the following example.

Example 4.1 Let X = Y = {a, b, c}, τ = { ϕ, {a, b}, X} and σ = {ϕ, {b, c}, Y}. Then the identity function f : (X, τ) → (Y, σ) is csg-continuous but it is not strongly sg-continuous.

Definition 4.2 *Let A be a subset of a topological space (X, τ). The set $\cap\{U \in \tau \mid A \subset U\}$ is called the Kernal of A [13] and is denoted by ker(A).*

Lemma 4.1 [8] *The following properties hold for subsets A, B of a space X:*

(i) $x \in ker(A)$ if and only if $A \cap F \neq \phi$ for any $F \in C(X, x)$;
(ii) $A \subset ker(A)$ and $A = ker(A)$ if A is open in X;
(iii) If $A \subset B$, then $ker(A) \subset ker(B)$.

Theorem 4.1 *Assume that arbitrary union of sg-open sets is sg-open. The following are equivalent for a function $f : (X, \tau) \to (Y, \sigma)$:*

(i) f is csg-continuous;
(ii) for every closed subset F of Y, $f^{-1}(F) \in SG(X, \tau)$;
(iii) for each $x \in X$ and each $F \in C(Y, f(x))$, there exists $U \in SG(X, \tau)$ such that $f(U) \subset F$;
(iv) $f(sgcl(A)) \subset ker(f(A))$ for every subset A of X;
(v) $sgcl(f^{-1}(B)) \subset f^{-1}(ker(B))$ for every subset B of Y.

Proof. The implications (i) $\to$ (ii) and (ii) $\to$ (iii) are obvious.

(iii) $\to$ (ii). Let F be any closed set of Y and $x \in f^{-1}(F)$. Then $f(x) \in F$ and there exists $U_x \in SG(X, x)$ such that $f(U_x) \subset F$. Therefore, we obtain $f^{-1}(F) = \bigcup\{U_x \mid x \in f^{-1}(F)\} \in SG(X, \tau)$.

(ii) $\to$ (iv). Let A be any subset of X. Suppose that $y \notin ker(f(A))$. Then by Lemma 4.1 there exists $F \in C(X, y)$ such that $f(A) \cap F = \phi$. Thus, we have $A \cap f^{-1}(F) = \phi$ and $sgcl(A) \cap f^{-1}(F) = \phi$. Therefore, we obtain $f(sgcl(A)) \cap F = \phi$ and $y \notin f(sgcl(A))$. This implies that $f(sgcl(A)) \subset ker(f(A))$.

(iv) $\to$ (v). Let B be any subset of Y. By (iv) and Lemma 4.1, we have $f(sgcl(f^{-1}(B))) \subset ker(f(f^{-1}(B))) \subset ker(B)$ and $sgcl(f^{-1}(B)) \subset f^{-1}(ker(B))$.

(v) $\to$ (i). Let V be any open set of Y. Then by Lemma 4.1 we have $sgcl(f^{-1}(V)) \subset f^{-1}(ker(V)) = f^{-1}(V)$ and $sgcl(f^{-1}(V)) = f^{-1}(V)$. This show that $f^{-1}(V)$ is sg-closed in (X, τ).

Theorem 4.2 *Every contra semi-continuous function is csg-continuous.*

Proof. The proof follows from the definitions.

Remark 4.1 Contra sg-continuous need not be contra semi-continuous in general as shown in the following example.

Example 4.2 Let X = Y = {a, b, c }, τ = { ϕ, {a, c}, X} and σ = {ϕ, {b, c}, Y}. Then the identity function f : (X, τ) $\rightarrow$ (Y, σ) is csg-continuous. However, f is not contra-semi-continuous, since for the closed set F = {a}, $f^{-1}(F)$ is sg-open but not semi-open in (X, τ).

Corollary 4.1 *Every contra α-continuous (resp. contra-continuous) function is csg-continuous.*

Theorem 4.3 *Assume that arbitrary union of sg-open sets is sg-open. Let f : (X, τ) $\rightarrow$ (Y, σ) be a function. Then the following are equivalent.*

(i) f : (X, τ) $\rightarrow$ (Y, σ) is sg-continuous.
(ii) for each x in X and each open set V in Y with f(x) $\in$ V, there is a sg-open set U in X such that x $\in$ U, f(U) $\subset$ V.

Proof. (i) $\Rightarrow$ (ii). Let f(x) $\in$ V. Since f is sg-continuous we have x $\in$ $f^{-1}(V)$ $\in$ SG(X, τ). Let U = $f^{-1}(V)$. Then x $\in$ V and f(U) $\subset$ V.

