A Mathematical Bibliography of Signed and Gain Graphs and Allied Areas

Compiled by
Thomas Zaslavsky

Manuscript prepared with
Marge Pratt

Department of Mathematical Sciences
Binghamton University
Binghamton, New York, U.S.A. 13902-6000

E-mail: zaslav@math.binghamton.edu

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Colleagues:
HELP!
If you have any suggestions whatever for items to include in this bibliography, or for other changes, please let me hear from you. Thank you.

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Preface

A *signed graph* is a graph whose edges are labeled by signs. This is a bibliography of signed graphs and related mathematics.

Several kinds of labelled graph have been called "signed" yet are mathematically very different. I distinguish four types:

- *Group-signed graphs*: the edge labels are elements of a 2-element group and are multiplied around a polygon (or along any walk). Among the natural generalizations are larger groups and vertex signs.
- *Sign-colored graphs*, in which the edges are labelled from a two-element set that is acted upon by the sign group: $-$ interchanges labels, $+$ leaves them unchanged. This is the kind of "signed graph" found in knot theory. The natural generalization is to more colors and more general groups—or no group.
- *Weighted graphs*, in which the edge labels are the elements $+1$ and -1 of the integers or another additive domain. Weights behave like numbers, not signs; thus I regard work on weighted graphs as outside the scope of the bibliography—except (to some extent) when the author calls the weights "signs".
- Labelled graphs where the labels have no structure or properties but are called "signs" for any or no reason.

Each of these categories has its own theory or theories, generally very different from the others, so in a logical sense the topic of this bibliography is an accident of terminology. However, narrow logic here leads us astray, for the study of true signed graphs, which arise in numerous areas of pure and applied mathematics, forms the great majority of the literature. Thus I regard as fundamental for the bibliography the notions of *balance* of a polygon (sign product equals $+$, the sign group identity) and the vertex-edge incidence matrix (whose column for a negative edge has two $+1$'s or two -1's, for a positive edge one $+1$ and one -1, the rest being zero); this has led me to include work on *gain graphs* (where the edge labels are taken from any group) and "consistency" in *vertex-signed graphs*, and generalizable work on two-graphs (the set of unbalanced triangles of a signed complete graph) and on even and odd polygons and paths in graphs and digraphs.

Nevertheless, it was not always easy to decide what belongs. I have employed the following principles:

Only works with mathematical content are entered, except for a few representative purely applied papers and surveys. I do try to include:

- Any (mathematical) work in which signed graphs are mentioned by name or signs are put on the edges of graphs, regardless of whether it makes essential use of signs. (However, due to lack of time and in order to maintain "balance" in the bibliography, I have included only a limited selection of items concerning binary clutters and postman theory, two-graphs, signed digraphs in qualitative matrix theory, and knot theory. For clutters, see Cornuéjols (20xxa) when it appears; for postman theory, A. Frank (1996a). For two-graphs, see any of the review articles by Seidel. For qualitative matrix theory, see e.g. Maybee and Quirk (1969a) and Brualdi and Shader (1995a). For knot theory there are uncountable books and surveys.)

- Any work in which the notion of balance of a polygon plays a role. Example: gain graphs. (Exception: purely topological papers concerning ordinary graph embedding.)
- Any work in which ideas of signed graph theory are anticipated, or generalized, or transferred to other domains. Examples: vertex-signed graphs; signed posets and matroids.
- Any mathematical structure that is an example, however disguised, of a signed or gain graph or generalization, and is treated in a way that seems in the spirit of signed graph theory. Examples: even-cycle and bicircular matroids; bidirected graphs; binary clutters (which are equivalent to signed binary matroids); some of the literature on two-graphs and double covering graphs.
- And some works that have suggested ideas of value for signed graph theory or that have promise of doing so in the future.

As for applications, besides works with appropriate mathematical content I include a few (not very carefully) selected representatives of less mathematical papers and surveys, either for their historical importance (e.g., Heider (1946a)) or as entrances to the applied literature (e.g., Taylor (1970a) and Wasserman and Faust (1993a) for psychosociology and Trinajstic (1983a) for chemistry). Particular difficulty is presented by spin glass theory in statistical physics—that is, Ising models and generalizations. Here one usually averages random signs and weights over a probability distribution; the problems and methods are rarely graph-theoretic, the topic is very specialized and hard to annotate properly, but it clearly is related to signed (and gain) graphs and suggests some interesting lines of graph-theoretic research. See Mézard, Parisi, and Virasoro (1987a) and citations in its annotation.

Plainly, judgment is required to apply these criteria. I have employed mine freely, taking account of suggestions from my colleagues. Still I know that the bibliography is far from complete, due to the quantity and even more the enormous range and dispersion of work in the relevant areas. I will continue to add both new and old works to future editions and I heartily welcome further suggestions.

There are certainly many errors, some of them egregious. For these I hand over responsibility to Sloth, Pride, Ambition, Envy, and Confusion. As Diedrich Knickerbocker says:

> Should any reader find matter of offense in this [bibliography], I should heartily grieve, though I would on no acount question his penetration by telling him he was mistaken, his good nature by telling him he was captious, or his pure conscience by telling him he was startled at a shadow. Surely when so ingenious in finding offense where none was intended, it were a thousand pities he should not be suffered to enjoy the benefit of his discovery.

Corrections, however, will be gratefully accepted by me.

Bibliographical Data. Authors' names are given usually in only one form, even should the name appear in different (but recognizably similar) forms on different publications. Journal abbreviations follow the style of *Mathematical Reviews* (MR) with minor 'improvements'. Reviews and abstracts are cited from MR and its electronic form MathSciNet, from *Zentralblatt für Mathematik* (Zbl.) and its electronic version (For early volumes, "Zbl. VVV, PPP" denotes printed volume and page; the electronic item number is "(e VVV.PPPNN)".), and occasionally from *Chemical Abstracts* (CA) or *Computing Reviews* (CR). A review marked (q.v.) has significance, possibly an insight, a criticism, or a viewpoint orthogonal to mine.

