Gen. Math. Notes, Vol. 5, No. 2, August 2011, pp. 8-23
ISSN 2219-7184; Copyright © ICSRS Publication, 2011
www.i-csrs.org
Available free online at http://www.geman.in

E-Cordial and Z_3-Magic Labelings in Extended Triplicate Graph of a Path

E. Bala[1] and K. Thirusangu[2]

[1, 2] Department of Mathematics
S.I.V.E.T. College, Gowrivakkam, Chennai–73.
[1]E-mail: balasankarasubbu@yahoo.com
[2]E-mail: kthirusangu@gmail.com

(Received: 19-3-11/ Accepted: 2-7-11)

Abstract

In this paper we prove that the extended triplicate graph (ETG) of finite paths admits product E-cordial, total product E-cordial labelings. We show that ETG of finite paths of length n where $n \notin \{4m-3 | m \in N\}$ *admits E-Cordial, total E-cordial labelings and also we prove the existence of* Z_3 *– magic labeling for the modified Extended Triplicate graph.*

Keywords: *Graph labeling, A- magic, Path, E-cordial.*

1 Introduction

The concept of graph labeling was introduced by Rosa in 1967 [8]. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. Labeled graphs serve as useful models for broad range of applications such as coding theory, X-ray, crystallography, radar, astronomy, circuit design, communication networks and data base management and models for constraint programming over finite domain . Hence in the intervening years various labeling of graphs such as graceful labeling, harmonious labeling, magic labeling, antimagic labeling, bimagic labeling, prime labeling, cordial labeling, total cordial labeling, k-graceful labeling and odd graceful labeling etc., have been studied in over 1100 papers [7].

Cahit has introduced cordial labeling [5]. In [6], it is proved that every tree is cordial; K_n is cordial if and only if $n \leq 3$, $K_{m,n}$ is cordial for all m and n. Friendship graph $C_3^{(t)}$ is cordial if and only if $t \equiv 2$ (mod 2) and all fans are cordial. In [1], Andaretal proved that the t-ply graph $P_t(u,v)$ is cordial except when it is Eulerian and the number of edges is congruent to 2(mod 4). In [13], Youssef proved that every Skolem-graceful graph is cordial.

They proved the following graphs are E-cordial: trees with n vertices if and only if $n \neq 2 \pmod 4$; K_n if and only if $n \neq 2 \pmod 4$; $K_{m,n}$ if and only if $m+ n \neq 2 \pmod 4$; C_n if and only if $n \neq 2 \pmod 4$; regular graphs of degree 1 on 2n vertices if and only if n is even; friendship graphs $C_3^{(n)}$ for all n; fans F_n if and only if $n \neq 1 \pmod 4$; and wheels W_n if and only if $n \neq 1 \pmod 4$. They also observed that with $n \equiv 2$ (mod 4) vertices can not be E-cordial. More over the graph labelings on digraphs has been extensively studied in literature.

The original concept of an A-magic graph is due to dedlack, who defined it to be a graph with real-valued edge labeling such that distinct edges have distinct non-negative labels which satisfies the condition that the sum of the labels of the edges incident to a particular vertex is the same for all vertices. It is easy to verify whether a graph is Z_3-magic or not. In [4], it is proved that the class of even cycles, Bistar, ladder,biregular graphs admits Z_3-magic labeling. It is also shown that the certain class of Cayley's digraphs are Z_3-magic. In [2], some labelings for digraphs such as E-cordial, total E-cordial, Product E-Cordial, Product total E-cordial labelings has been introduced and shown the existence of the same for some class of Cayley digraphs.

The concept of extended duplicate graph was introduced by K.Thirusangu, et al in [11] and they proved that EDG(P_m) is cordial. In [3], it is proved that the extended triplicate graph of a path P_n admits cordial, total cordial, product cordial and total product cordial labelings. In this paper we prove that the extended triplicate graph (ETG) of finite paths admits product E-cordial , total product E-cordial labelings. We show that ETG of finite paths of length n where $n \notin \{4m-3|m\in N\}$ admits E-Cordial, total E-cordial labelings and also we prove the existence of Z_3 – magic labeling for the modified Extended Triplicate graph.

2 Preliminaries

Let G = G(V,E) be a finite, simple and undirected graph with p vertices and q edges. By a labeling we mean a one-to-one mapping that carries a set of graph elements onto a set of numbers called labels(usually the set of integers). In this paper we deal with the labeling with domain either the set of all vertices or the set of all edges or the set of all vertices and edges. We call these labelings as the vertex labeling or the edge labeling or the total labeling respectively.

Definition 2.1: *Let G(V,E) be a simple graph. A Duplicate graph of G is* $DG=(V_1,E_1)$ *where the vertex set* $V_1 = V \cup V'$ and $V \cap V' = \varphi$ *and* $f: V \to V'$ *is bijective (for* $v \in V$, *we write* $f(v) = v'$ *for convenience) and the edge set* E_1 *of DG is defined as follows: The edge ab is in E if and only if both* ab' *and* $a'b$ *are edges in* E_1. *Clearly duplicate graph of a path is disconnected.*

Definition 2.2: *Let* $DG=(V_1,E_1)$*be a duplicate graph of a path G(V,E). Add an edge between any one vertex from V to any other vertex in* V', *except the terminal vertices of V and* V'. *For convenience Let us take* $v_2 \in V$ *and* $v_2' \in V'$ *and thus the edge* (v_2, v_2') *is formed. This graph is called the Extended Duplicate of the path* P_m *and it is denoted by* $EDG(P_m)$.

