

## OSCILLATION AND NONOSCILLATION CRITERIA FOR SECOND-ORDER LINEAR DIFFERENTIAL EQUATIONS

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ABSTRACT. Sufficient conditions for oscillation and nonoscillation of second-order linear equations are established.

### 1. STATEMENT OF THE PROBLEM AND FORMULATION OF BASIC RESULTS

Consider the differential equation

$$u'' + p(t)u = 0, \quad (1)$$

where  $p : [0, +\infty[ \rightarrow [0, +\infty[$  is an integrable function. By a solution of equation (1) is understood a function  $u : [0, +\infty[ \rightarrow ]-\infty, +\infty[$  which is locally absolutely continuous together with its first derivative and satisfies this equation almost everywhere.

Equation (1) is said to be oscillatory if it has a nontrivial solution with an infinite number of zeros, and nonoscillatory otherwise.

It is known (see [1]) that if for some  $\lambda < 1$  the integral  $\int^{+\infty} s^\lambda p(s) ds$  diverges, then equation (1) is oscillatory. Therefore, we shall always assume below that

$$\int^{+\infty} s^\lambda p(s) ds < +\infty \quad \text{for } \lambda < 1.$$

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Introduce the notation

$$\begin{aligned}
 h_\lambda(t) &= t^{1-\lambda} \int_t^\infty s^\lambda p(s) ds \quad \text{for } t > 0 \quad \text{and } \lambda < 1, \\
 h_\lambda(t) &= t^{1-\lambda} \int_1^t s^\lambda p(s) ds \quad \text{for } t > 0 \quad \text{and } \lambda > 1, \\
 p_*(\lambda) &= \liminf_{t \rightarrow +\infty} h_\lambda(t), \quad p^*(\lambda) = \limsup_{t \rightarrow +\infty} h_\lambda(t).
 \end{aligned} \tag{2}$$

In [1] it is proved that equation (1) is oscillatory if  $p^*(0) > 1$  or  $p_*(0) > \frac{1}{4}$ , and nonoscillatory if  $p^*(0) < \frac{1}{4}$ . The oscillation criteria for equation (1) written in terms of the numbers  $p_*(\lambda)$  and  $p^*(\lambda)$  have been established in [2]. Below we shall give the sufficient conditions for oscillation and nonoscillation of equation (1) which make the above-mentioned results of papers [1, 2] more precise and even extend them in some cases.

First of all, for the completeness of the picture we give a proposition, which slightly generalizes one of E. Hille's theorems [1].

**Proposition.** *Let either  $p_*(0) > \frac{1}{4}$  or  $p_*(2) > \frac{1}{4}$ . Then equation (1) is oscillatory.*

**Theorem 1.** *Let  $p_*(0) \leq \frac{1}{4}$  and  $p_*(2) \leq \frac{1}{4}$ . Moreover, let either*

$$p^*(\lambda) > \frac{\lambda^2}{4(1-\lambda)} + \frac{1}{2} \left( 1 + \sqrt{1 - 4p_*(2)} \right) \tag{3}$$

for some  $\lambda < 1$  or

$$p^*(\lambda) > \frac{\lambda^2}{4(\lambda-1)} - \frac{1}{2} \left( 1 - \sqrt{1 - 4p_*(0)} \right) \tag{4}$$

for some  $\lambda > 1$ . Then equation (1) is oscillatory.

**Corollary 1.** *Let either*

$$\lim_{\lambda \rightarrow 1^-} (1-\lambda)p^*(\lambda) > \frac{1}{4} \tag{5}$$

or

$$\lim_{\lambda \rightarrow 1^+} (\lambda-1)p^*(\lambda) > \frac{1}{4} \tag{6}$$

Then equation (1) is oscillatory.

**Corollary 2 ([2]).** For some  $\lambda \neq 1$  let

$$|1 - \lambda|p_*(\lambda) > \frac{1}{4} \quad (7)$$

Then equation (1) is oscillatory.

*Remark 1.* Inequalities (5)–(7) are exact and cannot be weakened. Indeed, let  $p(t) = \frac{1}{4t^2}$  for  $t \geq 1$ . Then  $|1 - \lambda|p_*(\lambda) = \frac{1}{4}$ , and equation (1) has oscillatory solution  $u(t) = \sqrt{t}$  for  $t > 1$ .