Some—not all—of the most fundamental works are marked with a ††; some almost as fundamental have a †. This is a personal selection.

Annotations. I try to describe the relevant content in a consistent terminology and notation, in the language of signed graphs despite occasional clumsiness (hoping that this will suggest generalizations), and sometimes with my [bracketed] editorial comments. I sometimes try also to explain idiosyncratic terminology, in order to make it easier to read the original item. Several of the annotations incorporate open problems (of widely varying degrees of importance and difficulty).

I use these standard symbols:

- Γ is a graph (undirected), possibly allowing loops and multiple edges. It is normally finite unless otherwise indicated.
- Σ is a signed graph. Its vertex and edge sets are V and E; its order is $n = |V|$. E_+, E_- are the sets of positive and negative edges and Σ_+, Σ_- are the corresponding spanning subgraphs (unsigned).
- $[\Sigma]$ is the switching class of Σ.
- $A(\ )$ is the adjacency matrix.
- Φ is a gain graph.
- Ω is a biased graph.
- $l(\ )$ is the frustration index (= line index of imbalance).
- $G(\ )$ is the bias matroid of a signed, gain, or biased graph.
- $L(\ ), L_0(\ )$ are the lift and extended lift matroids.

Some standard terminology (much more will be found in the *Glossary* (Zaslavsky 1998c)):

- polygon, circle: The graph of a simple closed path, or its edge set.
- cycle: In a digraph, a coherently directed polygon, i.e., "dicycle". More generally: in an oriented signed, gain, or biased graph, a matroid circuit (usually, of the bias matroid) oriented to have no source or sink.

Acknowledgement. I cannot name all the people who have contributed advice and criticism, but many of the annotations have benefited from suggestions by the authors or others and a number of items have been brought to my notice by helpful correspondents. I am very grateful to you all. Thanks also to the people who maintain the invaluable MR and Zbl. indices (and a special thank-you for creating our very own MSC classification: 05C22). However, I insist on my total responsibility for the final form of all entries, including such things as my restatement of results in signed or gain graphic language and, of course, all the praise and criticism (but not errors; see above) that they contain.

Subject Classification Codes

A code in *lower case* means the topic appears implicitly but not explicitly. A suffix **w** on **S**, **SG**, **SD**, **VS** denotes signs used as weights, i.e., treated as the numbers $+1$ and -1, added, and (usually) the sum compared to 0. A suffix **c** on **SG**, **SD**, **VS** denotes signs used as colors (often written as the numbers $+1$ and -1), usually permuted by the sign group. In a string of codes a colon precedes subtopics. A code may be refined through being suffixed by a parenthesised code, as **S**(**M**) denoting signed matroids (while **S**: **M** would indicate matroids of signed objects; thus **S**(**M**): **M** means matroids of signed matroids).

A Adjacency matrix, eigenvalues.
Alg Algorithms.
Appl Applications other than (**Chem**), (**Phys**), (**PsS**) (partial coverage).
Aut Automorphisms, symmetries, group actions.
B Balance (mathematical), cobalance.
Bic Bicircular matroids.
Chem Applications to chemistry (partial coverage).
Cl Clusterability.
Col Vertex coloring.
Cov Covering graphs, double coverings.
D Duality (graphs, matroids, or matrices).
E Enumeration of types of signed graphs, etc.
EC Even-cycle matroids.
ECol Edge coloring.
Exp Expository.
Exr Interesting exercises (in an expository work).
Fr Frustration (imbalance); esp. frustration index (line index of imbalance).
G Connections with geometry, including toric varieties, complex complement, etc.
GD Digraphs with gains (or voltages).
Gen Generalization.
GG Gain graphs, voltage graphs, biased graphs; includes Dowling lattices.
GN Generalized or gain networks. (Multiplicative real gains.)
Hyp Hypergraphs with signs or gains.
I Incidence matrix, Kirchhoff matrix.
K Signed complete graphs.
Knot Connections with knot theory (sparse coverage if signs are purely notational).
LG Line graphs.
M Matroids and geometric lattices, chain-groups, flows.
N Numerical and algebraic invariants of signed graphs, etc.
O Orientations, bidirected graphs.
OG Ordered gains.
P All-negative or antibalanced signed graphs; parity-biased graphs.
p Includes problems on even or odd length of paths or polygons (partial coverage).
Phys Applications in physics (partial coverage).
PsS Psychological, sociological, and anthropological applications (partial coverage).
QM Qualitative (sign) matrices: sign stability, sign solvability, etc. (sparse coverage).
Rand Random signs or gains, signed or gain graphs.
Ref Many references.
S Signed objects other than graphs and hypergraphs: mathematical properties.
SD Signed digraphs: mathematical properties.
SG Signed graphs: mathematical properties.

Sol Sign solvability, sign nonsingularity (partial coverage).
Sta Sign stability (partial coverage).
Str Structure theory.
Sw Switching of signs or gains.
T Topology applied to graphs; surface embeddings. (Not applications to topology.)
TG Two-graphs, graph (Seidel) switching (partial coverage).
VS Vertex-signed graphs ("marked graphs"); signed vertices and edges.
WD Weighted digraphs.
WG Weighted graphs.
X Extremal problems.

Robert P. Abelson

See also M.J. Rosenberg.

1967a Mathematical models in social psychology. In: Leonard Berkowitz, ed., *Advances in Experimental Social Psychology*, Vol. 3, pp. 1–54. Academic Press, New York, 1967.