(ii) $\Rightarrow$ (i). Let V be an open set in (Y, σ) and let x $\in$ $f^{-1}(V)$. Then f(x) $\in$ V and thus there exists a sg-open set U_x such that x $\in$ U_x and $f(U_x) \subset V$. Now x $\in$ U_x $\subset$ $f^{-1}(V)$ and $f^{-1}(V) = \cup_{x \in f-1(V)} U_x$. Therefore $f^{-1}(V)$ is sg-open in (X, τ) and consequently, f is sg-continuous.

Theorem 4.4 *Assume that arbitrary union of sg-open sets is sg-open. If a function f : (X, τ) $\rightarrow$ (Y, σ) is csg-continuous and Y is regular, then f is sg-continuous.*

Proof. Let x be an arbitrary point of X and V an open set of Y containing f(x). Since Y is regular, there exists an open set W in Y containing f(x) such that cl(W) $\subset$ V. Since f is csg-continuous, so by Theorem 4.1 there exists U$\in$ SG(X, x) such that f(U) $\subset$ cl(W). Then f(U) $\subset$ cl(W) $\subset$ V. Hence, by Theorem 4.3 f is sg-continuous.

Theorem 4.5 *Assume that arbitrary union of sg-open sets is sg-open. Let f : (X, τ) $\rightarrow$ (Y, σ) be a function and g : X $\rightarrow$ X $\times$ Y the graph function, given by g(x) = (x, f(x)) for every x $\in$ X. Then f is csg-continuous if and only if g is csg-continuous.*

Proof. Let x$\in$ X and let W be a closed subset of X $\times$ Y containing g(x). Then W$\cap$({x}$\times$Y) is closed in {x} $\times$ Y containing g(x). Also {x} $\times$ Y is homeomorphic to Y. Hence {y $\in$ Y ∣ (x, y) $\in$ W } is a closed subset of Y. Since f is csg-continuous, $\bigcup$ { $f^{-1}(y)$ ∣ (x, y) $\in$ W } is a sg-open subset of X. Further, x $\in$ $\bigcup$ { $f^{-1}(y)$ ∣ (x, y) $\in$ W } $\subset$ $g^{-1}(W)$. Hence $g^{-1}(W)$ is sg-open. Then g is csg-continuous.
Conversely, let F be a closed subset of Y. Then X $\times$ F is a closed subset of X $\times$ Y. Since g is csg-continuous, $g^{-1}(X \times F)$ is a sg-open subset of X. Also, $g^{-1}(X \times F)$ = $f^{-1}(F)$. Hence f is csg-continuous.

Theorem 4.6 *Assume that arbitrary union of sg-open sets is sg-open. If X is a topological space and for each pair of distinct points x_1 and x_2 in X there exists a map f into a Urysohn topological space Y such that $f(x_1) \neq f(x_2)$ and f is csg-continuous at x_1 and x_2, then X is sg-T_2.*

Proof. Let x_1 and x_2 be any distinct points in X. Then by hypothesis there is a Urysohn space Y and a function f : (X, τ) → (Y, σ) , which satisfies the conditions of the theorem. Let $y_i = f(x_i)$ for i =1,2. Then $y_1 \neq y_2$. Since Y is Urysohn, there exist open neighbourhoods U_{y1} and U_{y2} of y_1 and y_2 respectively in Y such that $cl(U_{y1}) \cap cl(U_{y2}) = \phi$. Since f is csg-continuous at x_i, there exists a sg-open neighbourhoods W_{xi} of x_i in X such that $f(W_{xi}) \subset cl(U_{yi})$ for i =1,2. Hence we get $W_{x1} \cap W_{x2} = \phi$ because $cl(Uy_1) \cap cl(Uy_2) = \phi$. Then X is sg-$T_2$.

Corollary 4.2 *Assume that arbitrary union of sg-open sets is sg-open. If f is a csg-continuous injection of a topological space X into a Urysohn space Y, then X is sg-T_2.*

Proof. For each pair of distinct points x_1 and x_2 in X, f is csg-continuous function of X into Urysohn space Y such that $f(x_1) \neq f(x_2)$ because f is injective. Hence by Theorem 4.6, X is sg-T_2.

Corollary 4.3 *If f is a csg-continuous injection of a topological space X into Ultra Hausdorff space Y, then X is sg-T_2.*

Proof. Let x_1 and x_2 be any distinct points in X. Then since f is injective and Y is Ultra Hausdorff $f(x_1) \neq f(x_2)$ and there exist $V_1, V_2 \in CO(Y, \sigma)$ such that $f(x_1) \in V_1$, $f(x_2) \in V_2$ and $V_1 \cap V_2 = \phi$. Then $x_1 \in f^{-1}(V) \in SG(X, \tau)$ for i = 1, 2 and $f^{-1}(V_1) \cap f^{-1}(V_2) = \phi$. Thus, X is sg-$T_2$.

