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WEAKLY PERIODIC SEQUENCES OF BOUNDED LINEAR TRANSFORMATIONS: A SPECTRAL CHARACTERIZATION

A. R. SOLTANI AND Z. SHISHEBOR

ABSTRACT. Let X and Y be two Hilbert spaces, and $\mathcal{L}(X,Y)$ the space of bounded linear transformations from X into Y. Let $\{A_n\} \subset \mathcal{L}(X,Y)$ be a weakly periodic sequence of period T. Spectral theory of weakly periodic sequences in a Hilbert space is studied by H. L. Hurd and V. Mandrekar (1991). In this work we proceed further to characterize $\{A_n\}$ by a positive measure μ and a number T of $\mathcal{L}(X,X)$-valued functions $a_0,\dots,a_{T-1}$; in the spectral form $A_n = \int_0^{2\pi} e^{-i\lambda n}\Phi(d\lambda)V_n(\lambda)$, where $V_n(\lambda) = \sum_{k=0}^{T-1} e^{-i\frac{2\pi kn}{T}} a_k(\lambda)$ and Φ is an $\mathcal{L}(X,Y)$-valued Borel set function on $[0,2\pi)$ such that

$$(\Phi(\Delta)x, \Phi(\Delta')x')_Y = (x,x')_X\,\mu(\Delta\cap\Delta').$$

1. INTRODUCTION

Let X and Y be Hilbert spaces, and let $\mathcal{L}(X,Y)$ stand for bounded linear transformations from X into Y. Suppose $\{A_n,\ n\in Z\}$, Z is the set of integers, is a sequence in $\mathcal{L}(X,Y)$. The sequence $\{A_n\}$ is said to be *weakly periodic,* if for every $n,m\in\mathbb{Z}$, $(A_nx, A_mx')_Y = (A_{n+T}x, A_{m+T}x')_Y$, $x,x'\in X$, for some $T>0$. The smallest T indicates the period, and in this case we use the notation T-WP. We call the sequence $\{A_n\}\subset\mathcal{L}(X,Y)$ strongly periodic if $A_n = A_{n+T}$, $n\in\mathbb{Z}$. Such a sequence is denoted by T-SP. Every 1-WP is stationary. Spectral theory of T-WP sequences in Hilbert spaces, and in certain Banach spaces, was first studied by H. L. Hurd and V. Mandrekar [1]. If X is finite dimensional and $Y = L^2(\Omega,\mathcal{F},P)$, where $(\Omega,\mathcal{F},P)$ is a probability space, then every T-WP sequence is a periodically correlated second order process. Such a process was first introduced and studied by Gladyshev [2]. For more on these kind of processes see [3], [4].

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The significance of a spectral representation for a given sequence is realized in probability theory. It amounts to considering the elements of the sequence as the Fourier coefficients of a certain (random) measure. Let Ψ be a set function on Borel sets in $[0, 2\pi)$ with values in $\mathcal{L}(X, Y)$. Among the following properties;

(i) for every $x \in X$, $\Psi(\cdot)x$ is an Y-valued measure,

(ii) for every $x, x' \in X$, $(\Psi(\Delta)x,\ \Psi(\Delta^{'})x^{'})_Y = 0$ if $\Delta \cap \Delta^{'} = \varnothing$ and $(\Psi(\cdot)x, \Psi(\cdot)x^{'})_Y$ defines a complex measure on $[0, 2\pi)$;

if (i) is satisfied, then we call Ψ a random spectrum, RS; if (i) and (ii) both are satisfied, then we call Ψ a random spectrum with orthogonal increments, RSOI.