§II: "Mathematical models of social structure." Part B: "The balance principle." Reviews basic notions of balance and clusterability in signed (di)graphs and measures of degree of balance or clustering. Notes that signed K_n is balanced iff $I + A = vv^{\mathrm{T}}$, $v = \pm 1$-vector. Proposes: degree of balance $= \lambda_1/n$, where $\lambda_1 =$ largest eigenvalue of $I + A(\Sigma)$ and $n =$ order of the (di)graph. [Cf. Phillips (1967a).] Part C, 3: "Clusterability revisited."
(SG, SD: B, Cl, Fr, A)

Robert P. Abelson and Milton J. Rosenberg

†1958a Symbolic psycho-logic: a model of attitudinal cognition. *Behavioral Sci.* 3 (1958), 1–13.

Basic formalism: the "structure matrix", an adjacency matrix $R(\Sigma)$ with entries o, p, n [corresponding to $0, +1, -1$] for nonadjacency and positive and negative adjacency and a for simultaneous positive and negative adjacency. Defines addition and multiplication of these symbols (p. 8) so as to decide balance of Σ via $\operatorname{per}(I + R(\Sigma))$. [See Harary, Norman, and Cartwright (1965a) for more on this matrix.] Analyzes switching, treated as Hadamard product of $R(\Sigma)$ with "passive T-matrices" [essentially, matrices obtained by switching the square all-1's matrix]. Thm. 11: Switching preserves balance.

Proposes (p. 12) "complexity" [frustration index] $l(\Sigma)$ as measure of imbalance. [Cf. Harary (1959b).] Thm. 12: Switching preserves frustration index. Thm. 14: $\max l(\Sigma)$, over all Σ of order n, equals $\lfloor (n-1)^2/4 \rfloor$. (Proof omitted. [Proved by Petersdorf (1966a) and Tomescu (1973a) for signed K_n's and hence for all signed simple graphs of order n.]) **(PsS)(SG: A, B, sw, Fr)**

B. Devadas Acharya

See also M.K. Gill.

1973a On the product of p-balanced and l-balanced graphs. *Graph Theory Newsletter* 2, No. 3 (Jan., 1973), Results Announced No. 1. **(SG, VS: B)**

1979a New directions in the mathematical theory of balance in cognitive organizations. MRI Tech. Rep. No. HCS/DST/409/76/BDA (Dec., 1979). Mehta Research Institute of Math. and Math. Physics, Allahabad, India, 1979.
(SG, SD: B, A, Ref)(PsS: Exp, Ref)

1980a Spectral criterion for cycle balance in networks. *J. Graph Theory* 4 (1980), 1–11. MR 81e:05097(q.v.). Zbl. 445.05066. **(SD, SG: B, A)**

1980b An extension of the concept of clique graphs and the problem of K-convergence to signed graphs. *Nat. Acad. Sci. Letters (India)* 3 (1980), 239–242. Zbl. 491.05052.
(SG: LG, Clique graph)

1981a On characterizing graphs switching equivalent to acyclic graphs. *Indian J. Pure Appl. Math.* 12 (1981), 1187-1191. MR 82k:05089. Zbl. 476.05069.

Begins an attack on the problem of characterizing by forbidden induced subgraphs the simple graphs that switch to forests. Among them are K_5 and C_n, $n \geq 7$. *Problem.* Find any others that may exist. [Forests that switch to forests are characterized by Hage and Harju (1998a).] **(TG)**

1982a Connected graphs switching equivalent to their iterated line graphs. *Discrete Math.* 41 (1982), 115–122. MR 84b:05078. Zbl. 497.05052. **(LG, TG)**

1983a Even edge colorings of a graph. *J. Combin. Theory Ser. B* 35 (1983), 78–79. MR 85a:05034. Zbl. 505.05032, (515.05030).

Find the fewest colors to color the edges so that in each polygon the number of edges of some color is even. [Possibly, inspired by §2 of Acharya and Acharya (1983a).] **(b: Gen)**

1983b A characterization of consistent marked graphs. *Nat. Acad. Sci. Letters (India)* 6 (1983), 431–440. Zbl. 552.05052.

Converts a vertex-signed graph (Γ, μ) into a signed graph Σ such that (Γ, μ) is consistent iff every polygon in Σ is all-negative or has an even number of all-negative components. [See S.B. Rao (1984a) and Hoede (1992a) for the definitive results on consistency.] **(VS, SG: b)**

1984a Some further properties of consistent marked graphs. *Indian J. Pure Appl. Math.* 15 (1984), 837–842. MR 86a:05101. Zbl. 552.05053.

Notably: nicely characterizes consistent vertex-signed graphs in which the subgraph induced by negative vertices is connected. [Subsumed by S.B. Rao (1984a).] **(VS: b)**

1984b Combinatorial aspects of a measure of rank correlation due to Kendall and its relation to social preference theory. In: B.D. Acharya, ed., *Proceedings of the National Symposium on Mathematical Modelling* (Allahabad, 1982). M.R.I. Lecture Notes in Appl. Math., 1. Mehta Research Institute of Math. and Math. Physics, Allahabad, India, 1984.

Includes an exposition of Sampathkumar and Nanjundaswamy (1973a). **(SG: K: Exp)**

1986a An extension of Katai-Iwai procedure to derive balancing and minimum balancing sets of a social system. *Indian J. Pure Appl. Math.* 17 (1986), 875–882. MR 87k:92037. Zbl. 612.92019.

Expounds the procedure of Katai and Iwai (1978a). Proposes a generalization to those Σ that have a certain kind of polygon basis. Construct a "dual" graph whose vertex set is a polygon basis supplemented by the sum of basic polygons. A "dual" vertex has sign as in Σ. Let $T =$ set of negative "dual" vertices. A T-join in the "dual", if one exists, yields a negation set for Σ. [A minimum T-join need not yield a minimum negation set. Indeed the procedure is unlikely to yield a minimum negation set (hence the frustration index $l(\Sigma)$) for all signed graphs, since it can be performed in polynomial time while $l(\Sigma)$ is NP-complete. *Questions.* To which signed graphs is the procedure applicable? For which ones does a minimum T-join yield a minimum negation set? Do the latter include all those that forbid an interesting subdivision or minor (cf. Gerards and Schrijver (1986a), Gerards (1988a, 1989a))?] **(SG: Fr: Alg)**

B. Devadas Acharya and Mukti Acharya [M.K. Gill]

1983a A graph theoretical model for the analysis of intergroup stability in a social system. Manuscript, 1983.