Definition 2.3.: *Let* $V = \{ v_1, v_2 \ldots, v_{n+1}\}$ *and* $E = \{ e_1, e_2 \ldots. , e_n\}$*be the vertex and Edge set of a path* P_n. *For every* $v_i \varepsilon V$, *if we write an ordered triple*$\{v_i , v_i', v_i''\}$ *where* $1 \leq i \leq n+1$ *and For every edge* $v_iv_j \varepsilon E$, *if we draw four edges* v_iv_j', $v_j' v_i''$, v_jv_i' *and* $v_i' v_j''$ *where* $j = i +1$, *then the graph with this vertex set and edge set is called a Triplicate Graph of a path* P_n. *It is dentoted by* $TG(P_n)$.

Definition 2.4: *The structure of Triplicate graph* $TG(P_n)$ *is defined as follows: From the construction, using definition 3.1 the* $TG(P_n)$ *has* $3(n+1)$ *vertices and 4n edges. Denote the vertex set as* $V = \{ v_1, v_2 \ldots, v_{3(n+1)}\}$ *and the edge set E as* $E = \{ e_1, e_2 \ldots. , e_{4n.}\}$
Clearly the Triplicate graph $TG(P_n)$ *is disconnected. To make this a connected graph, we construct the extended triplicate graph using definition 2.5.*

Definition 2.5: *In* $TG(P_n)$, *include new edges* (v_{n+1} , v_1) *in the edge set , if n is odd. Thus the edge set is constructed as follows:* $E = \{ ((v_{i-1}, v_i'), (v_{i-1}'', v_i'), (v_{i+1}, v_i'), (v_{i+1}'', v_i')$ *where* $2 \leq i \leq n) \cup (v_n, v_{n+1}') \cup (v_n'', v_{n+1}') \cup (v_1',v_2) \cup (v_1' , v_2'') \cup (v_{n+1},v_1)\}$. *Suppose if n is even, include new edge* (v_n ,v_1) *in the edge set of* $TG(P_n)$. *Thus the edges set is constructed as follows:* $E = \{ ((v_{i-1}, v_i'), (v_{i-1}'', v_i'), (v_{i+1}, v_i'), (v_{i+1}'', v_i')$ *where* $2 \leq i \leq n) \cup (v_n, v_{n+1}') \cup (v_n'', v_{n+1}') \cup (v_1',v_2) \cup (v_1',v_2'') \cup (v_n ,v_1)\}$. *Denote this graph as* $ETG(P_n)$. *Thus the Extended Triplicate graph of* P_n *has* $3(n+1)$ *vertices and* $4n +1$ *edges for all n.*

Definition 2.6: *The structure of Extended Triplicate for a Path* P_n *denoted by* $ETG(P_n)$ *is defined as follows. From the construction of Triplicate graph* $TG(P_n)$ *has* $3(n+1)$ *vertices and 4n edges. From the definition 2.5, the Extended Triplicate* $ETG(P_n)$ *has the same vertex set as in* $TG(P_n)$ *and the edge set has* $4n +1$*edges for all n and it is denoted by* $V=\{v_1,v_2..,v_{3(n+1)}\}$ *and* $E = \{ e_1, e_2 \ldots e_{4n+1}\}$.

3 E-Cordial and Total E-Cordial Labelings

Definition 3.1: *Let G be a graph with vertex set* V *and edge set* E *and let f be function from E to* {0,1}. *Define f* on V by* $f^*(v) = \{\sum f(uv)\} (\text{mod } 2)$ *where* $uv \in E$. *The function f is called E-cordial labeling of G if the number of vertices labeled 0 and the number of vertices labeled 1 differ atmost by 1, and number of edges labeled 0 and the number of edges labeled 1 differ atmost by 1. A graph that admits E-cordial labeling is called E-cordial.*

Definition 3.2: *A Graph G(V,E) is said to admit total E-cordial labeling if there exists a function f from E onto the set {0,1} such that the induced map f* on V is defined as* $f^*(v_i) = \{ \sum f(v_i\, v_j) \} (\text{mod } 2)$ *where* $v_i\, v_j \in E$ *satisfies the property that the number of vertices and arcs labeled with 0 and the number of vertices and arcs labeled with 1 differ atmost by 1.*

Now we present an algorithm to get E-cordial labeling of extended triplicate graph $ETG(P_n)$ for any n, where n > 0.

Algorithm 3.1.

Input: A finite Path P_n , $n \geq 1$ with n+1 vertices and n edges.
Begin

Step 1: Using definition 2.5, Construct the Extended triplicate graph $ETG(P_n)$

Step 2: Denote the vertex set and edge set of $ETG(P_n)$ as $V=\{v_1, v_2 .., v_{3(n+1)}\}$ and

$E = \{e_1, e_2 \ldots , e_{4n+1}\}$ for all n.

