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# On $\Lambda_e$-Sets and $V_e$-Sets and the Associated Topology $\tau^{\Lambda_e}$

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*Dedicated to Prof. Dr. Zekeriya GÜNEY on the occasion of his 67th birthday.*

**Abstract**

*The aim of this paper is to introduce and investigate the topologies defined through $\Lambda_e$-sets and $V_e$-sets. For this aim, we define the notions of $\Lambda_e$-sets and $V_e$-sets and investigate some of the fundamental properties of them.*

**Keywords:** *e-open sets, $\Lambda_e$-sets, $V_e$-sets, topology $\tau^{\Lambda_e}$, e-$T_1$, $T_{\frac{1}{2}}$.*

## 1 Introduction

In 1986, Maki [9] introduced and studied the notion of a generalized $\Lambda$-set in a topological space $X$. Also, he looked into the relationship between the given topology $\tau$ and the topology $\tau^{\Lambda}$ which is generated by the family of generalized $\Lambda$-sets. Recently, three generalizations of the notion of $\Lambda$-set are obtained and studied by Caldas and Dontchev [2], Ganster and Jafari and Noiri [7], Caldas and Jafari and Noiri [3]. They continued the work of Maki [9].

The main goal of this paper is to continue research along these directions but this time by utilizing the $e$-open sets. We introduce $\Lambda_e$-sets and $V_e$-sets in a given topological space and thus obtain new topologies defined by these families of sets. We also consider some of the fundamental properties of these new topologies.

We shall use the well-known accepted language almost in the whole of the article.

## 2 Preliminaries

In this section let us recall some definitions and results which are used in this paper. Throughout this paper, writing spaces $X$ and $Y$ we always mean topological spaces on which no separation axioms are assumed. For a subset $A$ of a space $X$, the closure of $A$ and the interior of $A$, are denoted by $cl(A)$ and $int(A)$, respectively. The set of all open neighborhoods of the point $x$ of $X$ is denoted by $\mathcal{U}(x)$. A subset $A$ of a space $X$ is called $\delta$-open [11] if $A = int_\delta(A)$, where $int_\delta(A) = \{x|(\exists U \in \mathcal{U}(x))(int(cl(U)) \subset A)\}$. The complement of a $\delta$-open set is called $\delta$-closed [11]. Equivalently, a subset $A$ of a space $X$ is called $\delta$-closed [11] if $A = cl_\delta(A)$, where $cl_\delta(A) = \{x|(\forall U \in \mathcal{U}(x))(int(cl(U)) \cap A \neq \emptyset)\}$. A subset $A$ of a space $X$ is called $e$-open [6] (resp. preopen [10]) if $A \subset cl(int_\delta(A)) \cup int(cl_\delta(A))$ (resp. $A \subset int(cl(A))$). The complement of an $e$-open (resp. preopen) set is called $e$-closed (resp. preclosed). The family of all regular open (resp. regular closed, $e$-open, $e$-closed, preopen, preclosed) subsets of $X$ is denoted by $RO(X)$ (resp. $RC(X)$, $eO(X)$, $eC(X), PO(X), PC(X)$).

**Lemma 2.1.** *[6] The family of all e-open sets in a given topological space $X$ is closed under arbitrary unions.*

**Lemma 2.2.** *Let $(X, \tau)$ be a topological space and $A \subset X$. If $\tau = RO(X)$, then the interior of $A$ and the closure of $A$ are equal to the $\delta$-interior of $A$ and the $\delta$-closure of $A$, respectively. Therefore $BO(X) = eO(X)$.*

## 3 $\Lambda_e$-Sets and $V_e$-Sets

In this section, we introduce the notions of $\Lambda_e$-sets and $V_e$-sets and study some of their fundamental properties.

**Definition 3.1.** *Let $(X, \tau)$ be a topological space and $A \subset X$. We define the subsets $A^{\Lambda_e}$ and $A^{V_e}$ as follows:*

$$A^{\Lambda_e} := \bigcap\{E|(A \subset E)(E \in eO(X))\} \text{ and } A^{V_e} := \bigcup\{F|(F \subset A)(F \in eC(X))\}.$$

**Theorem 3.2.** *Let $(X, \tau)$ be a topological space and $A \subset X$. Then:*

$(a)$ $A^{\Lambda_e} = \{x|(\forall E \in eO(X))(A \subset E)(x \in E)\}$,
$(b)$ $A^{V_e} = \{x|(\exists F \in eC(X))(F \subset A)(x \in F)\}$.