The first half (most of §1) was improved and published as (1986a).

The second half (§§2–3) appears to be unpublished. Given; a graph Γ, a vertex signing μ, and a covering $\mathcal{F}$ of $E(\Gamma)$ by cliques of size ≤ 3. Define a signed graph S by; $V(S) = \mathcal{F}$ and $QQ' \in E(S)$ when at least half the elements of Q or Q' lie in $Q \cap Q'$; sign QQ' negative iff there exist vertices $v \in Q \backslash Q'$, and $w \in Q' \backslash Q$ such that $\mu(v) \neq \mu(w)$. Suppose there is no edge QQ' in which $|Q| = 3$, $|Q'| = 2$, and the two members of $Q \backslash Q'$ have differing sign. [This seems a very restrictive supposition.] Main result (Thm. 7): S is balanced. The definitions, but not the theorem, are generalized to multiple vertex signs μ, general clique covers, and clique adjacency rules that differ slightly from that of the theorem. (**GG, VS, SG: B**)

1986a New algebraic models of social systems. *Indian J. Pure Appl. Math.* 17 (1986), 150–168. MR 87h:92087. Zbl. 591.92029.

Four criteria for balance in an arbitrary gain graph. [See also Harary, Lindstrom, and Zetterstrom (1982a).] (**GG: B, sw**)

B.D. Acharya, M.K. Gill, and G.A. Patwardhan

1984a Quasicospectral graphs and digraphs. In: *Proceedings of the National Symposium on Mathematical Modelling* (Allahabad, 1982), pp. 133–144. M.R.I. Lecture Notes Appl. Math., 1. Mehta Research Institute of Math. and Math. Physics, Allahabad, 1984. MR 86c:05087. Zbl. 556.05048.

A signed graph, or digraph, is "cycle-balanced" if every polygon, or every cycle, is positive. Graphs, or digraphs, are "quasicospectral" if they have cospectral signings, "strictly quasicospectral" if they are quasicospectral but not cospectral, "strongly cospectral" if they are cospectral and have cospectral cycle-unbalanced signings. There exist arbitrarily large sets of strictly quasicospectral digraphs, which moreover can be assumed strongly connected, weakly but not strongly connected, etc. There exist 2 unbalanced strictly quasicospectral signed graphs; existence of larger sets is not unsolved. There exist arbitrarily large sets of nonisomorphic, strongly cospectral connected graphs; also, weakly connected digraphs, which moreover can be taken to be strongly connected, unilaterally connected, etc. Proofs, based on ideas of A.J. Schwenk, are sketched. (**SD, SG: A**)

Mukti Acharya [Mukhtiar Kaur Gill]

See also B.D. Acharya and M.K. Gill.

1988a Switching invariant three-path signed graphs. In: M.N. Gopalan and G.A. Patwardhan, eds., *Optimization, Design of Experiments and Graph Theory* (Bombay, 1986), pp. 342–345. Indian Institute of Technology, Bombay, 1988. MR 90b:05102. Zbl. 744.05054. (**SG, Sw**)

L. Adler and S. Cosares

1991a A strongly polynomial algorithm for a special class of linear programs. *Oper. Res.* 39 (1991), 955–960. MR 92k:90042. Zbl. 749.90048.

The class is that of the transshipment problem with gains. Along the way, a time bound on the uncapacitated, demands-only flows-with-gains problem. (**GN: I(D), Alg**)

S.N. Afriat

1963a The system of inequalities $a_{rs} > X_r - X_s$. *Proc. Cambridge Philos. Soc.* 59 (1963), 125–133. MR 25 #5071. Zbl. 118, 149 (e: 118.14901).

See also Roy (1959a). (**GG: OG, Sw, b**)

1974a On sum-symmetric matrices. *Linear Algebra Appl.* 8 (1974), 129–140. MR 48 #11163. Zbl. 281.15017. (**GG: Sw, b**)

A.A. Ageev, A.V. Kostochka, and Z. Szigeti

1995a A characterization of Seymour graphs. In: Egon Balas and Jens Clausen, eds., *Integer Programming and Combinatorial Optimization* (4th Internat. IPCO Conf., Copenhagen, 1995, Proc.), pp. 364–372. Lecture Notes in Computer Sci., Vol. 920. Springer, Berlin, 1995. MR 96h:05157.

A Seymour graph satisfies with equality a general inequality between T-join size and T-cut packing. Thm.: A graph is not a Seymour graph iff it has a conservative ± 1-weighting such that there are two polygons with total weight 0 whose union is an antibalanced subdivision of $-K_n$ or $-Pr_3$ (the triangular prism). (**SGw: Str, B, P**)

1997a A characterization of Seymour graphs. *J. Graph Theory* 24 (1997), 357–364. MR 97m:05217. Zbl. 970.24507.

Virtually identical to (1995a). (**SGw: Str, B, P**)

J.K. Aggarwal

See M. Malek-Zavarei.

Ron Aharoni, Rachel Manber, and Bronislaw Wajnryb

1990a Special parity of perfect matchings in bipartite graphs. *Discrete Math.* 79 (1990), 221–228. MR 91b:05140. Zbl. 744.05036.

When do all perfect matchings in a signed bipartite graph have the same sign product? Solved. (**sg: b, Alg**)(**qm: Sol**)

R. Aharoni, R. Meshulam, and B. Wajnryb

1995a Group weighted matchings in bipartite graphs. *J. Algebraic Combin.* 4 (1995), 165–171. MR 96a:05111. Zbl. 950.25380.