**Theorem 2.** Let  $p_*(0) \leq \frac{1}{4}$  and  $p_*(2) \leq \frac{1}{4}$ . Moreover, let either

$$\begin{aligned} p_*(0) &> \frac{\lambda(2 - \lambda)}{4} \quad \text{and} \\ p^*(\lambda) &> \frac{p_*(0)}{1 - \lambda} + \frac{1}{2} \left( \sqrt{1 - 4p_*(0)} + \sqrt{1 - 4p_*(2)} \right) \end{aligned} \quad (8)$$

for some  $\lambda < 1$  or

$$\begin{aligned} p_*(2) &> \frac{\lambda(2 - \lambda)}{4} \quad \text{and} \\ p^*(\lambda) &> \frac{p_*(2)}{\lambda - 1} + \frac{1}{2} \left( \sqrt{1 - 4p_*(0)} + \sqrt{1 - 4p_*(2)} \right) \end{aligned} \quad (9)$$

for some  $\lambda > 1$ . Then equation (1) is oscillatory.

**Theorem 3.** Let  $p_*(0) \neq 0$  and  $p_*(2) \leq \frac{1}{4}$ . Moreover, for some  $0 < \lambda < 1$  let  $p_*(\lambda) < \frac{1 - \lambda^2}{4}$  and either

$$p_*(\lambda) > \frac{p_*(0)}{1 - \lambda} + \frac{\lambda}{2(1 - \lambda)} \left( \sqrt{1 - 4p_*(0)} + \sqrt{1 - 4p_*(2)} \right)$$

and

$$\begin{aligned} p^*(\lambda) &> p_*(\lambda) + \frac{1}{2} \left( \lambda + \sqrt{1 - 4p_*(2)} \right) + \\ &+ \sqrt{\lambda^2 + 1 - 4(1 - \lambda)p_*(\lambda) + 2\lambda\sqrt{1 - 4p_*(2)}} \end{aligned}$$

or

$$p_*(\lambda) < \frac{p_*(0)}{1 - \lambda} + \frac{\lambda}{2(1 - \lambda)} \left( \sqrt{1 - 4p_*(0)} + \sqrt{1 - 4p_*(2)} \right)$$

and

$$p^*(\lambda) > \frac{p_*(0)}{1 - \lambda} + \frac{1}{2(1 - \lambda)} \left( \sqrt{1 - 4p_*(0)} + \sqrt{1 - 4p_*(2)} \right). \quad (10)$$

Then equation (1) is oscillatory.

**Theorem 3'.** Let  $p_*(0) \leq \frac{1}{4}$  and  $p_*(2) \leq \frac{1}{4}$ . Moreover, for some  $0 < \lambda < 1$  let condition (10) be fulfilled, and let  $p_*(0) > \frac{1-\lambda^2}{4}$ . Then equation (1) is oscillatory.

**Corollary 3.** Let  $p_*(0) \leq \frac{1}{4}$ ,  $p_*(2) \leq \frac{1}{4}$  and

$$p^*(0) > p_*(0) + \frac{1}{2} \left( \sqrt{1 - 4p_*(0)} + \sqrt{1 - 4p_*(2)} \right).$$

Then equation (1) is oscillatory.

**Corollary 4.** For some  $\lambda \in [0, \frac{1}{4}[$  let

$$\frac{\lambda}{1-\lambda} < p_*(\lambda) < \frac{1}{4(1-\lambda)}$$

and

$$p^*(\lambda) > 1 + p_*(\lambda) - \frac{1}{2} \left( 1 - \lambda - \sqrt{(1+\lambda)^2 - 4(1-\lambda)p_*(\lambda)} \right).$$

Then equation (1) is oscillatory.

**Theorem 4.** For some  $\lambda \neq 1$  let

$$p^*(\lambda) > \frac{(2\lambda-1)(3-2\lambda)}{4|1-\lambda|} \quad \text{and} \quad p^*(\lambda) < \frac{1}{4|1-\lambda|}. \quad (11)$$

Then equation (1) is nonoscillatory.

*Remark 2.* As will be seen from the proof, Theorem 4 remains also valid when the function  $p$ , generally speaking, does not have a constant sign. For such a case this result for  $\lambda = 0$  is described in [3].

**Corollary 5.** Let  $p_*(0) < \frac{1}{4}$  and  $p_*(2) < \frac{1}{4}$ , and let the inequality

$$p^*(\lambda) < \frac{1}{4|1-\lambda|}$$

hold for some  $\lambda \in ] -\infty, 1 - \sqrt{\frac{1}{4} - p_*(0)}[ \cup ] 1 + \sqrt{\frac{1}{4} - p_*(2)}, +\infty[$ . Then equation (1) is nonoscillatory.