Theorem 4.7 *If f : (X, τ) → (Y, σ) is a contra sg-continuous function and g : (Y, σ) → (Z, η) is a continuous function, then (g o f) : (X, τ) → (Z, η) is csg-continuous.*

Theorem 4.8 *Let f : (X, τ) → (Y, σ) be surjective sg-irresolute and sg-open and g : (Y, σ) → (Z, η) be any function. Then (g o f) : (X, τ) → (Z, η) is csg-continuous if and only if g is csg-continuous.*

Proof. The "If" part is easy to prove. To prove the "only if" part, let (g o f): (X, τ) → (Z, η) be csg-continuous. Let F be a closed subset of Z. Then $(g \circ f)^{-1}(F)$ is a sg-open subset of X. That is $f^{-1}(g^{-1}(F))$ is sg-open. Since f is sg-open. $f(f^{-1}(g^{-1}(F)))$ is a sg-open subset of Y. So $g^{-1}(F)$ is sg-open in Y. Hence g is csg-continuous.

Theorem 4.9 *Let $\{X_i \mid i \in \wedge\}$ be any family of topological spaces. If $f : X \to \Pi X_i$ is a csg-continuous function. Then $\pi_i \circ f : X \to X_i$ is csg-continuous for each $i \in \wedge$, where π_i is the projection of ΠX_i onto X_i.*

Definition 4.3 *The graph G(f) of a function f : (X, τ) $\rightarrow$ (Y, σ) is said to be csg-closed in X $\times$ Y if for each (x, y) $\in$ (X $\times$ Y) – G(f), there exist U $\in$ SG(X, x) and V $\in$ C(Y, y) such that (U $\times$ V) $\cap$ G(f) = ϕ.*

Lemma 4.2 *The graph f : (X, τ) $\rightarrow$ (Y, σ) is contra sg-closed (briefly csg-closed) in X $\times$ Y if and only if for each (x, y) $\in$ (X $\times$ Y) – G(f), there exist U $\in$ SG(X, x) and V $\in$ C(Y, y) such that f(U) $\cap$ V = ϕ.*

Proof. The proof follows from the definition.

Theorem 4.10 *Assume that arbitrary union of sg-open sets is sg-open. If f : (X, τ) $\rightarrow$ (Y, σ) is csg-continuous and Y is Urysohn, then G(f) is contra-sg-closed in X $\times$ Y.*

Proof. Let (x, y) $\in$ (X $\times$ Y) – G(f). Then y $\neq$ f(x) and there exist open sets V, W such that f(x) $\in$ V, y $\in$ W and cl(U) $\cap$ cl(W) = ϕ. Since f is csg-continuous, there exists U $\in$ SG(X, x) such that f(U) $\subset$ cl(V). Therefore, we obtain f(U) $\cap$ cl(W) = ϕ. This shows that G(f) is contra-sg-closed.

Theorem 4.11 *A csg-continuous image of a sg-connected space is connected.*

Proof. Let f : (X, τ) $\rightarrow$ (Y, σ) be a contra-sg-continuous function of a sg-connected space X onto a topological space Y. Let Y be disconnected. Let A and B form a disconnected of Y. Then A and B are clopen and Y = A $\cup$ B where A $\cap$ B = ϕ. Since f is a contra-sg-continuous function X = f^{-1}(Y) = f^{-1}(A $\cup$ B) = f^{-1}(A) $\cup$ f^{-1}(B) where f^{-1}(A) and f^{-1}(B) are non-empty sg-open sets in X. Also f^{-1}(A) $\cap$ f^{-1}(B) = ϕ. Hence X is non sg-connected which is a contradiction. Therefore Y is connected.

Theorem 4.12 *Let X be sg-connected and Y be a T_1 space. If f is csg-continuous, then f is constant.*

Proof. Since Y is T_1 space, $\wedge$ = {f^{-1}({y}) : y $\in$ Y} is a disjoint sg-open partition of X. If |$\wedge$| $\geq$ 2, then X is the union of two non-empty sg-open sets. Since X is sg-connected, |$\wedge$| = 1. Hence, f is constant.

