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Similarity Flow Solution of MHD Boundary Layer Model for Non-Newtonian Power-Law Fluids over a Continuous Moving Surface

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Abstract

In this paper, the problem of laminar boundary layer flow for non-Newtonian power-law fluid over a continuous moving surface in the presence of transverse magnetic field is studied by appropriate similarity transformation. The governing partial differential equations are transformed into non linear ordinary differential equations using deductive group theoretic method.

Keywords: *Similarity analysis, Deductive group theoretic method, MHD boundary layer, Power-law fluid, transverse magnetic field.*

1 Introduction

Due to the wide applications in several technical and industrial processes, the boundary-layer flows over continuous moving surfaces have attracted researchers in many branches of engineering in recent years. For examples, in the extrusion of polymer sheet from a die, the lamination and melt-spinning process in the extrusion of polymers or the cooling of a large metallic plate in a bath, glass blowing continuous casting and spinning of fibers.

Sakiadis [1-3] studied the boundary layer behavior on a continuous solid surface moving on both flat and the cylindrical surface. Wu [4] presented the effects of suction or injection in a steady two-dimensional MHD boundary layer flow of on a flat plate. Takhar et al [5] obtained MHD asymmetric flow over a semi-infinite moving surface and numerical solution. Erickson et al [6] studied the cooling of a moving continuous flat sheet. Vajravelu and Rollins [7] presented the analysis of heat and mass transfer characteristics in an electrically conducting fluid over a linearly stretching sheet with variable wall temperature. Acrivos et al [8] and Pakdemirli [9] derived the boundary layer equations of power-fluids. Char and Chen [10] studied the temperature field of such fluid over a stretching plate with varied heat flux. Chiam [11] derived MHD boundary layer flow over continuously flat plate. Kumari and Nath [12] presented the problem of MHD boundary layer flow of a non-Newtonian fluid over a continuously moving surface with a parallel free stream while the non-similar solution is obtained by Jeng et al [13].

Recently, the numbers of researchers are motivated towards the problem of MHD boundary layer flow due to their application in areas like chemical engineering, food engineering, petroleum production, and power engineering, nuclear fusion, medicine. Guedda and Hammouch [14] present steady-state laminar boundary layer flow governed by the Ostwald-de Wael power-law model of an incompressible non-Newtonian fluid past a semi-infinite power-law stretched flat plate. Hoernel [15] investigated the similarity solutions for the steady laminar incompressible boundary layer governing MHD flow near forward stagnation-point of two-dimensional and axisymmetric bodies. Amkadni and Azzouzi [16] studied the steady flow of an incompressible electrically conducting fluid over a semi-infinite moving vertical cylinder. Patel and Timol [17] investigated steady, two dimensional laminar incompressible boundary layer flows past a moving continuous flat surface.

2 Governing Equations

Consider the steady, two-dimensional laminar boundary layer flow of power law fluid an electrically conducting, viscous, incompressible fluid past a continuously moving surface passing through with constant velocity U_w in the same or opposite direction to the free stream velocity U_∞. The $x-$axis extends parallel to the plate and $y-$axis perpendicular to the $x-$axis. A magnetic field of uniform strength B_0 is applied in the positive $y-$direction, which produce the magnetic field in the $x-$direction. The boundary layer equations governing the flow in a power-law fluid are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial \tau_{xy}}{\partial y}-\frac{\sigma B_0^2}{\rho}u \tag{2}$$

where u , v are the velocity components along x and y coordinates, τ_{xy} is the shear stress and ρ is the fluid density.

Together with the boundary conditions:

$$\begin{aligned} y=0: &\quad u=U_w,\ \ v=0 \\ y=\infty: &\quad u=U_\infty \end{aligned} \tag{3}$$

We apply power-law relation between the shear stress and the shear rate by

$$\tau_{xy}=K\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}$$

Where $\gamma\left|\frac{\partial u}{\partial y}\right|^{n-1}$ denotes the kinematics viscosity, K is the consistency coefficient, $\gamma=\frac{K}{\rho}$ and n is the power-law index, for $n<1$ pseudo plastic fluids, for $n=1$ the fluid is Newtonian, $n>1$ for dilatant fluids. Then equation (2) becomes