Given an edge weighting $w : E \to K$ where K is a finite abelian group. Main topic: perfect matchings M such that $\sum_{e \in M} w(e) = 0$ [I'll call them 0-weight matchings]. (Also, in §2, $= c$ where c is a constant.) Generalizes and extends Aharoni, Manber, and Wajnryb (1990a). Continued by Kahn and Meshulam (1998a). (**WG**)

Prop. 4.1 concerns vertex-disjoint polygons whose total sign product is + in certain signed digraphs. (**SD**)

Ravindra K. Ahuja, Thomas L. Magnanti, and James B. Orlin

1993a *Network Flows: Theory, Algorithms, and Applications.* Prentice Hall, Englewood Cliffs, N.J., 1993. MR 94e:90035.

§12.6: "Nonbipartite cardinality matching problem". Nicely expounds theory of blossoms and flowers (Edmonds (1965a), etc.). Historical notes and references at end of chapter. (**p: o, Alg: Exp, Ref**)

§5.5: "Detecting negative cycles"; §12.7, subsection "Shortest paths in directed networks". Weighted arcs with negative weights allowed. Techniques for detecting negative cycles and, if none exist, finding a shortest path. (**WD: OG, Alg: Exp**)

Ch. 16: "Generalized flows". Sect. 15.5: "Good augmented forests and linear programming bases", Thm. 15.8, makes clear the connection between flows with gains and the bias matroid of the underlying gain graph. Some terminology: "breakeven cycle" = balanced polygon; "good augmented forest" = basis of the bias matroid, assuming the gain graph is connected and unbalanced. (**GN: M(Bases), Alg: Exp, Ref**)

Martin Aigner

1979a *Combinatorial Theory.* Grundl. math. Wiss., Vol. 234. Springer-Verlag, Berlin, 1979. Reprinted: Classics in Mathematics. Springer-Verlag, Berlin, 1997. MR 80h:05002. Zbl. 415.05001, 858.05001 (reprint).

In §VII.1, pp. 333–334 and Exerc. 13–15 treat the Dowling lattices of $\mathrm{GF}(q)^{\times}$ and higher-weight analogs. (**GG, GG(Gen): M: N, Str**)

M. Aĭgner [Martin Aigner]

1982a *Kombinatornaya teoriya.* "Mir", Moscow, 1982. MR 84b:05002.

Russian translation of (1979a). Transl. V.V. Ermakov and V.N. Lyamin. Ed. and preface by G.P. Gavrilov. (**GG, GG(Gen): M: N, Str**)

J. Akiyama, D. Avis, V. Chvátal, and H. Era

††1981a Balancing signed graphs. *Discrete Appl. Math.* 3 (1981), 227–233. MR 83k:05059. Zbl. 468.05066.

Bounds for $D(\Gamma)$, the largest frustration index $l(\Gamma,\sigma)$ over all signings of a fixed graph Γ (not necessarily simple) of order n and size $m = |E|$. Main Thm.: $\frac{1}{2}m - \sqrt{mn} \leq D(\Gamma) \leq \frac{1}{2}m$. Thm. 4: $D(K_{t,t}) \leq \frac{1}{2}t^2 - c_0 t^{3/2}$, where c_0 can be taken $= \pi/480$. Probabilistic methods are used. Thus, Thm. 2: Given Γ, $\mathrm{Prob}(l(\Gamma,\sigma) > \frac{1}{2}m - \sqrt{mn}) \geq 1 - (\frac{2}{e})^n$. Moreover, let $n_{\mathrm{b}}(\Sigma)$ be the largest order of a balanced subgraph of Σ. Thm. 5: $\mathrm{Prob}(n_{\mathrm{b}}(K_n,\sigma) \geq k) \leq \binom{n}{k}/2^{\binom{k}{2}}$. (The problem of evaluating $n - n_{\mathrm{b}}$ was raised by Harary; see (1959b).) Finally, Thm. 1: If Σ has vertex-disjoint balanced induced subgraphs with m' edges, then $l(\Sigma) \leq \frac{1}{2}(m - m')$. [See Poljak and Turzík (1982a), Solé and Zaslavsky (1994a) for more on $D(\Gamma)$; Brown and Spencer (1971a), Gordon and Witsenhausen (1972a) for $D(K_{t,t})$; Harary, Lindström, and Zetterström (1982a) for a result similar to Thm. 1.] (**SG: Fr, Rand**)

S. Alexander and P. Pincus

1980a Phase transitions of some fully frustrated models. *J. Phys. A: Math. Gen.* 13, No. 1 (1980), 263–273. (**P: Phys**)

Kazutoshi Ando and Satoru Fujishige

1996a On structures of bisubmodular polyhedra. *Math. Programming* 74 (1996), 293–317. MR 97g:90102. Zbl. 855.68107. (**sg: O**)

Kazutoshi Ando, Satoru Fujishige, and Takeshi Naitoh

1997a Balanced bisubmodular systems and bidirected flows. *J. Oper. Res. Soc. Japan* 40 (1997), 437–447. MR 98k:05073. Zbl. 970.61830.

A balanced bisubmodular system corresponds to a bidirected graph that is balanced. The "flows" are arbitrary capacity-constrained functions, not satisfying conservation at a vertex. (**sg: O, B**)

Kazutoshi Ando, Satoru Fujishige, and Toshio Nemoto

1996a Decomposition of a bidirected graph into strongly connected components and its signed poset structure. *Discrete Appl. Math.* 68 (1996), 237–248. MR 97c:05096. Zbl. 960.53208. (**sg: O**)

1996b The minimum-weight ideal problem for signed posets. *J. Oper. Res. Soc. Japan* 39 (1996), 558-565. MR 98j:90084. Zbl. 874.90188. (**sg: O**)

Thomas Andreae

1978a Matroidal families of finite connected nonhomeomorphic graphs exist. *J. Graph Theory* 2 (1978), 149–153. MR 80a:05160. Zbl. 401.05070.