Step 3: For n = 3,7,11…...4k-1, where k > 0 is finite, define f such that

(i) For $2 \leq i \leq n+1$,

$$f(v_i'\, v_{i-1}'') = \begin{cases} 1, & i = 4m, m \in N \\ 0, & \text{otherwise} \end{cases}$$

(ii) For $1 \leq i \leq n$,

$$f(v_i'\, v_{i+1}'') = \begin{cases} 0, & i = 4m, m \in N \\ 1, & \text{otherwise} \end{cases}$$

(iii) For $1 \leq i \leq n$, $f(v_i\, v_{i+1}') = 1$

(iv) For $2 \leq i \leq n+1$, $f(v_i\, v_{i-1}') = 0$

(v) $f(v_1\, v_{n+1}) = 0$

Step 4: For all finite k, where k>0 and n = 2,6,10…..4k-2, define f such that

(i) For $2 \le i \le n+1$, $f(v_i' v_{i-1}'') = \begin{cases} 1, & i = 4m, m \in N \\ 0, & \text{otherwise} \end{cases}$

(ii) For $1 \le i \le n$, $f(v_i' v_{i+1}'') = \begin{cases} 0, & i = 4m, m \in N \\ 1, & \text{otherwise} \end{cases}$

(iii) For $2 \le i \le n+1$, $f(v_i v_{i-1}') = 0$

(iv) For $1 \le i \le n$, $f(v_i v_{i+1}') = 1$

(v) $f(v_n v_1) = 1$

Step 5: For all finite k, where k>0 and n = 4,8,12…..4k, define f such that

(i) For $2 \le i \le n+1$, $f(v_i' v_{i-1}'') = \begin{cases} 1, & i = 4m, m \in N \\ 0, & \text{otherwise} \end{cases}$

(ii) For $1 \le i \le n$, $f(v_i' v_{i+1}'') = \begin{cases} 0, & i = 4m, m \in N \\ 1, & \text{otherwise} \end{cases}$

(iii) For $2 \le i \le n+1$, $f(v_i v_{i-1}') = 0$

(iv) For $1 \le i \le n$, $f(v_i v_{i+1}') = 1$

(v) $f(v_n v_1) = 0$

End

Output : E-cordial labeling for $ETG(P_n)$ for any n, where n> 0.

Theorem 3.1: *For any $n \neq 4m-3$, where $n > 0$, $m \in N$, the Extended triplicate graph $ETG(P_n)$ is E-cordial.*

Proof: From the construction of Extended Triplicate graph, $ETG(P_n)$ has $3(n+1)$ vertices and $4n + 1$ edges. Denote the vertex set and edge set of $ETG(P_n)$ as $V=\{v_1,v_2..,v_{3(n+1)}\}$ and $E=\{e_1, e_2 \ldots, e_{4n+1}\}$.

To prove that for any $n \neq 4m-3$, where $n > 0$, $m \in N$, the Extended triplicate graph $ETG(P_n)$ is E-cordial, we have to show that there exists a function $f : E \to \{0,1\}$ such that the induced function f^* on V defined as $f^*(v_i) = \{\sum f(v_i v_j)\} \pmod 2$ where $v_i v_j \in E$, which satisfies the property that the number of vertices labeled 0 and the number of vertices labeled 1 differ by atmost 1 and the number of edges labeled 0 and the number of vertices labeled 1 differ by atmost1. Consider the arbitrary vertex $v_i \in V$.

Case (i) From step 3 of the above algorithm the edges are labeled for all finite k, where k>0 and n = 3,7,11…...4k-1 so that the number of edges labeled 0 is 2n and the number edges labeled 1 is 2n+1. In order to get the labels for the vertices, define the induced map $f^* : V \to \{0,1\}$ such that

(i) for all $1 \le i \le n$, $f^*(v_i) = \sum f(v_iv_j) = 1$ where $v_i \in V$ and v_j is adjacent with v_i

(ii) $f^*(v_{n+1}) = f(v_{n-1}' v_n) + f(v_n v_1) = 0$

(iii) $f^*(v_1') = f(v_1' v_2) + f(v_1' v_2'') = 1$

(iv) for $1 \le i \le n$ $f^*(v_i') = \sum f(v_iv_j) = 0$

(v) for all $1 \le i \le n+1$

$$f^*(v_i'') = \sum f(v_i'' v_j) = \begin{cases} 0, & i \equiv 1 \pmod 2 \\ 1, & i \equiv 0 \pmod 2 \end{cases}$$

The number of vertices labeled 0 is 3(n+1)/2 and the number of vertices labeled 1 is 3(n+1)/2. Thus the number of vertices labeled 0 and the number of vertices labeled 1 differ by atmost 1.

Case (ii) From Step 4 of the above algorithm, for all finite k, where k>0 and n=2,6,10…..4k-2, we define a map $f : E \to \{0,1\}$ such that the number of edges labeled 0 is 2n and the number edges labeled 1 is 2n+1. In order to get the labels for the vertices , define the induced map $f^* : V \to \{0,1\}$ such that

(i) for all $2 \le i \le n-1$, $f^*(v_i) = \sum f(v_iv_i) = 1$ where $v_i \in V$ and v_j is adjacent with v_i and $f^*(v_1) = f^*(v_n) = f^*(v_{n+1}) = 0$

(ii) $f^*(v_i') = 0, 2 \le i \le n$; $f^*(v_1') = f^*(v_{n+1}') = 1$

(iii) $f^*(v_i'') = \sum f(v_i'' v_j) = \begin{cases} 0, & i \equiv 1 \pmod 2 \\ 1, & i \equiv 0 \pmod 2 \end{cases}$

and $f^*(v_{n+1}'') = \sum f(v_{n+1}'' v_j) = 1$

The number of vertices labeled 0 is (3n+4)/2 and the number of vertices labeled 1 is (3n+2)/2. Thus the number of vertices labeled 0 and the number of vertices labeled 1 differ by atmost 1.