Definition 4.4 *A topological space (X, τ) is said to be sg-normal if each pair of non-empty disjoint closed sets can be separated by disjoint sg-open sets.*

Definition 4.5 [19] *A topological space (X, τ) is said to be ultra normal if each pair of non-empty disjoint closed sets can be separated by disjoint clopen sets.*

Theorem 4.13 *If f : (X, τ) $\rightarrow$ (Y, σ) is a csg-continuous, closed injection and Y is ultra normal, then X is sg-normal.*

Proof. Let F_1 and F_2 be a disjoint closed subsets of X. Since f is closed and injective, $f(F_1)$ and $f(F_2)$ are disjoint closed subsets of Y. Since Y is ultra normal $f(F_1)$ and $f(F_2)$ are separated by disjoint clopen sets V_1 and V_2 respectively. Hence $F_i \subset f^{-1}(V_i)$, $f^{-1}(V_i) \in SG(X, \tau)$ for i = 1,2 and $f^{-1}(V_1) \cap f^{-1}(V_2) = \phi$. Thus, X is sg-normal.

References

[1] S.P. Arya and T. Nour, Characterizations of s-normal spaces, *Indian J. Pure Appl. Math.*, 21(8) (1990), 717-719.

[2] P. Bhattacharya and B.K. Lahiri, Semi-generalized closed sets in topology, *Indian J. Math.*, 29(3) (1987), 375-382.

[3] M. Caldas, Semi-generalized continuous maps in topological spaces, *Port. Math.*, 52(4) (1995), 339-407.

[4] R. Devi, H. Maki and K. Balachandran, Semi-generalized closed maps and generalized semi-closed maps, *Mem. Fac. Sci. Kochi Univ.*, 14(1993), 41-54.

[5] J. Dontchev, Contra-continuous functions and strongly s-closed spaces, *Internat. J. Math. and Math. Sci.*, 19(1996), 303-310.

[6] J. Dontchev and T. Noiri, Contra-semicontinuous functions, *Math. Pannonica*, 10(1999), 159-168.

[7] S. Jafari and T. Noiri, Contra-α-continuous functions between topological spaces, *Iranian. Int. J. Sci.*, 2(2) (2001), 153-167.

[8] S. Jafari and T. Noiri, Contra-super-continuous functions, *Annales Univ. Sci. Budapest*, 42(1999), 27-34.

[9] R.C. Jain, *Ph.D. Thesis*, Meerut University, India (1980).

[10] N. Levine, Generalized closed sets in topology, *Rend. Circ. Math. Palermo*, 19(2) (1970), 89-96.

[11] N. Levine, Semi-open sets and semi-continuity in topological spaces, *Amer. Math. Monthly*, 70(1963), 36-41.

[12] N. Levine, Strong continuity in topological spaces, *Amer. Math. Monthly*, 67(1960), 269.

[13] M. Mrsevic, On pairwise R_0 and pairwise R_1 bitopological spaces, *Bull. Math. Soc. Math.*, R.S. Roumanie, 30(78) (1986), 141-148.

[14] G.B. Navalagi, Properties of gs-closed sets and sg-closed sets in topology, *Topology Atlas*.

[15] G.B. Navalagi, Semi-generalized separation axioms in topology, *Topology Atlas*.

[16] O. Njastad, On some classes of nearly open sets, *Pacific J. Math.*, 15(1965), 961-970.

[17] O. Ravi, M.L. Thivagar and R. Latha, Properties of contra sg-continuous functions, *Gen. Math. Notes*, (To appear).

[18] O. Ravi, G. Ramkumar and S. Chandrasekar, On quasi sg-open and quasi sg-closed functions, *Advances in Applied Mathematical Analysis*, 4(1) (2009), 73-78.

[19] R. Staum, The algebra of bounded continuous functions into a nonarchimedean field, *Pacific J. Math.*, 50(1974), 169-185.

[20] P. Sundaram, N. Rajesh, M.L. Thivagar and Z. Duszynski, $\tilde{g}$-semi-closed sets in topological spaces, *Mathematica Pannonica*, 18/1 (2007), 51-61.

[21] P. Sundaram, H. Maki and K. Balachandran, Semi-generalized continuous maps and semi-$T_{1/2}$ spaces, *Bull. Fukuoka Univ. Edu.*, 40(III) (1991), 33-40.

[22] M.K.R.S. Veera Kumar, Between g*-closed sets and g-closed sets, *Antarctica J. Math.*, (3)(1) (2006), 43-65.

[23] M.K.R.S. Veera Kumar, $\hat{g}$-closed sets in topological spaces, *Bulletin Allahabad Math. Soc.*, 18(2003), 99-112.

[24] M.K.R.S. Veera Kumar, $^{\#}$g-semi-closed sets in topological spaces, *Antarctica J. Math.*, 2(2) (2005), 201-222.