Partially anticipates the "count" matroids of graphs (see Whiteley (1996a)). (**Bic, EC: Gen**)

St. Antohe and E. Olaru

1981a Singned graphs homomorphism [*sic*]. [Signed graph homomorphisms.] *Bul. Univ. Galati Fasc. II Mat. Fiz. Mec. Teoret.* 4 (1981), 35–43. MR 83m:05057.

A "congruence" is an equivalence relation R on $V(\Sigma)$ such that no negative edge is within an equivalence class. The quotient Σ/R has the obvious simple underlying graph and signs $\bar\sigma(\bar x\bar y) = \sigma(xy)$ [which is ambiguous]. A signed-graph homomorphism is a function $f : V_1 \to V_2$ that is a sign-preserving homomorphism of underlying graphs. [This is inconsistent, since the sign of edge $f(x)f(y)$ can be ill defined. The defect might perhaps be remedied by allowing multiple edges with different signs or by passing entirely to multigraphs.] The canonical map $\Sigma \to \Sigma/R$ is such a homomorphism. Composition of homomorphisms is well defined and associative; hence one has a category $\mathrm{Graph}^{\mathrm{sign}}$. The categorial product is $\prod_{i\in I}\Sigma_i :=$ Cartesian product of the $|\Sigma_i|$ with the component-wise signature $\sigma((\ldots,u_i,\ldots)(\ldots,v_i,\ldots)) := \sigma_i(u_iv_i)$. Some further elementary properties of signed-graph homomorphisms and congruences are proved. [The paper is hard to interpret due to mathematical ambiguity and grammatical and typographical errors.] (**SG**)

Katsuaki Aoki

See M. Iri.

Julián Aráoz, William H. Cunningham, Jack Edmonds, and Jan Green-Krótki

1983a Reductions to 1-matching polyhedra. Proc. Sympos. on the Matching Problem: Theory, Algorithms, and Applications (Gaithersburg, Md., 1981). *Networks* 13 (1983), 455–473. MR 85d:90059. Zbl. 525.90068.

The "minimum-cost capacitated b-matching problem in a bidirected graph B" is to minimize $\sum_e c_ex_e$ subject to $0 \leq x \leq u \in \{0,1,\ldots,\infty\}^E$ and $I(B)x = b \in \mathbb{Z}^V$. The paper proves, by reduction to the ordinary perfect matching problem, Edmonds and Johnson's (1970a) description of the convex hull of feasible solutions. (**sg: O: I, Alg, G**)

Dan Archdeacon

1995a Problems in topological graph theory. Manuscript, 1995. WorldWideWeb URL (2/98) http://www.emba.uvm.edu/~archdeac/papers/papers.html

A compilation from various sources and contributors, updated every so often. "The genus sequence of a signed graph", p. 10: A conjecture due to Širáň (?) on the demigenus range (here called "spectrum" [though unrelated to matrices]) for orientation embedding of Σ, namely, that the answer to Question 1 under Širáň (1991b) is affirmative. (**SG: T**)

1996a Topological graph theory: a survey. Surveys in Graph Theory (Proc., San Francisco, 1995). *Congressus Numer.* 115 (1996), 5–54. Updated version: WorldWideWeb URL (2/98) http://www.emba.uvm.edu/~archdeac/papers/papers.html MR 98g:05044. Zbl. 897.05026.

§2.5 describes orientation embedding (called "signed embedding" [although there are other kinds of signed embedding]) and switching (called "sequence of local switches of sense") of signed graphs with rotation systems. §5.5, "Signed embeddings", briefly mentions Širáň (1991b), Širáň and Škoviera (1991a), and Zaslavsky (1993a, 1996a). **(SG: T: Exp)**

Dan Archdeacon and Jozef Širáň

1998a Characterizing planarity using theta graphs. *J. Graph Theory* 27 (1998), 17–20. MR 98j:05055. Zbl. 887.05016.

A "claw" consists of a vertex and three incident half edges. Let C be the set of claws in Γ and T the set of theta subgraphs. Fix a rotation of each claw. Call $t \in T$ an "edge" with endpoints c, c' if t contains c and c'; sign it $+$ or $-$ according as t can or cannot be embedded in the plane so the rotations of its trivalent vertices equal the ones chosen for c and c'. This defines, independently (up to switching) of the choice of rotations, the "signed triple graph" $T^{\pm}(\Gamma)$. Theorem: Γ is planar iff $T^{\pm}(\Gamma)$ is balanced. **(SG, Sw)**

Srinivasa R. Arikati and Uri N. Peled

1993a A linear algorithm for the group path problem on chordal graphs. *Discrete Appl. Math.* 44 (1993), 185–190. MR 94h:68084. Zbl. 779.68067.

Given a graph with edges weighted from a group. The weight of a path is the product of its edge weights in order (not inverted, as with gains). Problem: to determined whether between two given vertices there is a chordless path of given weight. This is NP-complete in general but for chordal graphs there is a fast algorithm (linear in $(|E|+|V|)\cdot$(group order)). [*Question.* What if the edges have gains rather than weights?] **(WG: p(Gen): Alg)**

1996a A polynomial algorithm for the parity path problem on perfectly orientable graphs. *Discrete Appl. Math.* 65 (1996), 5–20. MR 96m:05120. Zbl. 854.68069.

Problem: Does a given graph contain an induced path of specified parity between two prescribed vertices? A polynomial-time algorithm for certain graphs. (Cf. Bienstock (1991a).) [*Problem.* Generalize to paths of specified sign in a signed graph.] **(p: Alg)(Ref)**

Esther M. Arkin and Christos H. Papadimitriou

1985a On negative cycles in mixed graphs. *Oper. Res. Letters* 4 (1985), 113–116. MR 87h:68061. Zbl. 585.05017. **(WG: OG)**

E.M. Arkin, C.H. Papadimitriou, and M. Yannakakis

1991a Modularity of cycles and paths in graphs. *J. Assoc. Comput. Mach.* 38 (1991), 255–274. MR 92h:68068. Zbl. 799.68146.