Case (iii) From Step 5 of the above algorithm, for all finite k, where k > 0 and n = 4,8,12…..4k, we define a map $f:E \to \{0,1\}$ such that the number of edges labeled 0 is 2n and the number edges labeled 1 is 2n+1. In order to get the labels for the vertices, define the induced map $f^* : V \rightarrow \{0,1\}$ such that

(i) for all $1 \le i \le n$, $f^*(v_i) = \sum f(v_i v_j) = 1$ where $v_i \in V$ and v_j is adjacent with v_i and $f^*(v_{n+1})=0$

(ii) $f^*(v_i') = 0, 2 \le i \le n$; $f^*(v_1') = f^*(v_{n+1}') = 1$

(iii) for all $1 \le i \le n+1$, $f^*(v_i'') = \sum f(v_i'' v_j) = \begin{cases} 0, & i \equiv 1 \pmod 2 \\ 1, & i \equiv 0 \pmod 2 \end{cases}$

The number of vertices labeled 1 is (3n+4)/2 and the number of vertices labeled 0 is (3n+2)/2. Thus the number of vertices labeled 0 and the number of vertices labeled 1 differ by atmost 1. Hence for all the above cases, for any $n \neq 4m-3$, where $n > 0$, $m \in N$, the Extended triplicate graph ETG(P_n) is E-cordial.

Theorem 3.2: *For any $n \neq 4m-3$, where $n > 0$, $m \in N$, the Extended triplicate graph ETG(P_n) admits total E-cordial labeling.*

Proof: To prove that for any $n \neq 4m-3$, where $n > 0$, $m \in N$, the Extended triplicate graph ETG(P_n) admits total E-cordial labeling, we have to show that there exists a function $f: E \rightarrow \{0,1\}$ such that $f(v_i) = \sum f(v_i v_j) \pmod 2$, where $v_i v_i \in E$ which satisfies the property that that the number of zeroes on the vertices and edges taken together differ by atmost 1 with the number of one's on vertices and edges taken together.

In case (i) of the above theorem, by using the map f on E and there by the induced map f* on V, we have the number of edges labeled 0 is 2n and the number of vertices labeled 0 is 3(n+1)/2. Also, the number of edges labeled by 1 is 2n+1 and the number of vertices labeled by 1 is 3(n+1)/2. Thus the total number of one's on vertices and edges taken together is 3(n+1)/2 + 2n+1 =(7n+5)/2 and the the total number of zeroes on vertices and edges taken together is 3(n+1)/2+2n =(7n+3)/2.

In case (ii) of the above theorem, by using the map f on E and there by the induced map f* on V, we have the number of edges labeled 0 is 2n and the number of vertices labeled 0 is 3(n+4)/2. Also, the number of edges labeled by 1 is 2n+1 and the number of vertices labeled by 1 is 3(n+2)/2. Thus the total number of one's on vertices and edges taken together is 3(n+2)/2 + 2n+1 =(7n+4)/2 and the the total number of zeroes on vertices and edges taken together is 3(n+4)/2+2n =(7n+4)/2 .

Similarly for case (iii) of the above theorem, by using the map f on E and there by the induced map f* on V, we have the number of edges labeled 0 is 2n and the number of vertices labeled 0 is 3(n+4)/2. Also, the number of edges labeled by 1 is 2n+1 and the number of vertices labeled by 1 is 3(n+2)/2. Thus the total number of one's on vertices and edges taken together is 3(n+2)/2 + 2n+1

=(7n+4)/2 and the the total number of zeroes on vertices and edges taken together is 3(n+4)/2+2n = (7n+4)/2.

Thus in all the three cases, the number of zeroes on the vertices and edges taken together differ by atmost 1 with the number of one's on vertices and edges taken together. *Hencefor any $n \neq 4m-3$, where $n > 0$, $m \in \mathbb{N}$, the Extended triplicate graph ETG(P_n) admits total E-cordial labeling.*

Example 3.1: E-cordial labeling of ETG(P_7) , ETG(P_6), ETG(P_8) is shown in figure 1,2 and 3 respectively.