2. SOME AUXILIARY PROPOSITIONS

**Lemma 1.** *For equation (1) to be nonoscillatory, it is necessary and sufficient that for some  $\lambda \neq 1$  the equation*

$$v'' = \frac{1}{t^2} \left( -h_\lambda^2(t) + \lambda \operatorname{sgn}(1 - \lambda) h_\lambda(t) \right) v - \frac{2 \operatorname{sgn}(1 - \lambda)}{t} h_\lambda(t) v', \quad (12)$$

where  $h_\lambda$  is the function defined by (2), be nonoscillatory.

*Proof.* The equality  $\rho(t) = t^\lambda \frac{u'(t)}{u(t)} - t^{\lambda-1} h_\lambda(t) \operatorname{sgn}(1 - \lambda)$  determines the relation between the nonoscillatory solution  $u$  of equation (1) and the solution  $\rho$ , defined in some neighborhood of  $+\infty$ , of the equation

$$\begin{aligned} \rho' &= -t^{-\lambda} \rho^2 + \lambda t^{-1} \rho - 2 \operatorname{sgn}(1 - \lambda) t^{-1} h_\lambda^2(t) \rho - \\ &\quad - t^{\lambda-2} h_\lambda^2(t) + \lambda \operatorname{sgn}(1 - \lambda) t^{\lambda-1} h_\lambda(t). \end{aligned} \quad (13)$$

On the other hand, the equality  $\rho(t) = t^\lambda \frac{v'(t)}{v(t)}$  determines the relation between the nonoscillatory solution  $v$  of equation (11) and the solution  $\rho$  defined in some neighborhood  $+\infty$  of equation (13). Thus nonoscillation of either of equation (1) or (12) results in nonoscillation of the other.  $\square$

**Lemma 2.** *Let equation (1) be nonoscillatory. Then there exists  $t_0 > 0$  such that the equation*

$$\rho' + p(t)\rho + \rho^2 = 0 \quad (14)$$

has a solution  $\rho : ]t_0, +\infty[ \rightarrow ]0, +\infty[$ ; moreover,

$$\rho(t_0+) = +\infty, (t - t_0)\rho(t) < 1 \quad \text{for } t_0 < t < +\infty, \quad (15)$$

$$\lim_{t \rightarrow +\infty} t^\lambda \rho(t) = 0 \quad \text{for } \lambda < 1 \quad (16)$$

and

$$\liminf_{t \rightarrow +\infty} t\rho(t) \geq A, \quad \limsup_{t \rightarrow +\infty} t\rho(t) \leq B, \quad (17)$$

where

$$A = \frac{1}{2} \left( 1 - \sqrt{1 - 4p_*(0)} \right), \quad B = \frac{1}{2} \left( 1 + \sqrt{1 - 4p_*(2)} \right)^1. \quad (18)$$

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<sup>1</sup>Since equation (1) is nonoscillatory, we have  $p_*(0) \leq \frac{1}{4}$  and  $p_*(2) \leq \frac{1}{4}$ .

*Proof.* Since equation (1) is nonoscillatory, there exists  $t_0 > 0$  such that the solution  $u$  of equation (1) under the initial conditions  $u(t_0) = 0$ ,  $u'(t_0) = 1$  satisfies the inequalities

$$u(t) > 0, \quad u'(t) \geq 0 \quad \text{for } t_0 < t < +\infty.$$

Clearly, the function  $\rho(t) = \frac{u'(t)}{u(t)}$  for  $t_0 < t < +\infty$  is the solution of equation (14), and  $\lim_{t \rightarrow t_0+} \rho(t) = +\infty$ . From (14) we have

$$\frac{-\rho'(t)}{\rho^2(t)} > 1 \quad \text{for } t_0 < t < +\infty.$$

Integrating the above inequality from  $t_0$  to  $t$ , we obtain  $(t - t_0)\rho(t) < 1$  for  $t_0 < t < +\infty$ . In particular, equality (16) holds for any  $\lambda < 1$ .