Modular poise gains in digraphs (gain $+1$ on each oriented edge). **(gg: B)**

Christos A. Athanasiadis

1996a Characteristic polynomials of subspace arrangements and finite fields. *Adv. Math.* 122 (1996), 193–233.

See Headley (1997a) for definitions of the Shi arrangements. Here the characteristic polynomials of these and other arrangements are evaluated combinatorially. §3: "The Shi arrangements". §4: "The Linial arrangement": this represents $\mathrm{Lat^b}\,(K_n, \varphi_1)$ (see Stanley (1996a) for notation). §5: "Other interesting hyperplane arrangements", treats: the arrangement representing $\mathrm{Lat^b}\, L \cdot K_n$ where $L = \{-k, \ldots, k-1, k\}$, which is the semilattice of k-composed partitions (see Zaslavsky (20xxh), also Edelman and Reiner (1996a)) and several generalizations, including to arbitrary sign-symmetric gain sets L and to Weyl analogs; also, an antibalanced analog of the A_n Shi arrangment (Thm. 5.4); and more. **(sg, gg: G, M, N)**

1997a A class of labeled posets and the Shi arrangement of hyperplanes. *J. Combin. Theory Ser. A* 80 (1997), 158–162. MR 98d:05008. Zbl. 970.66662.

The Shi arrangement of hyperplanes [of type A_{n-1}] represents $\mathrm{Lat}^{\mathrm{b}}\,\Phi$ where $\Phi = (K_n, \varphi_0) \cup (K_n, \varphi_1)$ (see Stanley (1996a) for notation). **(gg: G, M, N)**

1998a On free deformations of the braid arrangement. *European J. Combin.* 19 (1998), 7–18.

The arrangements considered are the subarrangements of the projectivized Shi arrangements of type A_{n-1} that contain A_{n-1}. Thms. 4.1 and 4.2 characterize those that are free or supersolvable. Arrangements representing the extended lift matroid $L_0(\Phi)$ where $\Phi = \bigcup_{i=1-a}^{a} (K_n, \varphi_i)$ and $a \geq 1$ ($a = 1$ giving the Shi arrangement), and a mild generalization, are of use in the proof (see Stanley (1996a) for notation). **(gg: G, M, N)**

20xxa Deformations of Coxeter hyperplane arrangements and their characteristic polynomials. Submitted.

David Avis

See J. Akiyama.

Constantin P. Bachas

1984a Computer-intractibility of the frustration model of a spin glass. *J. Physics A* 17 (1984), L709–L712. MR 85j:82043.

The frustration index decision problem on signed (3-dimensional) cubic lattice graphs is NP-complete. [Proof is incomplete; completed and improved by Green (1987a).] [Cf. Barahona (1982a).] **(SG: Fr: Alg)**

G. David Bailey

20xxa Inductively factored signed-graphic arrangements of hyperplanes. Submitted.

Continues Edelman and Reiner (1994a). **(SG: G, M)**

V. Balachandran

1976a An integer generalized transportation model for optimal job assignment in computer networks. *Oper. Res.* 24 (1976), 742–759. MR 55 #12068. Zbl. 356.90028. **(GN: M(bases))**

V. Balachandran and G.L. Thompson

1975a An operator theory of parametric programming for the generalized transportation problem: I. Basic theory. II. Rim, cost and bound operators. III. Weight operators. IV. Global operators. *Naval Res. Logistics Quart.* 22 (1975), 79–100, 101–125, 297–315, 317–339. MR 52 ##2595, 2596, 2597, 2598. Zbl. 331.90048, 90049, 90050, 90051. **(GN: M)**

Egon Balas

1966a The dual method for the generalized transportation problem. *Management Sci.* 12 (1966), No. 7 (March, 1966), 555–568. MR 32 #7232. Zbl. 142, 166 (e: 142.16601). **(GN: M(bases))**

1981a Integer and fractional matchings. In: P. Hansen, ed., *Studies on Graphs and Discrete Programming*, pp. 1–13. North-Holland Math. Stud., 59. Ann. Discrete Math., 11. North-Holland, Amsterdam, 1981. MR 84h:90084.

Linear (thus "fractional", meaning half-integral) vs. integral programming solutions to maximum matching. The difference of their maxima = $\frac{1}{2}$(max number of matching-separable vertex-disjoint odd polygons). Also noted (p. 12): (max) fractional matchings in Γ correspond to (max) matchings in the

double covering graph of $-\Gamma$. [*Question.* Does this lead to a definition of maximum matchings in signed graphs?] **(p, o: I, G, Alg, cov)**

E. Balas and P.L. Ivanescu [P.L. Hammer]

1965a On the generalized transportation problem. *Management Sci.* 11 (1965), No. 1 (Sept., 1964), 188–202. MR 30 #4599. Zbl. 133, 425 (e: 133.42505). **(GN: M, B)**

K. Balasubramanian

1988a Computer generation of characteristic polynomials of edge-weighted graphs, heterographs, and directed graphs. *J. Computational Chem.* 9 (1988), 204–211.

Here a "signed graph" means, in effect, an acyclically oriented graph D along with the antisymmetric adjacency matrix $A_{\pm}(D) = A(+D \cup -D^*)$, D^* being the converse digraph. [That is, $A_{\pm}(D) = A(D) - A(D)^{\mathrm{t}}$. The "signed graphs" are just acyclic digraphs with an antisymmetric adjacency matrix and, correspondingly, what we may call the 'antisymmetric characteristic polynomial'.] Proposes an algorithm for the polynomial. Observes in some examples a relationship between the characteristic polynomial of Γ and the antisymmetric characteristic polynomial of an acyclic orientation. **(SD, wg: A: N: Alg, Chem)**

1991a Comments on the characteristic polynomial of a graph. *J. Computational Chem.* 12 (1991), 248–253. MR 92b:92057.