FIGURE 1: ETG (P_7)

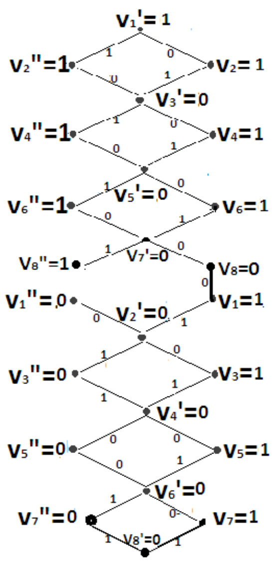


FIGURE 2: ETG (P_6)

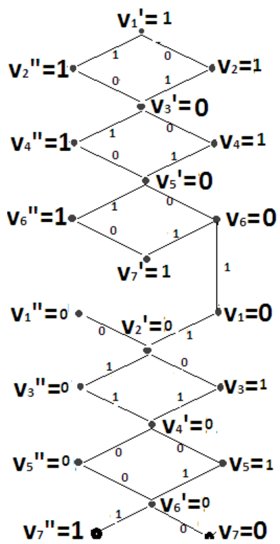
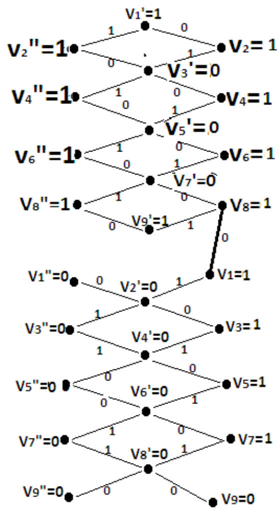


FIGURE 3: ETG(P_8)



4 Product E-Cordial and Total Product E-Cordial Labelings

Definition 4.1: *A Graph G(V,E)* is *said to admit product E-cordial labeling if there exists a function f from E onto the set {0,1} such that the induced map f* on V is defined as $f^*(v_i) = \{\Pi f(v_i\, v_j)\}(\text{mod } 2)$ where $v_i\, v_j \in E$ satisfies the property that if the number of vertices labeled 0 and the number of vertices labeled 1 differ atmost by 1, and number of edges labeled 0 and the number of arcs labeled 1 differ atmost by 1.*

Definition 4.2: *A Graph G(V,E) is said to admit total product E-cordial labeling if there exists a function f from E onto the set {0,1} such that the induced map f* on V is defined as $f^*(v_i) = \{\Pi f(v_i\, v_j)\}(\text{mod } 2)$ where $v_i\, v_j \in E$ satisfies the property that that the number of vertices and arcs labeled with 0 and the number of vertices and edges labeled with 1 differ atmost by 1.*

We present an algorithm to get Product E-cordial labeling of of Extended triplicate graph ETG(P_n) for all finite n and n>0.

Algorithm 4.1:

Input: A finite Path P_n with n+1 vertices and n edges where $n \geq 1$.
Begin

Step 1: Using definition 2.5,Construct the Extended triplicate graph ETG(P_n)

Step 2: Denote the vertex set and edge set as $V=\{v_1, v_2.., v_{3(n+1)}\}$and $E =\{e_1, e_2 \dots , e_{4n+1}\}$ for all finite n and $n \geq 1$.

Step 3: For odd n, define f from E on to the set {0,1} as follows:

(i) For $2 \leq i \leq n+1$, $f(v_i' v_{i-1}'') = \begin{cases} 0, & i \equiv 0 \pmod 2 \\ 1, & i \equiv 1 \pmod 2 \end{cases}$

(ii) For $1 \leq i \leq n$, $f(v_i' v_{i+1}'') = \begin{cases} 0, & i \equiv 0 \pmod 2 \\ 1, & i \equiv 1 \pmod 2 \end{cases}$

(iii) For $1 \leq i \leq n$, $f(v_i v_{i+1}') = \begin{cases} 0, & i \equiv 1 \pmod 2 \\ 1, & i \equiv 0 \pmod 2 \end{cases}$

(iv) For $2 \leq i \leq n+1$, $f(v_i v_{i-1}') = \begin{cases} 0, & i \equiv 1 \pmod 2 \\ 1, & i \equiv 0 \pmod 2 \end{cases}$

(v) $f(v_1 v_{n+1}) = 1$

Step 4: For even n,define f from E on to the set {0,1} as follows:

(i) For $2 \leq i \leq n+1$, $f(v_i' v_{i-1}'') = \begin{cases} 1, & i \equiv 1 \pmod 2 \\ 0, & i \equiv 0 \pmod 2 \end{cases}$

(ii) For $1 \leq i \leq n$, $f(v_i' v_{i+1}'') = \begin{cases} 1, & i \equiv 1 \pmod 2 \\ 0, & i \equiv 0 \pmod 2 \end{cases}$

(iii) For $2 \leq i \leq n+1$, $f(v_i v_{i-1}') = \begin{cases} 0, & i \equiv 1 \pmod 2 \end{cases}$

1, $i \equiv 0 \pmod 2$)

(iv) For $1 \le i \le n$, $f(v_i \, v_{i+1}') = \begin{cases} 0, & i \equiv 1 \pmod 2 \\ 1, & i \equiv 0 \pmod 2 \end{cases}$

(v) $f(v_n v_1) = 1$

End

Output: Product E-cordial labeling for the ETG(P_n).

Theorem 4.1: *For any n>0, the* Extended *Ttriplicate Graph ETG(P_n) is Product E-cordial..*

Proof: From the construction of Extended Triplicate graph ETG(P_n) for all finite n>0 , we have 3 (n+1) vertices and 4n +1 edges. Denote the vertex set and edge set of ETG(P_n) as V={$v_1, v_2 .., v_{3(n+1)}$}and E ={e_1, e_2 … , e_{4n+1}}. To prove ETG(P_n) is Product E-cordial for any n>0, we have to show that there exists a function f from E onto the set {0,1} such that the induced map f* on V defined as $f^*(v_i)$ = {$\Pi f(v_i \, v_j)$}(mod 2) where $v_i \, v_j \in E$ satisfies the property that, the number of vertices labeled 0 and the number of vertices labeled 1 differ atmost by 1 and number of edges labeled 0 and the number of edges labeled 1 differ atmost by 1.