**Proof:**

$$
\begin{aligned}
(a)\ \ x \notin A^{\Lambda_e} &\Leftrightarrow (\exists E \in eO(X))(A \subset E)(x \notin E)\\
&\Leftrightarrow x \notin \{x|(\forall E \in eO(X))(A \subset E)(x \in E)\}.\\
(b)\ \ x \in A^{V_e} &\Leftrightarrow (\exists F \in eC(X))(F \subset A)(x \in F)\\
&\Leftrightarrow x \in \{x|(\exists F \in eC(X))(F \subset A)(x \in F)\}.
\end{aligned}
$$

**Theorem 3.3.** *Let $(X, \tau)$ be a topological space. Then the following statements are satisfied:*

$(a)$ $A \subset X \Rightarrow A \subset A^{\Lambda_e}$,
$(b)$ $A \subset B \subset X \Rightarrow A^{\Lambda_e} \subset B^{\Lambda_e}$,
$(c)$ $A \subset X \Rightarrow (A^{\Lambda_e})^{\Lambda_e} = A^{\Lambda_e}$,
$(d)$ $A \in eO(X) \Rightarrow A = A^{\Lambda_e}$,
$(e)$ $\mathcal{A} \subset \mathcal{P}(X) \Rightarrow (\bigcup \mathcal{A})^{\Lambda_e} = \bigcup\limits_{A \in \mathcal{A}} A^{\Lambda_e}$,
$(f)$ $\mathcal{A} \subset \mathcal{P}(X) \Rightarrow (\bigcap \mathcal{A})^{\Lambda_e} \subset \bigcap\limits_{A \in \mathcal{A}} A^{\Lambda_e}$,
$(g)$ $A \subset X \Rightarrow (X \setminus A)^{\Lambda_e} = X \setminus A^{V_e}$.

**Proof:** $(a)$ Clear.
$(b)$ Let $A \subset B \subset X$.

$$
\left.\begin{array}{r} x \notin B^{\Lambda_e} \Rightarrow (\exists E \in eO(X))(B \subset E)(x \notin E) \\ A \subset B \end{array}\right\} \Rightarrow
$$

$\Rightarrow (\exists E \in eO(X))(A \subset E)(x \notin E) \Rightarrow x \notin A^{\Lambda_e}$.
$(c)$ Let $A \subset X$.

$$
\left.\begin{array}{r} x \in \left(A^{\Lambda_e}\right)^{\Lambda_e} \Rightarrow (\forall E \in eO(X))\left(A^{\Lambda_e} \subset E\right)(x \in E) \\ A \subset A^{\Lambda_e} \end{array}\right\} \Rightarrow
$$

$\Rightarrow (\forall E \in eO(X))(A \subset E)(x \in E) \Rightarrow x \in A^{\Lambda_e}$.
$(d)$ Let $A \in eO(X)$.

$$
\left.\begin{array}{r} A^{\Lambda_e} = \cap \{E|(A \subset E)(E \in eO(X))\} \\ A \in eO(X) \end{array}\right\} \Rightarrow A^{\Lambda_e} = A.
$$

$(e)$ Let $\mathcal{A} \subset \mathcal{P}(X)$.

$$
\begin{array}{rcl}
x \notin (\bigcup \mathcal{A})^{\Lambda_e} & \Rightarrow & (\exists E \in eO(X))(\bigcup \mathcal{A} \subset E)(x \notin E)\\
& \Rightarrow & (\exists E \in eO(X))(\forall A \in \mathcal{A})(A \subset E)(x \notin E)\\
& \Rightarrow & (\forall A \in \mathcal{A})(\exists E \in eO(X))(A \subset E)(x \notin E)\\
& \Rightarrow & (\forall A \in \mathcal{A})\left(x \notin A^{\Lambda_e}\right)\\
& \Rightarrow & x \notin \bigcup\limits_{A \in \mathcal{A}} A^{\Lambda_e}.
\end{array}
$$

$$\begin{aligned}
x \notin \bigcup_{A\in\mathcal{A}} A^{\Lambda_e} &\Rightarrow (\forall A\in\mathcal{A})\left(x\notin A^{\Lambda_e}\right) \\
&\Rightarrow (\forall A\in\mathcal{A})\,(\exists E\in eO(X))\,(A\subset E)\,(x\notin E) \\
&\Rightarrow \left(\bigcup_{\substack{(A\in\mathcal{A})(A\subset E)\\(x\notin E\in eO(X))}} E\in eO\left(X\right)\right)\left(\bigcup\mathcal{A}\subset \bigcup_{\substack{(A\in\mathcal{A})(A\subset E)\\(x\notin E\in eO(X))}} E\right)\left(x\notin \bigcup_{\substack{(A\in\mathcal{A})(A\subset E)\\(x\notin E\in eO(X))}} E\right) \\
&\Rightarrow x\notin \left(\bigcup\mathcal{A}\right)^{\Lambda_e}.
\end{aligned}$$