If Ψ is a RS, then

$$B_n = \int_0^{2\pi} e^{-in\lambda}\Psi(d\lambda), \quad n \in \mathbb{Z}, \tag{1.1}$$

is well-defined in the sense that

$$\begin{aligned}(B_n x, B_m x^{'})_Y &= \int_0^{2\pi}\int_0^{2\pi} e^{-i(n\lambda - m\lambda^{'})}(\Psi(d\lambda)x, \Psi(d\lambda^{'})x^{'})_Y = \\ &= \int_0^{2\pi}\int_0^{2\pi} e^{-i(n\lambda - m\lambda^{'})}\Gamma_{x,x'}(d\lambda, d\lambda^{'}),\end{aligned}$$

where $\Gamma_{x,x'}(\cdot,\cdot)$ is a bimeasure on $[0, 2\pi) \times [0, 2\pi)$. Any sequence $\{B_n\} \subset \mathcal{L}(X, Y)$ given by (1.1) is called harmonizable [4]. Every harmonizable sequence in $\mathcal{L}(X, Y)$, for which its Ψ also satisfies (ii), is a stationary sequence [5]. By using a certain Tth root of a unitary operator Hurd and Mandrekar [1] provided the Gladyshev's type representation for a T-WP sequence in a Hilbert space. It can be deduced from their work that $\{A_n\} \subset \mathcal{L}(X, Y)$ is a T-WP sequence if and only if $A_n = \int_0^{2\pi} e^{-in\lambda}\mathcal{Z}(d\lambda)$, $n \in \mathbb{Z}$, where $\mathcal{Z}$ is a RS on $[0, 2\pi)$, that its restriction to each $[\frac{2\pi p}{T}, \frac{2\pi(p+1)}{T})$, $p = 0, \dots, T-1$, is a RSOI; more precisely,

$$\mathcal{Z}(\Delta) = \sum_{p=0}^{T-1} W_p\Big(\Delta \bigcap \Big[\frac{2\pi p}{T}, \frac{2\pi(p+1)}{T}\Big)\Big), \tag{1.2}$$

$$\Big(W_p\Big(\Delta + \frac{2\pi p}{T}\Big)x,\ W_q\Big(\Delta' + \frac{2\pi q}{T}x'\Big)\Big)_Y = 0$$

for every $\Delta, \Delta' \subseteq [0, \frac{2\pi}{T})$ with $\Delta \cap \Delta' = \varnothing$, and every $p, q = 0, \dots, T-1$. This representation was also independently derived by the authors in [6] by using a different approach.

In this work we proceed further to identify each T-WP sequence $\{A_n\}$ in $\mathcal{L}(X, Y)$ with a positive measure μ as a number T of $\mathcal{L}(X, X)$-valued functions, $a_0, \dots, a_{T-1}$. The identification is given in Theorem 3.4, which is a refinement of Gladyshev's result on periodically correlated sequences. The relationship between the functions $a_0, \dots, a_{T-1}$ and the matrix f, given in (2.3), introduced by Gladyshev is also provided in Theorem 3.4. An application of our representation to the regularity of periodically correlated sequences is as follows.

A periodically correlated sequence X_n, $n \in \mathbb{Z}$, is regular, i.e., $\bigcap_n \overline{\text{span}}\{X_t, t \leq n\} = \{0\}$, if and only if $\int_0^{2\pi/T} \log\det\ f(\lambda)\ d\lambda > -\infty$, with f as in (2.3) (see [2]). It follows from (3.3) that $\det f(\lambda) = \prod_{j=0}^{T-1} a_0\Big(\lambda + \frac{2\pi j}{T}\Big)^2$, $\lambda \in [0, 2\pi/T)$. Therefore the sequence is regular if and only if $\int_0^{2\pi} \log a_0(\lambda)\ d\lambda > -\infty$. Theorem 3.1 in [3] easily follows from this observation, as f_0 in [3] dominates a_0.

We summarize our characterization by the spectral representation $A_n = \int_0^{2\pi} e^{-in\lambda}\Phi(d\lambda)V_n(\lambda)$, where, Φ is a certain RSOI, as in the abstract, and for each $\lambda \in [0, 2\pi)$, $\{V_n(\lambda)\} \subset \mathcal{L}(X, X)$ is a T-SP in n, as is defined in Theorem 3.4.