Case(i) For odd n, from step 3 of the above algorithm the edges are labeled , so that the number of edges labeled 0 is 2n and the number edges labeled 1 is 2n+1. In order to get the labels for the vertices , define the induced map $f^* : V \rightarrow \{0,1\}$ such that

(i) For all $1 \le i \le n+1$, $f^*(v_i) = \Pi f(v_i v_j)(\bmod 2) = \begin{cases} 0(\bmod 2), & i \equiv 1 \pmod 2 \\ 1(\bmod 2), & i \equiv 0 \pmod 2 \end{cases}$

where $v_i \in V$ and v_j is adjacent with v_i.

(ii)For $1 \le i \le n+1$, $f^*(v_i') = \Pi f(v_i'' v_j)(\bmod 2) = \begin{cases} 1, & i \equiv 1 \pmod 2 \\ 0, & i \equiv 0 \pmod 2 \end{cases}$

(iii) For all $1 \le i \le n+1$, $f^*(v_i'') = \Pi f(v_i'' v_j)(\bmod 2) = \begin{cases} 0(\bmod 2), & i \equiv 1 \pmod 2 \\ 1(\bmod 2), & i \equiv 0 \pmod 2 \end{cases}$

The number of vertices labeled 1 is 3(n+1)/2 and the number of vertices labeled 0 is 3(n+1)/2. Thus the number of vertices labeled 0 and the number of vertices labeled 1 differ by atmost 1.

Case(ii) For even n , from step 4 of the above algorithm the edges are labeled , so that the number of edges labeled 0 is 2n and the number edges labeled 1 is 2n+1. In order to get the labels for the vertices , define the induced map $f^* : V \rightarrow \{0,1\}$ such that

(i) For all $1 \leq i \leq n+1$, $f^*(v_i) = \Pi f(v_i v_j)(\bmod 2) = \begin{cases} 1(\bmod 2), & i \equiv 1 \pmod 2 \\ 0(\bmod 2), & i \equiv 0 \pmod 2 \end{cases}$

where $v_i \in V$ and v_j is adjacent with v_i.

(ii)For $1 \leq i \leq n+1$, $f^*(v_i') = \Pi f(v_i'' v_j)(\bmod 2) = \begin{cases} 1, & i \equiv 1 \pmod 2 \\ 0, & i \equiv 0 \pmod 2 \end{cases}$

(iii) For all $1 \leq i \leq n+1$, $f^*(v_i'') = \Pi f(v_i'' v_j)(\bmod 2) = \begin{cases} 0(\bmod 2), & i \equiv 1 \pmod 2 \\ 1(\bmod 2), & i \equiv 0 \pmod 2 \end{cases}$

The number of vertices labeled 1 is 3(n+1)/2 and the number of vertices labeled 0 is 3(n+1)/2. Thus in both the cases the number of vertices labeled 0 and the number of vertices labeled 1 differ by atmost 1. Hence the ETG(P_n) admits product E-cordial labeling for any n>0.

Theorem 4.2: *For any n>0, ETG(P_n) admits total product E-cordial labeling.*

Proof: To prove ETG(P_n) admits total product E-cordial labeling n>0, it is enough to show that there exists a function $f: E \rightarrow \{0,1\}$ such that the induced function f* on V defined by $f^*(v_i) = \Pi f(v_i v_j)(\bmod 2)$ where $v_i v_j \in E$ satisfies the property that the number of zeroes on the vertices and edges taken together differ by atmost 1 with the number of one's on vertices and edges taken together.
By case (i) of the above theorem, using the map f on E and there by the induced map f* on V, we have the number of edges labeled 0 is 2n and the number of vertices labeled 0 is 3(n+1)/2. Thus the total number of zeroes on vertices and edges taken together is 3(n+1)/2 + 2n = (7n+3)/2 Also, the number of edges labeled by 1 is 2n+1 and the number of vertices labeled by 1 is 3(n+1)/2. Thus the total number of one's on vertices and edges taken together is 3(n+1)/2 + 2n +1 = (7n+5)/2.
Similarly by case (ii), using the map f on E and there by the induced map f* on V, we have the number of edges labeled 0 is 2n and the number of vertices labeled 0 is (3n+4)/2. Thus the total number of zeroes on vertices and edges taken together is (3n+4)/2 + 2n = (7n+4)/2 Also, the number of edges labeled by 1 is 2n+1 and the number of vertices labeled by 1 is (3n+2)/2. Thus the total number of one's on vertices and edges taken together is (3n+2)/2 + 2n +1 = (7n+4)/2.

Thus in both the cases the number of zeroes on the vertices and edges taken together differ by atmost 1 with the number of one's on vertices and aedges taken together. Hence ETG(P_n) admits total product E-cordial labeling for any n>0.