$(f)$ Let $\mathcal{A}\subset\mathcal{P}(X)$.

$$\begin{aligned}
x \notin \bigcap_{A\in\mathcal{A}} A^{\Lambda_e} &\Rightarrow (\exists A\in\mathcal{A})(x\notin A^{\Lambda_e}) \\
&\Rightarrow (\exists A\in\mathcal{A})(\exists E\in eO(X))(A\subset E)(x\notin E) \\
&\Rightarrow (\exists E\in eO(X))(\exists A\in\mathcal{A})(A\subset E)(x\notin E) \\
&\Rightarrow (\exists E\in eO(X))\left(\bigcap\mathcal{A}\subset E\right)(x\notin E) \\
&\Rightarrow x\notin \left(\bigcap\mathcal{A}\right)^{\Lambda_e}.
\end{aligned}$$

$(g)$ Let $A\subset X$.

$$X\setminus A^{V_e} = \bigcap\{\,X\setminus F|\,(X\setminus A\subset X\setminus F)\,(X\setminus F\in eO\,(X))\} = (X\setminus A)^{\Lambda_e}.$$

**Theorem 3.4.** *Let $(X,\tau)$ be a topological space. Then the following statements are satisfied:*

$(a)$ $A\subset X\Rightarrow A^{V_e}\subset A$,
$(b)$ $A\subset B\subset X\Rightarrow A^{V_e}\subset B^{V_e}$,
$(c)$ $A\subset X\Rightarrow (A^{V_e})^{V_e}=A^{V_e}$,
$(d)$ $A\in eC(X)\Rightarrow A=A^{V_e}$,
$(e)$ $\mathcal{A}\subset\mathcal{P}(X)\Rightarrow \left(\bigcap\mathcal{A}\right)^{V_e}=\bigcap_{A\in\mathcal{A}} A^{V_e}$,
$(f)$ $\mathcal{A}\subset\mathcal{P}(X)\Rightarrow \bigcup_{A\in\mathcal{A}} A^{V_e}\subset\left(\bigcup\mathcal{A}\right)^{V_e}$,

**Proof:** It follows directly from Definition 3.1 and Theorem 3.3.

**Definition 3.5.** *Let $(X,\tau)$ be a topological space and $A\subset X$. Then we define the notions $\Lambda_e$-set and $V_e$-set as follows:*

*$A$ is called $\Lambda_e$-set* $:\Leftrightarrow A=A^{\Lambda_e}$ *and* $\Lambda_e(X):=\{A|(A\subset X)(A=A^{\Lambda_e})\}$

*$A$ is called $V_e$-set* $:\Leftrightarrow A=A^{V_e}$ *and* $V_e(X):=\{A|(A\subset X)(A=A^{V_e})\}$.

**Proposition 3.6.** *Let $(X,\tau)$ be a topological space. Then the following statements are satisfied:*

$(a)$ $eO(X)\subset\Lambda_e(X)$,
$(b)$ $eC(X)\subset V_e(X)$.

**Proof:** It follows directly from Definition 3.1 and Definition 3.5.

It is not difficult to see that every $\Lambda_e$-set need not to be $e$-open as shown by the following example.

**Example 3.7.** *Let $X = \{a, b, c, d\}$ be and $\tau = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}, \{a, b\}, \{a, b, c\}, \{a, c, d\}\}$. The set $\{d\}$ is $\Lambda_e$-set but it is not e-open.*

**Theorem 3.8.** *Let $(X, \tau)$ be a topological space. Then the following statements are satisfied:*

$(a)$ $\emptyset, X \in \Lambda_e(X) \cap V_e(X)$,
$(b)$ $A \in \Lambda_e(X) \Leftrightarrow X \setminus A \in V_e(X)$,
$(c)$ $\mathcal{A} \subset \Lambda_e(X) \Rightarrow (\bigcup \mathcal{A} \in \Lambda_e(X))\, (\bigcap \mathcal{A} \in \Lambda_e(X))$,
$(d)$ $\mathcal{A} \subset V_e(X) \Rightarrow (\bigcup \mathcal{A} \in V_e(X))\, (\bigcap \mathcal{A} \in V_e(X))$.