Let us now show that inequalities (17) are valid. Assume  $p_*(0) \neq 0$  and  $p_*(2) \neq 0$  (inequalities (17) are trivial, otherwise). Let us introduce the notation

$$r = \liminf_{t \rightarrow +\infty} t\rho(t), \quad R = \limsup_{t \rightarrow +\infty} t\rho(t).$$

From (14) we easily find that for any  $t_1 > t_0$

$$\begin{aligned} t\rho(t) &= t \int_t^{+\infty} p(s)ds + t \int_t^{+\infty} \rho^2(s)ds, \\ t\rho(t) &= \frac{t_1^2 \rho(t_1)}{t} - t^{-1} \int_{t_1}^t s^2 p(s)ds + t^{-1} \int_{t_1}^t s\rho(s)(2 - s\rho(s))ds \end{aligned} \quad (19)$$

for  $t_1 < t < +\infty$ .

This implies that  $r \geq p_*(0)$  and  $R \leq 1 - p_*(2)$ .

It is easily seen that for any  $0 < \varepsilon < \min\{r, 1 - R\}$  there exists  $t_\varepsilon > t_1$  such that

$$\begin{aligned} r - \varepsilon < t\rho(t) < R + \varepsilon, \quad t \int_t^{+\infty} p(s)ds > P_*(0) - \varepsilon, \\ \frac{1}{t} \int_{t_1}^t s^2 p(s)ds > p_*(2) - \varepsilon \quad \text{for } t_\varepsilon < t < +\infty. \end{aligned}$$

Taking into account the above argument, from (19) we have

$$\begin{aligned} t\rho(t) &> p_*(0) - \varepsilon + (r - \varepsilon)^2 \quad \text{for } t_\varepsilon < t < +\infty, \\ t\rho(t) &< \frac{t_\varepsilon^2 \rho(t_\varepsilon)}{t} - p_*(2) + \varepsilon + (R + \varepsilon)(2 - R - \varepsilon) \quad \text{for } t_\varepsilon < t < +\infty, \end{aligned}$$

whence

$$r \geq p_*(0) + r^2, \quad R \leq -p_*(2) + R(2 - R),$$

that is,  $r \geq A$  and  $R \leq B$ , where  $A$  and  $B$  are the numbers defined by equalities (18). Hence (17) holds.  $\square$

**Lemma 3.** *Let the functions  $g, q : [a, +\infty[ \rightarrow \mathbb{R}$  be integrable in every finite interval, and let  $v : [a, +\infty[ \rightarrow ]0, +\infty[$  be absolutely continuous together with its first derivative on every compactum contained in  $[a, +\infty[$ . Moreover, let the inequality*

$$v''(t) \leq g(t)v(t) + q(t)v'(t) \tag{20}$$

hold almost everywhere in  $[a, +\infty[$ . Then the equation  $u'' = g(t)u + q(t)u'$  is nonoscillatory.

### 3. PROOF OF THE BASIC RESULTS

*Proof of Theorem 1.* Assume the contrary. Let equation (1) be nonoscillatory. Then, according to Lemma 2, equation (14) has the solution  $\rho : ]t_0, +\infty[ \rightarrow ]0, +\infty[$  satisfying conditions (15)–(17). Suppose  $\lambda < 1$  ( $\lambda > 1$ ). Because of (17) we have that for any  $\varepsilon > 0$  there exists  $t_\varepsilon > t_0$  such that

$$A - \varepsilon < t\rho(t) < B + \varepsilon \quad \text{for } t_\varepsilon < t < +\infty. \tag{21}$$

Multiplying equality (14) by  $t^\lambda$ , integrating it from  $t$  to  $+\infty$  (from  $t_\varepsilon$  to  $t$ ), and taking into account (15)–(17), we get

$$\begin{aligned} \int_t^{+\infty} s^\lambda p(s) ds &= t^\lambda \rho(t) + \frac{\lambda^2 t^{\lambda-1}}{4(1-\lambda)} - \int_t^{+\infty} (s^{\frac{\lambda}{2}} \rho(s) - \frac{1}{2} s^{\frac{\lambda}{2}-1})^2 ds < \\ &< t^{\lambda-1} \left( B + \varepsilon + \frac{\lambda^2}{4(1-\lambda)} \right) \quad \text{for } t_\varepsilon < t < +\infty \\ \left( \int_{t_\varepsilon}^t s^\lambda p(s) ds &< t^{\lambda-1} \left( \frac{\lambda^2}{4(1-\lambda)} - A + \varepsilon + t_\varepsilon^\lambda \rho(t_\varepsilon) \right) \right. \\ &\left. \text{for } t_\varepsilon < t < +\infty \right), \end{aligned}$$

whence we have  $p^*(\lambda) \leq \frac{\lambda^2}{4(1-\lambda)} + \frac{1}{2}(1 + \sqrt{1 - 4p_*(2)})$  ( $p^*(\lambda) \leq \frac{\lambda^2}{4(\lambda-1)} - \frac{1}{2}(1 - \sqrt{1 - 4p_*(0)})$ ), which contradicts equality (3) ((4)).  $\square$