Example 4.1: Product E-cordial labeling for ETG(P_5) and ETG(P_7) is shown in figure 4 and 5 respectively.

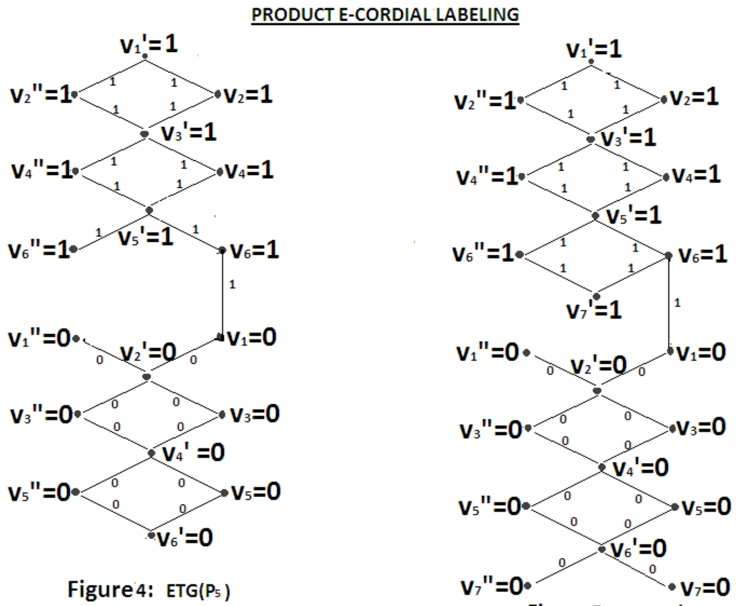


Figure4: ETG(P_5) Figure 5: ETG(P_6)

5 Z_3 –Magic Labeling

Definition 5.1 *A graph G(V,E) is said to admit Z_3-magic labeling if there exists a function f from E onto the set { 1,2} such that the induced map f* on V defined by $f^*(v_i) = \sum f(e) \pmod 3 = k$, a constant, where $e = (v_iv_j) \in E$.*

Now we present an algorithm to get a modified version of Extended triplicate graph and prove that ETG(P_n) admits Z_3-magic labeling for any n>0..

Algorithm 5.1:

Input: A finite Path P_n, n≥1 with n+1 vertices and n edges.
Begin
Step 1: Using definition 2.5, Construct the Extended triplicate graph ETG(P_n)

Step 2: Denote the vertex set and edge set as $V=\{v_1,v_2..,v_{3(n+1)}\}$and $E =\{e_1, e_2 …., e_{4n+1}\}$ for any n>0.

Step 3: Add new edge $(v_{n+1}'' v_1'')$ for odd n and $(v_{n+1}'' v_1'')$, (v_{n+1}'', v_1'), (v_{n+1}, v_1') for even n.

Step 4: For odd n, define the function f from E on to the set {1,2}such that

(i) For $2 \le i \le n+1$, $f(v_i' v_{i-1}'') = \begin{cases} 2, & i \equiv 0 \pmod 2 \\ 1, & i \equiv 1 \pmod 2 \end{cases}$

(ii) For $1 \le i \le n$, $f(v_i' v_{i+1}'') = \begin{cases} 1, & i \equiv 0 \pmod 2 \\ 2, & i \equiv 1 \pmod 2 \end{cases}$

(iii) For $1 \le i \le n$, $f(v_i v_{i+1}') = \begin{cases} 1, & i \equiv 1 \pmod 2 \\ 2, & i \equiv 0 \pmod 2 \end{cases}$

(iv) For $2 \le i \le n+1$, $f(v_i v_{i-1}') = \begin{cases} 2, & i \equiv 1 \pmod 2 \\ 1, & i \equiv 0 \pmod 2 \end{cases}$

(v)$f(v_1 v_{n+1}) = 2$

(vi) $f(v_1'' v_{n+1}'') = 1$

Step 5: For even n, define the function f from E on to the set {1,2}such that

(i) For $3 \le i \le n$, $f(v_i' v_{i-1}'') = \begin{cases} 2, & i \equiv 1 \pmod 2 \\ 1, & i \equiv 0 \pmod 2 \end{cases}$

and $f(v_2' v_1'') = 2$

(ii) For $1 \le i \le n$, $f(v_i' v_{i+1}'') = \begin{cases} 1, & i \equiv 1 \pmod 2 \\ 2, & i \equiv 0 \pmod 2 \end{cases}$

(iii) For $2 \le i \le n+1$, $f(v_i v_{i-1}') = \begin{cases} 1, & i \equiv 1 \pmod 2 \\ 2, & i \equiv 0 \pmod 2 \end{cases}$

(iv) For $2 \le i \le n-1$, $f(v_i v_{i+1}') = \begin{cases} 2, & i \equiv 1 \pmod 2 \\ 1, & i \equiv 0 \pmod 2 \end{cases}$

and $f(v_1 v_2') = 1$

(v) $f(v_n v_1) = f(v_1' v_{n+1}) = 2$

(vi) $f(v_1'' v_n'') = f(v_1' v_{n+1}'') = f(v_{n+1}' v_1'') = 1$

End

Output: Modified Extended triplicate graph with z_3-magic labeling for all finite n.