2. Notation and Preliminaries

Suppose $\mathcal{Z}$ is given by (1.4); let F be the $\mathcal{L}(X, X)$-valued Borel set function on $[0, 2\pi) \times [0, 2\pi)$ for which

$$(x, F(\Delta, \Delta')x')_X = (\mathcal{Z}(\Delta)x, \mathcal{Z}(\Delta')x')_Y, \quad x, x' \in X. \tag{2.1}$$

We call F the spectral measure of the sequence $\{A_n\}$. Note that the support of F lies on the lines d_k: $s - t = \frac{2\pi k}{T}$, $k = -T+1, \dots, T-1$. Furthermore $(A_n x, A_m x')_Y = \int_0^{2\pi}\int_0^{2\pi} e^{-int+ims}(x, F(dt, ds)x')_X$, $x, x' \in X$. Let F_k be the restriction of F to the line d_k, $k = -T+1, \dots, T-1$. Let μ be a positive finite measure for which each F_k is μ-continuous and

$$\mu_{\frac{2\pi k}{T}}(dx) = \mu(dx) \ \text{ on } \ [0, 2\pi/T), \ \ k = 1-T, \dots, T-1, \tag{2.2}$$

and $(x, F_k(\Delta)x^{'}) = \int_\Delta (x, f_k(s)x^{'})\mu(ds)$, where f_k are $\mathcal{L}(X, X)$-valued functions on $[0, 2\pi)$, $k = 1-T, \dots, T-1$, [7].

To see the existence of such a measure, note that corresponding to each F_k there is a positive finite measure μ_k such that F_k is μ_k-continuous [8]. Now let $\nu = \sum_{k=1-T}^{T-1} \mu_k$, and $\nu_{\frac{2\pi k}{T}}(E) = \nu((E + \frac{2\pi k}{T}) \cap [0, 2\pi))$, then define $\mu(E) = \sum_{j=1}^{T}\sum_{k=1-T}^{T-1} \nu_{\frac{2\pi k}{T}}(E \cap [\frac{2\pi(j-1)}{T}, \frac{2\pi j}{T}))$.

Now

$$f(s) = \Big[f_{p-l}\Big(s + \frac{2\pi p}{T}\Big)\ \Big]_{l,p=0,\dots,T-1}, \quad s \in \Big[0, \frac{2\pi}{T}\Big), \tag{2.3}$$

defines an operator-matrix; we refer to a matrix whose entries are operators as an *operator-matrix.* We call f the spectral density of $\{A_n\}$ with respect to μ. Recall that an operator-matrix $T = [T_{i,j}]$ $i, j = 0, \ldots, T-1$ is called positive definite if and only if the matrix $[(x_{kj}, T_{j,l}x_{k_l})]_{j,l=0,\ldots,T-1}$, is positive definite for each $x_{k_0}, \ldots, x_{k_{T-1}}$. We recall that $L_2(\mu, X)$ stands for μ-Bochner integrable functions $g : (0, 2\pi] \to X$ such that $\int_0^{2\pi} \|g(\lambda)\|_X^2 \mu(d\lambda) < \infty$ the proof of which is omitted [9].

3. Spectral Characterization

H_n integral (stochastic integral) of the form $\int_0^{2\pi} \Phi(d\lambda)g(\lambda)$, where $g \in L_2(\mu, X)$ and Φ is the RSOI given below in (3.1), can be defined by using the well known result on a existence of the weak stochastic integral. The detail is given in the following theorem. For more on a stochastic integral we refer the readers to [10] and [1].

Theorem 3.1. *Let μ be a positive finite Borel measure on $[0, 2\pi)$ and Φ be a RSOI for which*

$$(\Phi(\Delta)x, \Phi(\Delta^{'})x^{'})_Y = (x, x')_X \mu(\Delta \cap \Delta^{'}). \tag{3.1}$$

Then for every $g \in L_2(\mu, X)$, $\int_0^{2\pi} \Phi(d\lambda)g(\lambda)$ is well defined as an element in Y and

$$\left\| \int_0^{2\pi} \Phi(d\lambda)g(\lambda) \right\|_Y = \left\{ \int_0^{2\pi} \|g(\lambda)\|_X^2 \mu(d\lambda) \right\}^{\frac{1}{2}}.$$

Moreover, the integral satisfies the following properties:

(i) $\int_0^{2\pi} \Phi(d\lambda)(g_1(\lambda)+g_2(\lambda)) = \int_0^{2\pi} \Phi(d\lambda)g_1(\lambda)+\int_0^{2\pi} \Phi(d\lambda)g_2(\lambda)$, $g_1, g_2 \in L_2(\mu, X)$;

(ii) *if $g_n \to g$ in $L_2(\mu, X)$, then $\int_0^{2\pi} \Phi(d\lambda)g_n(\lambda) \to \int_0^{2\pi} \Phi(d\lambda)g(\lambda)$ in Y as $n \to \infty$.*

Corollary 3.2. *Let μ and Φ be as in Theorem 3.1. Let*

$$\mathcal{A} = \left\{ a;\ a : [0, 2\pi) \to \mathcal{L}(X, X),\ \int \|a(\lambda)x\|_X^2 \mu(d\lambda) < \infty,\ x \in X \right\}.$$

Then for every $a \in \mathcal{A}$, $x \in X$, $\int_0^{2\pi} \Phi(d\lambda)a(\lambda)x$ is well defined as an element in Y, and

$$\left\| \int_0^{2\pi} \Phi(d\lambda)a(\lambda)x \right\|_Y = \left\{ \int_0^{2\pi} \|a(\lambda)x\|_X^2 \mu(d\lambda) \right\}^{\frac{1}{2}}.$$

Moreover, for every $x, x' \in X$ and every $a, b \in \mathcal{A}$,

(i) $\int_0^{2\pi} \Phi(d\lambda)(a(\lambda) + b(\lambda))x = \int_0^{2\pi} \Phi(d\lambda)a(\lambda)x + \int_0^{2\pi} \Phi(d\lambda)b(\lambda)_x$,

(ii) $\int_0^{2\pi} \Phi(d\lambda)a(\lambda)(x + x^{'}) = \int_0^{2\pi} \Phi(d\lambda)a(\lambda)x + \int_0^{2\pi} \Phi(d\lambda)a(\lambda)x^{'}$,

(iii) *for a given $x \in X$, if $a_n(\cdot)x \to a(\cdot)x$ in $L_2(\mu, X)$, then $\int_0^{2\pi} \Phi(d\lambda)a_n(\lambda)x \to \int_0^{2\pi} \Phi(d\lambda)a(\lambda)x$ in Y.*

Proof. For each fix $x \in X$, $a(\cdot)x$ is an X-valued function. If $a \in \mathcal{A}$, then $a(\cdot)x \in L_2(\mu, X)$. Now apply Theorem 3.1 and use the linear property of $a(\cdot)$. $\square$

The following lemma concerns the Cholesky decomposition for a positive definite operator-matrix (Lemma 3.5 in [11]) and it plays a crucial role in our characterization.

Lemma 3.3. *Let M_n be an $n \times n$ positive operator-matrix, then $M_n = U_n^* U_n$, where U_n is an $n \times n$ upper triangular matrix.*

The following theorem is the essential result of this paper.

Theorem 3.4. *Let $\{A_n\} \subset \mathcal{L}(X, Y)$. Then $\{A_n\}$ is a T-WP if and only if*

$$A_n = \int_0^{2\pi} e^{-ins} \Phi(ds) V_n(s), \tag{3.2}$$

where

(i) *Φ is a RSOI that satisfies* (3.1) *and μ is a finite measure satisfying* (2.2)*;*

(ii) $V_n(s) = \sum_{k=0}^{T-1} e^{-i\frac{2\pi kn}{T}} a_k(s + \frac{2\pi k}{T})$, $s \in [0, 2\pi)$, $n \in \mathbb{Z}$, *where each a_k is an $\mathcal{L}(X,X)$-valued function on $[0, 2\pi)$ with $a_k(s) = 0$ for $s \in [0, \frac{2\pi k}{T})$.*