**Proof:** $(a)$ and $(b)$ are obvious.

$(c)$ Let $\mathcal{A} \subset \Lambda_e(X)$.

$$\begin{aligned} A \in \mathcal{A} \subset \Lambda_e(X) &\Rightarrow A = A^{\Lambda_e} \\ &\Rightarrow \bigcup_{A \in \mathcal{A}} A = \bigcup_{A \in \mathcal{A}} A^{\Lambda_e} = \left(\bigcup_{A \in \mathcal{A}} A\right)^{\Lambda_e} \\ &\Rightarrow \bigcup \mathcal{A} = (\bigcup \mathcal{A})^{\Lambda_e} \\ &\Rightarrow \bigcup \mathcal{A} \in \Lambda_e(X). \end{aligned}$$

$$\begin{aligned} A \in \mathcal{A} \subset \Lambda_e(X) &\Rightarrow A = A^{\Lambda_e} \\ &\Rightarrow \bigcap_{A \in \mathcal{A}} A = \bigcap_{A \in \mathcal{A}} A^{\Lambda_e} \supset \left(\bigcap_{A \in \mathcal{A}} A\right)^{\Lambda_e} \\ &\Rightarrow \bigcap \mathcal{A} \supset (\bigcap \mathcal{A})^{\Lambda_e} \\ &\Rightarrow \bigcap \mathcal{A} \in \Lambda_e(X). \end{aligned}$$

$(d)$ Let $\mathcal{A} \subset \Lambda_e(X)$.

$$\begin{aligned} A \in \mathcal{A} \subset V_e(X) &\Rightarrow A = A^{V_e} \\ &\Rightarrow \bigcup_{A \in \mathcal{A}} A = \bigcup_{A \in \mathcal{A}} A^{V_e} \subset \left(\bigcup_{A \in \mathcal{A}} A\right)^{V_e} \\ &\Rightarrow \bigcup \mathcal{A} \subset (\bigcup \mathcal{A})^{V_e} \\ &\Rightarrow \bigcup \mathcal{A} \in V_e(X). \end{aligned}$$

$$\begin{aligned} A \in \mathcal{A} \subset V_e(X) &\Rightarrow A = A^{V_e} \\ &\Rightarrow \bigcap_{A \in \mathcal{A}} A = \bigcap_{A \in \mathcal{A}} A^{V_e} = \left(\bigcap_{A \in \mathcal{A}} A\right)^{V_e} \\ &\Rightarrow \bigcap \mathcal{A} = (\bigcap \mathcal{A})^{V_e} \\ &\Rightarrow \bigcap \mathcal{A} \in V_e(X). \end{aligned}$$

**Remark 3.9.** *$\Lambda_e(X)$ (resp.$V_e(X)$) defines a topology on $X$ containing all e-open (resp. e-closed) sets. Clearly, $(X, \Lambda_e(X))$ and $(X, V_e(X))$ are Alexandroff spaces [1], i.e. arbitrary intersections of open sets are open.*

Recall that a space $(X, \tau)$ is said to be $e$-$T_1$ [5] if for each pair of distinct points $x$ and $y$ of $X$ there exists an $e$-open set containing $x$ but not $y$. It is not difficult to show that a space $(X, \tau)$ is $e$-$T_1$ if and only if singletons are $e$-closed. We now compare additional characterizations of $e$-$T_1$ spaces.

**Theorem 3.10.** *Let $(X, \tau)$ be a topological space. Then the following properties are equivalent:*

$(a)$ *$(X, \tau)$ is $e$-$T_1$,*
$(b)$ *$\mathcal{P}(X) = \Lambda_e(X)$,*
$(c)$ *$\mathcal{P}(X) = V_e(X)$.*

**Proof:** It is obvious that $(b) \Leftrightarrow (c)$.
$(a) \Rightarrow (c)$ : Let $(X, \tau)$ be $e$-$T_1$ space.

$$\left.\begin{array}{r} x \in A \subset X \\ (X,\tau) \text{ is } e\text{-}T_1 \end{array}\right\} \begin{array}{ll} \Rightarrow & (\{x\} \subset A)(\{x\} \in eC(X)) \\ \Rightarrow & A = \bigcup\limits_{x \in A} \{\{x\} | (\{x\} \subset A)(\{x\} \in eC(X))\} \\ \Rightarrow & A = A^{V_e} \\ \Rightarrow & A \in V_e(X). \end{array}$$