Theorem 5.1: *For any n>0, Modified Extended triplicate graph admits z_3-magic labeling..*

Proof: Using step 3 of the above algorithm, construct the modified extended triplicate graph ETG(P_n) and also by step 4 and 5 , the edges of modified ETG(P_n) are labeled for both odd and even n. In order to prove that the modified EDG(P_n) is Z_3-magic, define the induced map $f^*:V \rightarrow \{0,1,2\}$ such that for any vertex v_i , $f^*(v_i)=\sum f(v_i v_j) \pmod 3$ = k,a constant for all i.

Now consider the arbitrary vertex $v_i \in V$.
$f^*(v_i)=\sum f(v_i v_j) \pmod 3 = 0 \pmod 3$ where v_i is adjacent with v_j
Thus $f^*(v_i) = 0$ (mod3) which is a constant for all i.
Hence the modified Extended triplicate graph admits z_3-magic labeling for any n>0.

Example 5.1 z_3-magic labeling for the modified version of Extended triplicate graph ETG(P_5) and ETG(P_6)is shown in figure 6 and 7 respectively.

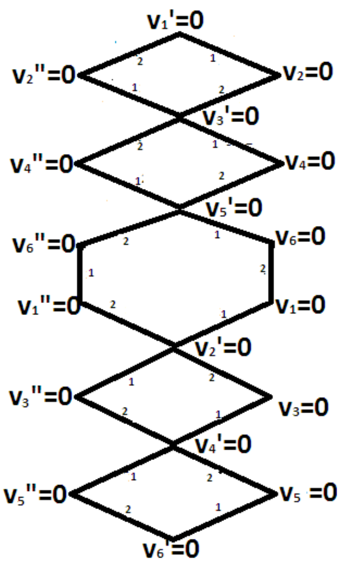


Figure 6: ETG(P5)

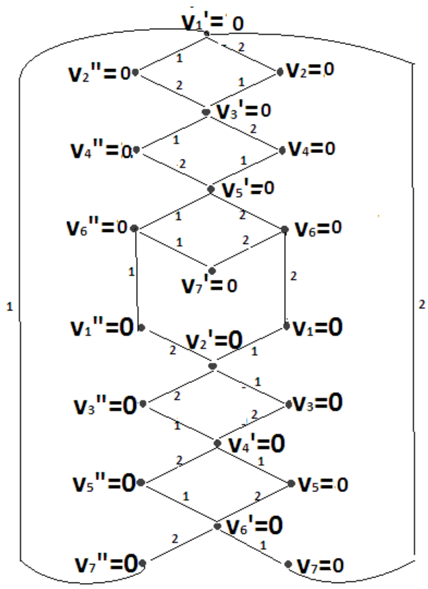


Figure 7: ETG(P6)

6 Conclusion

In this paper we have proved that the extended triplicate graph (ETG) of finite paths admits product E-cordial , total product E-cordial labelings. We have shown the ETG of finite paths of length n where $n \notin \{4m-3 | m \in N\}$ admits E-Cordial, total E-cordial labelings and also we proved the existence of Z_3 – magic labeling for the modified Extended Triplicate graph.

References

[1] M. Andar, S. Boxwala and N. Limaye, On the cordiality of the t-ply $P_t(u,v)$, *Ars Combin*, 77(2005), 245-259.

[2] E. Bala and K. Thirusangu, Some graph labelings in competition graph of Cayley digraphs, *International Journal of Combinatorial Graph Theory and Applications*, To appear, (2011).

[3] E. Bala and K. Thirusangu, Some graph labelings in extended triplicate graph of a path P_n, *International Review of Applied Engineering Research*, To appear, (2011)

[4] J.B. Babujee and L. Shobana, On Z_3-magic labeling and Cayley digraphs, *Int. J. Contemp. Math. Sciences*, 5(48) (2010), 2357-2368.

[5] J. Cahit, Cordial graphs: A weaker version of graceful and harmonious graphs, *Ars Combin*, 23(1987), 201-207.

[6] J. Cahit, On cordial and 3-equitable labelings of graphs, *Util.Math.*, 37 (1990), 189-198.

[7] J.A. Gallian, A dynamic survey of graph labeling, *The Electronic Journal of Combinatrics*, 16(2009), DS6.

[8] A. Rosa, On certain valuations of the vertices of a graph, *Theory of Graphs (Internat. Symposium, Rome, July 1996)*, Gordon and Breach, N.Y. and Dunod Paris, (1967).

[9] K. Thirusangu, J.B. Babujee and R. Rajeswari, On antimagic labelings in Cayley's digraphs, *International Journal of Mathematics and Applications*, 2(1-2) (2009), 11-16.

[10] K. Thirusangu, A.K. Nagar and R. Rajeswari, Labelings in Cayley's digraphs, *European Journal of Combinatrics*, 32(1) (2011), 133 -139.

[11] K. Thirusangu, P.P. Ulaganathan and B. Selvam, Cordial labeling in duplicate graphs, *International Journal of Computer, Mathematical Sciences and Applications*, 4(1-2) (2010), 179-186.

[12] R. Yilmaz and I. Cahit, E-cordial graphs, *Ars Combin.*, 46(1997), 251-266.

[13] M.Z. Youssef, On Skolem-graceful and cordial graphs, *Ars Combin.*, 78(2006), 167-177.