Furthermore, the triangular operator-matrix $A(x) = [a^*_{j-k}(x + \frac{2\pi j}{T})]_{k \le j}$, $k, j = 0, \dots, T-1$, satisfies

$$f(x) = A^*(x) A(x), \quad x \in \left[0, \frac{2\pi}{T}\right), \tag{3.3}$$

i.e., it is the Cholesky decomposition of the density $f(x)$ given by (2.3)*, $a^*(x)$ stands for the adjoint of $a(x)$.*

Proof. Let $\{A_n\}$ be a sequence for which (3.2) holds, i.e.,

$$A_n = \sum_{k=0}^{T-1} \int_0^{2\pi} e^{-ins} \Phi_{\frac{-2\pi k}{T}}(ds) a_k(s). \tag{3.4}$$

Thus

$$(A_n x,\ A_m x')_Y =$$

$$\sum_{k=0}^{T-1} \sum_{l=0}^{T-1} \int_0^{2\pi} \int_0^{2\pi} e^{-ins+imt} (\Phi_{\frac{-2\pi k}{T}}(ds) a_k(s) x, \Phi_{\frac{-2\pi l}{T}}(dt) a_l(t) x')_Y. \tag{3.5}$$

It also follows from (3.1) that

$$(\Phi_{\frac{-2\pi k}{T}}(ds)a_k(s)x,\ \Phi_{\frac{-2\pi l}{T}}(dt)a_l(t)x')_Y =$$
$$= \begin{cases} (a_k(s)x,\ a_{p+k}(s+\frac{2\pi p}{T})x')_X\mu_{\frac{-2\pi p}{T}}(ds), & s-\frac{2\pi k}{T}=t-\frac{2\pi l}{T} \\ 0, & \text{otherwise} \end{cases},$$

where $p = l-k$. Since $\mu_{\frac{-2\pi p}{T}}(ds) = \mu(ds)$, $p = 1-T,\dots,T-1$. We obtain

$$(A_nx, A_mx')_Y =$$
$$= \sum_{p=-(T-1)}^{-1}\int_0^{2\pi} e^{-ins+im(s+\frac{2\pi p}{T})}\sum_{k=-p}^{T-1}\big(a_k(s)x, a_{k+p}(s+2\pi p/T)x'\big)_X\mu(ds)+$$
$$+\int_0^{2\pi} e^{-ins+ims}\sum_{k=0}^{T-1}(a_k(s)x,\ a_k(s)x')_X\mu(ds)+$$
$$+\sum_{p=1}^{T-1}\int_0^{2\pi} e^{-ins+im(s+\frac{2\pi p}{T})}\sum_{k=0}^{T-1-p}\big(a_k(s)x, a_{k+p}(s+2\pi p/T)x'\big)_X\mu(ds). \quad (3.6)$$

It is clear from (3.6) that $\{A_n\}$ is a T-WP sequence. Conversely, let $\{A_n\}$ be a T-WP sequence; then it follows from (2.1) that

$$\begin{aligned}
(A_nx, A_mx')_Y &= \int_0^{2\pi}\int_0^{2\pi} e^{-ins+imt}(\mathcal{Z}(ds)x,\ \mathcal{Z}(dt)x')_Y = \\
&= \int_0^{2\pi}\int_0^{2\pi} e^{-ins+imt}(x, F(ds,dt)x')_X = \\
&= \sum_{p=0}^{T-1}\sum_{l=0}^{T-1}\int_{\frac{2\pi p}{T}}^{\frac{2\pi(p+1)}{T}}\int_{\frac{2\pi l}{T}}^{\frac{2\pi(l+1)}{T}} e^{-ins+imt}(x, F(ds,dt)x')_X = \\
&= \sum_{p=0}^{T-1}\sum_{l=0}^{T-1}\int_{\frac{2\pi p}{T}}^{\frac{2\pi(p+1)}{T}} e^{-i(n-m)s-i\frac{2\pi(p-l)m}{T}}(x, F_{p-l}(ds)y)_X = \\
&= \sum_{k=-T+1}^{-1}\sum_{p=0}^{T+k-1}\int_{\frac{2\pi p}{T}}^{\frac{2\pi(p+1)}{T}} e^{-i(n-m)s-i\frac{2\pi km}{T}}(x, f_k(ds)y)_X\mu(ds)+ \\
&+ \sum_{p=0}^{T-1}\int_{\frac{2\pi p}{T}}^{\frac{2\pi(p+1)}{T}} e^{-i(n-m)s}(x, f_0(ds)y)_X\mu(ds)+ \\
&+ \sum_{k=1}^{T-1}\sum_{p=k}^{T-l}\int_{\frac{2\pi p}{T}}^{\frac{2\pi(p+1)}{T}} e^{-i(m-n)s-i\frac{2\pi km}{T}}(x, f_k(ds)y)_X\mu(ds) =
\end{aligned}$$

