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Certain Aspects of Some Arithmetic Functions in Number Theory [1]

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Abstract

The purpose of this paper is to present several inequalities about the arithmetic functions $\sigma^{(e)}$, $\tau^{(e)}$, $\sigma^{(e)*}$, $\tau^{(e)*}$ and other well-known arithmetic functions. Among these, we have the following:

$$\frac{\sqrt{\sigma_k^*(n)\cdot\sigma_l^*(n)}}{\sigma^*_{\frac{k-l}{2}}(n)}\leq\frac{n^{\frac{l-k}{4}}\cdot\sigma_k^*(n)+n^{\frac{k-l}{4}}\cdot\sigma_l^*(n)}{2\cdot\sigma_{\frac{k-l}{2}}(n)}\leq n^{\frac{l-k}{4}}\cdot\frac{n^{\frac{k+l}{2}}+1}{2},$$

for any $n,k,l\in\mathbb{N}^*$,

$$\frac{\sqrt{\sigma_k^{(e)*}(n)\cdot\tau^{(e)*}(n)}}{\sigma^{(e)*}_{\frac{k-l}{2}}(n)}\leq\frac{n^{\frac{l-k}{4}}\cdot\sigma_k^{(e)*}(n)+n^{\frac{k-l}{4}}\cdot\tau^{(e)*}(n)}{2\cdot\sigma^{(e)*}_{\frac{k-l}{2}}(n)}\leq$$

$\leq n^{\frac{l-k}{4}}\cdot\frac{n^{\frac{k+l}{2}}+1}{2}$, for any $n,k,l\in\mathbb{N}^*$, $\sigma_k^{(e)}(n)\cdot\sigma_l^{(e)}(n)\leq\tau^{(e)}(n)\cdot\sigma_{k+l}^{(e)}(n)$, for any $n,k,l\in\mathbb{N}^*$ and $\frac{\sigma_{k+1}^{(e)*}(n)}{\sigma_k^{(e)*}(n)}\geq\frac{\sigma^{(e)*}(n)}{\tau^{(e)*}(n)}\geq\tau(n)$, for any $n,k\in\mathbb{N}^*$, where $\tau(n)$ is the number of the natural divisors of n and $\sigma(n)$ is the sum of the divisors of n.

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