$(c) \Rightarrow (a)$ : Let $A$ be $V_e$-set for all subsets of $X$.

$$x \in X \Rightarrow \{x\} \subset X \Rightarrow \{x\} = \{x\}^{V_e} \Rightarrow \{x\} \in eC(X).$$

Recall that a subset $A$ of a topological space $(X, \tau)$ is said to be generalized closed (briefly $g$-closed) [8] if $cl(A) \subset U$ whenever $A \subset U$ and $U \in \tau$. A topological space $(X, \tau)$ is said to be $T_{\frac{1}{2}}$ if every $g$-closed subset of $X$ is closed. Dunham [4] pointed out that $(X, \tau)$ is $T_{\frac{1}{2}}$ if and only if for each $x \in X$ the singleton $\{x\}$ is open or closed.

**Theorem 3.11.** *Let $(X, \tau)$ be a topological space. Then the following properties hold:*

$(a)$ *$(X, \Lambda_e(X))$ and $(X, V_e(X))$ are $T_{1/2}$,*
$(b)$ *If $(X, \tau)$ is an $e$-$T_1$ space, then $\Lambda_e(X) = V_e(X) = \mathcal{P}(X)$.*

**Proof:**

$$\begin{array}{ll} (a) & x \in X \Rightarrow (\{x\} \in PC(X) \vee \{x\} \in \tau) \\ & \qquad \Rightarrow (\{x\} \in eC(X) \vee \{x\} \in eO(X)) \\ & \qquad \Rightarrow (X \setminus \{x\} \in eO(X) \vee \{x\} \in eO(X)) \\ & \qquad \Rightarrow (X \setminus \{x\} \in \Lambda_e(X) \vee \{x\} \in \Lambda_e(X))(\{x\} \in V_e(X) \vee X \setminus \{x\} \in V_e(X)). \end{array}$$

$(b)$ This follows from Theorem 3.10.

# 4 Generalized $\Lambda_e$-Sets and Generalized $V_e$-Sets

**Definition 4.1.** *Let $(X,\tau)$ be a topological space and $A \subset X$. Then we define the notion of generalized $\Lambda_e$-set (br. g-$\Lambda_e$-set) as follows:*

$$A \text{ is called generalized } \Lambda_e\text{-set} :\Leftrightarrow [(A \subset F)(F \in eC(X)) \Rightarrow A^{\Lambda_e} \subset F]$$

$$g\text{-}\Lambda_e(X) := \{A | (A \subset X)(A \text{ is } g\text{-}\Lambda_e\text{-set})\}$$

**Definition 4.2.** *Let $(X,\tau)$ be a topological space and $A \subset X$. Then we define the notion of generalized $V_e$-set (br. g-$V_e$-set) as follows:*

$$A \text{ is generalized } V_e\text{-set} :\Leftrightarrow X \setminus A \text{ is generalized } \Lambda_e\text{-set}$$

$$g\text{-}V_e(X) := \{A | (A \subset X)(A \text{ is } g\text{-}V_e\text{-set})\}$$

**Theorem 4.3.** *Let $(X,\tau)$ be a topological space. Then the following properties hold:*

$(a)$ $g\text{-}\Lambda_e(X) = \Lambda_e(X)$,
$(b)$ $g\text{-}V_e(X) = V_e(X)$.

**Proof:** $(a)$ Let $A \in \Lambda_e(X)$.

$$\left.\begin{array}{r} (A \subset F)(F \in eC(X)) \\ A \in \Lambda_e(X) \Rightarrow A = A^{\Lambda_e} \end{array}\right\} \Rightarrow A^{\Lambda_e} \subset F \Rightarrow A \in g\text{-}\Lambda_e(X).$$

Suppose that $A \in g\text{-}\Lambda_e(X)$ but $A \notin \Lambda_e(X)$.

$$\begin{array}{rcl} A \notin \Lambda_e(X) & \Rightarrow & A \neq A^{\Lambda_e} \Rightarrow A \subsetneqq A^{\Lambda_e} \Rightarrow (\exists x \in X)(x \in A^{\Lambda_e} \setminus A) \\ & \Rightarrow & (\{x\} \in \tau \vee \{x\} \in PC(X)) \\ & \Rightarrow & (\{x\} \in eO(X) \vee \{x\} \in eC(X)) \end{array}$$

First case.