$$= \sum_{k=-T+1}^{-1} e^{-i\frac{2\pi mk}{T}} \int_0^{\frac{2\pi(T+k)}{T}} e^{-i(n-m)s}(x, f_k(ds)y)_X \mu(ds) +$$
$$+ \int_0^{2\pi} e^{-i(n-m)s}(x, f_0(ds)y)_X \mu(ds) +$$
$$+ \sum_{k=1}^{T-1} e^{-i\frac{2\pi mk}{T}} \int_{\frac{2\pi k}{T}}^{2\pi} e^{-i(n-m)s}(x, f_k(ds)y)_X \mu(ds).$$

By using (2.3) and comparing the last equality with (3.6) we obtain that if $a_k,\ k = 1-T, \ldots, T-1$, satisfy

$$\begin{aligned}
&\sum_{k=-p}^{T-1} \Big(a_k(s)x, a_{p+k}\Big(s + \frac{2\pi p}{T}\Big)x'\Big)_X = (x, f_{-p}(s)x')_X,\ p = -T+1, \ldots, -1,\\
&\sum_{k=0}^{T-1} (a_k(s)x, a_k(s)x')_X = (x, f_0(s)x')_X, \qquad (3.7)\\
&\sum_{k=0}^{T-1-p} \Big(a_k(s)x, a_{k+p}\Big(s + \frac{2\pi p}{T}\Big)x'\Big)_X = (x, f_{-p}(s)x')_X,\ p = 1, \ldots, T-1,
\end{aligned}$$

then $\{A_n\}$ will satisfy (3.2), as (3.2), (3.4), (3.5) and (3.6) are equivalent. But it easily follows that (3.7) and (3.3) are identical. On the other hand, (3.3) follows by applying Lemma 3.3 to f. □

Remark 3.5. We were led by the referee to the representation that $\{A_n\} \subset \mathcal{L}(X, Y)$ is T-WP if and only if

$$A_n = \int_0^{2\pi} e^{-i\lambda n} \Psi(d\lambda) D_n, \qquad (3.8)$$

where Ψ is an $\mathcal{L}(X, Y)$-valued measure and $\{D_n\} \subset \mathcal{L}(X, X)$ is a T-SP sequence. In (3.8) Ψ lacks property (3.1), but instead each $\{D_n\}$ is independent of λ.

Let us provide the referee's proof which is a nice application of the symbolic calculus to unitary operators.

Proof. Since $\{A_n\}$ is T-weakly periodic, $A_{n+T} = UA_n$ for some unitary U in Y and each n and T. We define $B_n = U^{-n/T}A_n$. It is easy to make sure that $\{B_n\} \subset \mathcal{L}(X, Y)$ is T-strongly periodic and $A_n = U^{n/T}B_n$. Writing the spectral representation for the unitary operator $U^{-1/T}$ we get $A_n = \int_0^{2\pi} e^{-i\lambda n}E(d\lambda)B_n$, where $E(d\lambda)$ is a solution of the identity of Y. Now if X is infinite-dimensional, one can establish an isomorphism (without loss of generality one can assume that X and Y are separable) $j \in \mathcal{L}(Y, X)$ and we come to (3.8) with $\Psi(d\lambda) = E(d\lambda)j^{-1}$ and $D_n = jB_n$. □

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Authors' addresses:

Center for Theoretical Physics
and Mathematics AEOI
P.O. Box 11365-8486, Tehran
Iran

Department of Statistics
Shiraz University
Shiraz 71454
Iran