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial}{\partial y}\left(\gamma\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right)-\frac{\sigma B_0^2}{\rho}u \tag{4}$$

Introducing the stream function $\psi(x,y)$ such that $u=\frac{\partial \psi}{\partial y},\quad v=-\frac{\partial \psi}{\partial x}$ which satisfies the continuity equation (1) identically. On the other hand we have

$$\left(\frac{\partial \psi}{\partial y}.\frac{\partial^2 \psi}{\partial y \partial x}-\frac{\partial \psi}{\partial x}.\frac{\partial^2 \psi}{\partial y^2}\right)=\gamma\frac{\partial}{\partial y}\left(\left|\frac{\partial^2 \psi}{\partial y^2}\right|^{n-1}\frac{\partial^2 \psi}{\partial y^2}\right)-\frac{\sigma B_0^2}{\rho}\frac{\partial \psi}{\partial y} \tag{5}$$

And the boundary conditions are

$$\frac{\partial \psi}{\partial y}(x,0)=U_w,\ \ \frac{\partial \psi}{\partial x}(x,0)=0,\ \ \ \frac{\partial \psi}{\partial y}(x,\infty)=U_\infty \tag{6}$$

3 Group Theoretic Treatment

Similarity analysis by the deductive group-theoretic method is derived from theory of continuous group transformations. Birkhoff [18] was introduced the basic concept of this method and later on number of authors has contributed much to the development of the theory. Recently, this theory is found to give more adequate treatment of boundary layer equations (Refer Seshadri and Na [19]). Consider the following transformation:

$$\psi(x,y) = ax^{\alpha} f(\eta), \quad \eta = b\frac{y}{x^{\beta}} \tag{7}$$

Where a, b, α and β are real numbers, η is similarity variable, $f(\eta)$ is the transformed dimensionless stream function.

Applying this similarity variable η we derive

$$\left.\begin{aligned} \psi_x &= ax^{\alpha-1}\left[\alpha f - \eta\beta f'\right] \\ \psi_y &= abf' x^{\alpha-\beta} \\ \psi_{yy} &= ab^2 x^{\alpha-2\beta} f'' \\ \psi_{yx} &= abx^{\alpha-\beta-1}\left[\alpha f' - \beta f' - \beta\eta f''\right] \end{aligned}\right\} \tag{8}$$

Using equation (7) along with the equation (8) into equation (5) we get the transformed non linear ordinary differential of the form.

$$\left(|f''|^{n-1} f''\right)' - Mf' - (\alpha-\beta) f'^2 + \alpha ff'' = 0 \tag{9}$$

if $(2-n)\alpha + (2\mathrm{n}-1)\beta = 1$ and $\gamma \mathrm{a}^{2n-1} b^{n-2} = 1$ holds, where prime denotes differentiation with respect to η .Taking the boundary conditions (6) into consideration we have $ab = U_{\infty}$ and $\alpha - \beta = 0$.

Finally the equation (9) transformed to

$$\left(|f''|^{n-1} f''\right)' - Mf' + \frac{1}{n+1} ff'' = 0 \tag{10}$$

With the transformed boundary conditions:

$$f(0) = 0, \quad f'(0) = \in, \quad f'(\infty) = 1$$

$n = 1$ gives Newtonian fluid, then the equation (10) becomes

$f''' - Mf' + \frac{1}{2} ff'' = 0$ with the boundary conditions $f(0) = 0,\ f'(0) = \in,\ f'(\infty) = 1$

Where $\in = \frac{U_w}{U_\infty}$ is the velocity parameter and $M = \frac{\sigma B_0^2}{\rho U_\infty} x$ is the magnetic parameter. Here we note that when $\in = 0$ the plate is stationary, when $\in < 0$ fluid and plate moves in opposite direction, when $\in > 0$ fluid and plate moves in same direction, for $0 < \in < 1$ the speed of the plate is less than the fluid and for $\in = 1$ the plate and fluid moves with same velocity.

4 Conclusion

Similarity solution of laminar boundary layer flow for non-Newtonian power-law fluid over a continuous moving surface in the presence of transverse magnetic field is investigated. The similarity transformations obtained are unique in their form and the reduced system is in most general form. The governing flow situation transformed may be use for further numerical analysis.

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