$$\left.\begin{array}{r} \{x\} \in eO(X) \Rightarrow X \setminus \{x\} \in eC(X) \\ x \in A^{\Lambda_e} \setminus A \Rightarrow x \notin A \Rightarrow A \subset X \setminus \{x\} \end{array}\right\} \Rightarrow A^{\Lambda_e} \subset X \setminus \{x\} \text{ which is a contradiction.}$$

Second case.

$$\left.\begin{array}{r} \{x\} \in eC(X) \Rightarrow X \setminus \{x\} \in eO(X) \\ x \in A^{\Lambda_e} \setminus A \Rightarrow x \notin A \Rightarrow A \subset X \setminus \{x\} \end{array}\right\} \Rightarrow A^{\Lambda_e} \subset X \setminus \{x\} \text{ which is a contradiction.}$$

$(b)$ This is proved in a similar way.

# 5 The Associated Topology $\tau^{\Lambda_e}$

**Definition 5.1.** *Let $(X,\tau)$ be a topological space and $A \subset X$. Then we define the subsets $C^{\Lambda_e}(A)$ and $I^{V_e}(A)$ as follows:*

$$C^{\Lambda_e}(A) := \bigcap\{U|(A\subset U)(U\in\Lambda_e(X))\},$$
$$I^{V_e}(A) := \bigcup\{V|(V\subset A)(V\in V_e(X))\}.$$

**Theorem 5.2.** *Let $(X,\tau)$ be a topological space and $A \subset X$. Then the following statements are satisfied.*

$(a)$ $C^{\Lambda_e}(\emptyset)=\emptyset$,
$(b)$ $A\subset C^{\Lambda_e}(A)$,
$(c)$ $\bigcup\limits_{A\in\mathcal{A}} C^{\Lambda_e}(A) = C^{\Lambda_e}(\bigcup\mathcal{A})$,
$(d)$ $A\subset B \Rightarrow C^{\Lambda_e}(A)\subset C^{\Lambda_e}(B)$,
$(e)$ $C^{\Lambda_e}(C^{\Lambda_e}(A)) = C^{\Lambda_e}(A)$,
$(f)$ $A\in\Lambda_e(X)\Rightarrow C^{\Lambda_e}(A)=A$,
$(g)$ $A\in V_e(X)\Rightarrow I^{V_e}(A)=A$,
$(h)$ $C^{\Lambda_e}(X\setminus A) = X\setminus I^{V_e}(A)$.

**Proof:** $(a)$ and $(b)$ are clear from Definition 5.1.
$(c)$ Let $x\notin C^{\Lambda_e}(\bigcup\mathcal{A})$.

$$\begin{array}{rcl} x\notin C^{\Lambda_e}(\bigcup\mathcal{A}) & \Rightarrow & (\exists E\in\Lambda_e(X))\,(\bigcup\mathcal{A}\subset E)\,(x\notin E)\\ & \Rightarrow & (\exists E\in\Lambda_e(X))\,(\forall A\in\mathcal{A})\,(A\subset E)\,(x\notin E)\\ & \Rightarrow & (\forall A\in\mathcal{A})\,(\exists E\in\Lambda_e(X))\,(A\subset E)\,(x\notin E)\\ & \Rightarrow & (\forall A\in\mathcal{A})\,\big(x\notin C^{\Lambda_e}(A)\big)\\ & \Rightarrow & x\notin \bigcup\limits_{A\in\mathcal{A}} C^{\Lambda_e}(A). \end{array}$$

$$\begin{array}{rcl} x\notin \bigcup\limits_{A\in\mathcal{A}} C^{\Lambda_e}(A) & \Rightarrow & (\forall A\in\mathcal{A})\,\big(x\notin C^{\Lambda_e}(A)\big)\\ & \Rightarrow & (\forall A\in\mathcal{A})\,(\exists E\in\Lambda_e(X))\,(A\subset E)\,(x\notin E)\\ & \Rightarrow & \left(\bigcup\limits_{\substack{(A\in\mathcal{A})(A\subset E)\\(x\notin E\in\Lambda_e(X))}} E\in\Lambda_e(X)\right)\left(\bigcup\mathcal{A}\subset \bigcup\limits_{\substack{(A\in\mathcal{A})(A\subset E)\\(x\notin E\in\Lambda_e(X))}} E\right)\left(x\notin \bigcup\limits_{\substack{(A\in\mathcal{A})(A\subset E)\\(x\notin E\in\Lambda_e(X))}} E\right)\\ & \Rightarrow & x\notin C^{\Lambda_e}(\bigcup\mathcal{A}). \end{array}$$

$(d)$ Let $x\notin C^{\Lambda_e}(B)$.