To convince ourselves that Corollary 1 is valid, let us note that (5) ((6)) imply

$$\begin{aligned} \lim_{\lambda \rightarrow 1+} \left[ (1-\lambda)p^*(\lambda) - \frac{\lambda^2}{4} - \frac{1-\lambda}{2}(1 + \sqrt{1-4p_*(2)}) \right] &> 0 \\ \left( \lim_{\lambda \rightarrow 1-} \left[ (\lambda-1)p^*(\lambda) - \frac{\lambda^2}{4} - \frac{\lambda-1}{2}(1 + \sqrt{1-4p_*(0)}) \right] > 0 \right). \end{aligned}$$

Consequently, (3) ((4)) is fulfilled for some  $\lambda < 1$  ( $\lambda > 1$ ). Thus, according to Theorem 1, equation (1) is oscillatory. As for Corollary 2, taking into account that the mapping  $\lambda \mapsto (1-\lambda)p_*(\lambda)$  for  $\lambda < 1$  ( $\lambda \mapsto (\lambda-1)p_*(\lambda)$  for  $\lambda > 1$ ) is non-decreasing (non-increasing), we easily find from (7) that (5) ((6)) is fulfilled for some  $\lambda$ .

*Proof of Theorem 2.* Assume the contrary. Let equation (1) be nonoscillatory. Then according to Lemma 2, equation (14) has the solution  $\rho : ]t_0, +\infty[ \rightarrow ]0, +\infty[$  satisfying conditions (15)–(17). Suppose  $\lambda < 1$  ( $\lambda > 1$ ). By the conditions of the theorem,  $p_*(0) > \frac{\lambda(2-\lambda)}{4}$  ( $p_*(2) > \frac{\lambda(2-\lambda)}{4}$ ), which implies that  $A > \frac{\lambda}{2}$  ( $B < \frac{\lambda}{2}$ ). On account of (17), for any  $0 < \varepsilon < A - \frac{\lambda}{2}$  ( $0 < \varepsilon < \frac{\lambda}{2} - B$ ) there exists  $t_\varepsilon > t_0$  such that (21) holds.

Multiplying equality (14) by  $t^\lambda$ , integrating it from  $t$  to  $+\infty$  (from  $t_\varepsilon$  to  $t$ ), and taking into account (15)–(17), we easily find that

$$\begin{aligned} t^{1-\lambda} \int_t^{+\infty} s^\lambda p(s) ds &= t\rho(t) + t^{1-\lambda} \int_t^{+\infty} s^{\lambda-2} s\rho(s)(\lambda - s\rho(s)) ds < \\ &< B + \varepsilon + \frac{1}{1-\lambda}(A - \varepsilon)(\lambda - A + \varepsilon) \quad \text{for } t_\varepsilon < t < +\infty \\ \left( t^{1-\lambda} \int_{t_\varepsilon}^t s^\lambda p(s) ds &< \varepsilon - A + \frac{1}{\lambda-1}(B + \varepsilon)(\lambda - B - \varepsilon) + \right. \\ &\left. + t^{1-\lambda} t_\varepsilon \rho(t_\varepsilon) \quad \text{for } t_\varepsilon < t < +\infty \right). \end{aligned}$$

This implies

$$\begin{aligned} p^*(\lambda) &\leq \frac{p_*(0)}{1-\lambda} + \frac{1}{2} \left( \sqrt{1-4p_*(0)} + \sqrt{1-4p_*(2)} \right) \\ \left( p^*(\lambda) &\leq \frac{p_*(2)}{\lambda-1} + \frac{1}{2} \left( \sqrt{1-4p_*(0)} + \sqrt{1-4p_*(2)} \right) \right), \end{aligned}$$