$$\left.\begin{array}{r} x\notin C^{\Lambda_e}(B)\Rightarrow(\exists E\in\Lambda_e(X))\,(B\subset E)\,(x\notin E)\\ A\subset B\end{array}\right\}\Rightarrow$$
$$\Rightarrow (\exists E\in\Lambda_e(X))\,(A\subset E)\,(x\notin E)\Rightarrow x\notin C^{\Lambda_e}(A).$$

$(e)$ Let $x\notin C^{\Lambda_e}(A)$.

$$\begin{array}{rcl} x\notin C^{\Lambda_e}(A) & \Rightarrow & (\exists E\in\Lambda_e(X))\,(A\subset E)\,(x\notin E)\\ & \Rightarrow & (\exists E\in\Lambda_e(X))\,\big(C^{\Lambda_e}(A)\subset C^{\Lambda_e}(E)=E\big)\,(x\notin E)\\ & \Rightarrow & x\notin C^{\Lambda_e}(C^{\Lambda_e}(A)). \end{array}$$

$(f),(g)$ and $(h)$ are clear from Definition 5.1.

**Corollary 5.3.** *Let $X$ be a nonempty set. The map $C^{\Lambda_e}:\mathcal{P}(X)\to\mathcal{P}(X)$ is a Kuratowski closure operator.*

**Definition 5.4.** *Let $\tau^{\Lambda_e}$ be the topology on $X$ generated by $C^{\Lambda_e}$ in the usual manner, i.e., $\tau^{\Lambda_e}=\{A|(A\subset X)(C^{\Lambda_e}(X\setminus A)=X\setminus A)\}$. We define the family $\rho^{\Lambda_e}$ by $\rho^{\Lambda_e}:=\{A|(A\subset X)(C^{\Lambda_e}(A)=A)\}$.*

**Theorem 5.5.** *Let $(X,\tau)$ be a topological space. Then the following properties hold.*

$(a)$ $\tau^{\Lambda_e}=\{A|(A\subset X)(I^{V_e}(A)=A)\}$,
$(b)$ $\Lambda_e(X)=\rho^{\Lambda_e}$,
$(c)$ $V_e(X)=\tau^{\Lambda_e}$,
$(d)$ $eC(X)=\tau^{\Lambda_e}\Rightarrow\Lambda_e(X)=eO(X,\tau)$,
$(e)$ $\Lambda_e(X)\subset eO(X)\Rightarrow\tau^{\Lambda_e}=\{A|(A\subset X)(A=A^{V_e})\}$,
$(f)$ $\Lambda_e(X)\subset eC(X)\Rightarrow eO(X)=\tau^{\Lambda_e}$.

**Proof:** $(a)$ This is clear from Definition 5.4 and Theorem 5.2(h).

$$\begin{aligned}A\in\{A|(A\subset X)(C^{\Lambda_e}(X\setminus A)=X\setminus A)\} &\Leftrightarrow (A\subset X)(C^{\Lambda_e}(X\setminus A)=X\setminus A)\\ &\Leftrightarrow (A\subset X)(X\setminus I^{\Lambda_e}(A)=X\setminus A)\\ &\Leftrightarrow (A\subset X)(I^{\Lambda_e}(A)=A)\\ &\Leftrightarrow A\in\{A|(A\subset X)(I^{\Lambda_e}(A)=A)\}.\end{aligned}$$

$(b)$ Let $A\in\Lambda_e(X)$. $A\in\Lambda_e(X)\Leftrightarrow C^{\Lambda_e}(A)=A\Leftrightarrow A\in\rho^{\Lambda_e}$.
$(c)$ Let $A\in V_e(X)$. $A\in V_e(X)\Leftrightarrow I^{V_e}(A)=A\Leftrightarrow A\in\tau^{V_e}$.
$(d)$ Let $eC(X)=\tau^{\Lambda_e}$.

$$A\in\Lambda_e(X)\Rightarrow A\in\rho^{\Lambda_e}\Rightarrow X\setminus A\in\tau^{\Lambda_e}\Rightarrow X\setminus A\in eC(X)\Rightarrow A\in eO(X).$$

$(e)$ Let $\Lambda_e(X)\subset eO(X)$.

$$\begin{aligned}A\in\tau^{\Lambda_e} &\Leftrightarrow (A\subset X)(X\setminus A=C^{\Lambda_e}(X\setminus A))\\ &\Leftrightarrow (A\subset X)(X\setminus A=\bigcap\{U|(X\setminus A\subset U)(U\in\Lambda_e(X))\})\\ &\Leftrightarrow (A\subset X)(X\setminus A=\bigcap\{U|(X\setminus A\subset U)(U\in eO(X))\})\\ &\Leftrightarrow (A\subset X)(X\setminus A=(X\setminus A)^{\Lambda_e})\\ &\Leftrightarrow (A\subset X)(X\setminus A=X\setminus A^{V_e})\\ &\Leftrightarrow (A\subset X)(A=A^{V_e})\\ &\Leftrightarrow A\in\{A|(A\subset X)(A=A^{V_e})\}.\end{aligned}$$

$(f)$ Let $\Lambda_e(X)\subset eC(X)$.

$$\left.\begin{array}{r}A\in\tau^{\Lambda_e}\Rightarrow A\in V_e(X)\Rightarrow X\setminus A\in\Lambda_e(X)\\ \Lambda_e(X)\subset eC(X)\end{array}\right\}\Rightarrow X\setminus A\in eC(X)\Rightarrow A\in eO(X).$$

$$\left.\begin{array}{r}A\in eO(X)\Rightarrow A\in\Lambda_e(X)\\ \Lambda_e(X)\subset eC(X)\end{array}\right\}\Rightarrow A\in eC(X)\Rightarrow A\in V_e(X)\Rightarrow A\in\tau^{\Lambda_e}.$$

**Theorem 5.6.** *Let $(X, \tau)$ be a topological space.*

$$eO(X) = \tau^{\Lambda_e} \Rightarrow \tau^{\Lambda_e} = \mathcal{P}(X).$$

**Proof:** Let $eO(X) = \tau^{\Lambda_e}$.

$$\left.\begin{array}{r} \{x\} \notin eO(X) \Rightarrow \{x\} \in eC(X) \Rightarrow \{x\} \in V_e(X) \Rightarrow \{x\}^{V_e} = \{x\} \Rightarrow \{x\} \in \tau^{\Lambda_e} \\ \{x\} \in eO(X) \Rightarrow \{x\} \in \tau^{\Lambda_e} \end{array}\right\} \Rightarrow$$

$$\Rightarrow (\forall x \in X)(\{x\} \in \tau^{\Lambda_e}) \Rightarrow \tau^{\Lambda_e} = \mathcal{P}(X).$$

# 6 Conclusion

If the open sets and the regular open sets coincide in a topological space, then all of the results which are obtained in this paper are the same in [3] because $eO(X) = BO(X)$ when the open sets and the regular open sets are the same as we stated in Lemma 2.2.

# References

[1] P. Alexandroff, Diskrete raume, *Math. Sb.*, 2(1937), 501-508.

[2] M. Caldas and J. Dontchev, $G.\Lambda_s$-sets and $g.V_s$-sets, *Mem. Fac. Sci. Kochi Univ. (Math.)*, 21(2000), 21-30.

[3] M. Caldas, S. Jafari and T. Noiri, On $\Lambda_b$-sets and the associated topology $\tau^{\Lambda_b}$, *Acta Math. Hungar.*, 100(4) (2006), 337-345.

[4] W. Dunham, $T_{\frac{1}{2}}$ spaces, *Kyungpook Math. J.*, 17(1977), 161-169.

[5] E. Ekici, New forms of contra-continuity, *Carpathian J. Math.*, 24(1) (2008), 37-45.

[6] E. Ekici, On $e$-open sets, $\mathcal{DP}^*$-sets and $\mathcal{DPE}^*$-sets and decompositions of continuity, *Arab. J. Sci. Eng. Sect. A Sci.*, 33(2) (2008), 269-282.

[7] M. Ganster, S. Jafari and T. Noiri, On pre-$\Lambda$-sets and pre-$V$-sets, *Acta Math. Hungar.*, 95(4) (2002), 337-343.

[8] N. Levine, Generalized closed sets in topological spaces, *Rend. Circ. Mat. Palermo*, 19(1970), 89-96.

[9] H. Maki, Generalized $\Lambda$-sets and the associated closure operator, *The Special Issue in Commemoration of Prof. Kazusada IKEDA's Retirement Oct 1*, (1986), 139-146.

[10] A.S. Mashhour, M.E.A. El-Monsef and S.N. El-Deep, On precontinuous and weak precontinuous mappings, *Proc. Math. Phys. Soc., Egypt*, 53(1982), 47-53.

[11] N.V. Veličko, $H$-closed topological spaces, *Am. Math. Soc. Trans.*, 78(1968), 103-108.