which contradicts condition (8) ((9)).  $\square$



*Proof of Theorems 3 and 3'.* Assume the contrary. Let equation (1) be nonoscillatory. Then according to Lemma 2, equation (14) has the solution  $\rho : ]t_0, +\infty[ \rightarrow [0, +\infty[$  satisfying conditions (15)–(17). Multiplying equality (14) by  $t^\lambda$ , integrating it from  $t$  to  $+\infty$ , and taking into account (16), we easily obtain

$$\begin{aligned} t\rho(t) &= h_\lambda(t) - \lambda t^{1-\lambda} \int_t^{+\infty} s^{\lambda-1} \rho(s) ds + \\ &+ t^{1-\lambda} \int_t^{+\infty} s^\lambda \rho^2(s) ds \quad \text{for } t_0 < t < +\infty, \end{aligned} \quad (22)$$

where  $h_\lambda$  is the function defined by equality (2).

Introduce the notation

$$r = \liminf_{t \rightarrow +\infty} t\rho(t).$$

On account of (17) we have  $r > 0$ . Therefore for any  $0 < \varepsilon < \max\{r, p_*(\lambda)\}$  there exists  $t_\varepsilon > t_0$  such that

$$r - \varepsilon < t\rho(t) < B + \varepsilon, h_\lambda(t) > p_*(\lambda) - \varepsilon \quad \text{for } t_\varepsilon < t < +\infty.$$

Owing to the above arguments, we find from (22) that

$$(1 - \lambda)h_\lambda < B + \varepsilon - (r - \varepsilon)^2 \quad \text{for } t_\varepsilon < t < +\infty,$$

$$t\rho(t) > p_*(\lambda) - \varepsilon - \frac{\lambda}{1 - \lambda}(B + \varepsilon) + \frac{1}{1 - \lambda}(r - \varepsilon)^2 \quad \text{for } t_\varepsilon < t < +\infty,$$

which implies

$$p^*(\lambda) \leq \frac{B - r^2}{1 - \lambda}, \quad (23)$$

$$r \geq p_*(\lambda) - \frac{\lambda}{1 - \lambda}B + \frac{r^2}{1 - \lambda}.$$

The latter inequality results in  $r \geq x_1$ , where  $x_1$  is the least root of the equation

$$\frac{1}{1 - \lambda}x^2 - x + p_*(\lambda) - \frac{\lambda}{1 - \lambda}B = 0.$$

Thus  $r \geq \max\{A, x_1\}$ . From (23) we have that if  $A < x_1$ , then

$$p^*(\lambda) \leq B + p_*(\lambda) - x_1,$$

but if  $A \geq x_1$ , then

$$p^*(\lambda) \leq \frac{1}{1 - \lambda}B - \frac{1}{1 - \lambda}A^2,$$

which contradicts the conditions of the theorem.  $\square$

*Proof of Theorem 4.* From (11) we have that for some  $t_0 > 0$

$$\frac{(2\lambda - 1)(3 - 2\lambda)}{4|1 - \lambda|} < h_\lambda(t) < \frac{1}{4|1 - \lambda|} \quad \text{for } t_0 < t < +\infty,$$

whence

$$h_\lambda^2(t) + \frac{2\lambda^2 - 4\lambda + 1}{2|1 - \lambda|} h_\lambda(t) + \frac{(2\lambda - 1)(3 - 2\lambda)}{16(1 - \lambda)^2} < 0 \quad \text{for } t_0 < t < +\infty.$$

Taking into consideration the latter inequality, we can easily see that (20) holds, where

$$v(t) = t^{\frac{1-2\lambda}{4(1-\lambda)}} \quad \text{for } t_0 < t < +\infty,$$

$$g(t) = -\frac{1}{t^2} (h_\lambda^2(t) - \lambda \operatorname{sgn}(1 - \lambda) h_\lambda(t)) \quad \text{for } t_0 < t < +\infty,$$

and

$$q(t) = -\frac{2}{t} \operatorname{sgn}(1 - \lambda) h_\lambda(t) \quad \text{for } t_0 < t < +\infty.$$

Consequently, according to Lemmas 1 and 3, equation (1) is nonoscillatory.  $\square$

#### REFERENCES

1. E. Hille, Non-oscillation theorems. *Trans. Amer. Math. Soc.*, **64**(1948), 234–252.
2. Z. Nehari, Oscillation criteria for second order-linear differential equations. *Trans. Amer. Math. Soc.*, **85**(1957), 428–445.
3. A. Wintner, On the non-existence of conjugate points. *Amer. J. Math.*, **73**(1951), 368–380.

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