Argues (heuristically) that a certain algorithm is superior to another, in particular for the antisymmetric polynomial defined in (1988a). **(SD: A: N: Alg)**

1992a Characteristic polynomials of fullerene cages. *Chemical Physics Letters* 198 (1992), 577–586.

Computed for graphs of six different cages of three different orders, in both ordinary and "signed" (see (1988a)) versions. Observes a property of the "signed graph" polynomials [which is due to antisymmetry, as explained by P.W. Fowler (Comment on "Characteristic polynomials of fullerene cages". *Chemical Physics Letters* 203 (1993), 611–612)]. **(SD: A: N: Chem)**

1994a Are there signed cospectral graphs? *J. Chemical Information and Computer Sciences* 34 (1994), 1103–1104.

The "signed graphs" are as in (1988a). Simplified contents: It is shown by example that the antisymmetric characteristic polynomials of two nonisomorphic acyclic orientations of a graph (see (1988a)) may be equal or unequal. [Much smaller examples are provided by P.W. Fowler (Comment on "Characteristic polynomials of fullerene cages". *Chemical Physics Letters* 203 (1993), 611–612).] [*Question.* Are there examples for which the underlying (di)graphs are nonisomorphic?] [For cospectrality of other kinds of signed graphs, see Acharya, Gill, and Patwardhan (1984a) (signed K_n's).] **(SD: A: N)**

R. Balian, J.M. Drouffe, and C. Itzykson

1975a Gauge fields on a lattice. II. Gauge-invariant Ising model. *Phys. Rev. D* 11 (1975), 2098–2103. **(SG: Phys, Sw, B)**

Jørgen Bang-Jensen and Gregory Gutin

1997a Alternating cycles and paths in edge-coloured multigraphs: A survey. *Discrete Math.* 165/166 (1997), 39–60. MR 98d:05080. Zbl. 876.05057.

A rich source for problems on bidirected graphs. An edge 2-coloration of a graph becomes an all-negative bidirection by taking one color class to con-

sist of introverted edges and the other to consist of extroverted edges. An alternating path becomes a coherent path; an alternating polygon becomes a coherent polygon. [*General Problem.* Generalize to bidirected graphs the results on edge 2-colored graphs mentioned in this paper. (See esp. §5.) *Question.* To what digraph properties do they specialize by taking the underlying signed graph to be all positive?] [See e.g. Bánkfalvi and Bánkfalvi (1968a) (q.v.), Bang-Jensen and Gutin (1998a), Das and Rao (1983a), Grossman and Häggqvist (1983a), Mahadev and Peled (1995a), Saad (1996a).]
(p: o: Paths, Polygons)

1998a Alternating cycles and trails in 2-edge-colored complete multigraphs. *Discrete Math.* 188 (1998), 61–72. MR 99g:05072.

The longest coherent trail, having degrees bounded by a specified degree vector, in a bidirected all-negative complete multigraph that satisfies an extra hypothesis. Generalization of Das and Rao (1983a) and Saad (1996a), thus ultimately of Thm. 1 of Bánkfalvi and Bánkfalvi (1968a) (q.v.). Also, a polynomial-time algorithm. **(p: o: Paths, Alg)**

M. Bánkfalvi and Zs. Bánkfalvi

1968a Alternating Hamiltonian circuit in two-coloured complete graphs. In: P. Erdős and G. Katona, eds., *Theory of Graphs* (Proc. Colloq., Tihany, 1966), pp. 11–18. Academic Press, New York, 1968. MR 38 #2052. Zbl. 159, 542 (e: 159.54202).

Let Σ be a bidirected $-K_{2n}$ which has a coherent 2-factor. ("Coherent" means that, at each vertex in the 2-factor, one edge is directed inward and the other outward.) Thm. 1: B has a coherent Hamiltonian polygon iff, for every $k \in \{2, 3, \ldots, n-2\}$, $s_k > k^2$, where $s_k :=$ the sum of the k smallest indegrees and the k smallest outdegrees. Thm. 2: The number of k's for which $s_k = k^2$ equals the smallest number p of polygons in any coherent 2-factor of B. Moreover, the p values of k for which equality holds imply a partition of V into p vertex sets, each inducing B_i consisting of a bipartite [i.e., balanced] subgraph with a coherent Hamiltonian polygon and in one color class only introverted edges, while in the other only extroverted edges. [*Problem.* Generalize these remarkable results to an arbitrary bidirected complete graph. The all-negative case will be these theorems; the all-positive case will give the smallest number of cycles in a covering by vertex-disjoint cycles of a tournament that has any such covering.] [See Bang-Jensen and Gutin (1997a) for further developments on alternating walks.] **(p: o: Polygons)**

Zs. Bánkfalvi

See M. Bánkfalvi.

C. Bankwitz

1930a Über die Torsionszahlen der alternierenden Knotes. *Math. Ann.* 103 (1930), 145–161.

Introduces the sign-colored graph of a link diagram. [Further work by numerous writers, e.g., S. Kinoshita *et al.* and esp. Kauffman (1989a) and successors.] **(Knot: SGc)**

Francisco Barahona

1981a Balancing signed toroidal graphs in polynomial-time. Unpublished manuscript, 1981.

Given a 2-connected Σ whose underlying graph is toroidal, polynomial-time algorithms are given for calculating the frustration index $l(\Sigma)$ and the generating function of switchings Σ^μ by $|E_-(\Sigma^\mu)|$. The technique is to solve a Chinese postman (T-join) problem in the toroidal dual graph, T corresponding to the frustrated face boundaries. Generalizes (1982a). [See (1990a), p. 4, for a partial description.] **(SG: Fr, Alg)**

1982a On the computational complexity of Ising spin glass models. *J. Phys. A: Math. Gen.* 15 (1982), 3241–3253. MR 84c:82022.

The frustration-index problem, that is, minimization of $|E_-(\Sigma^\eta)|$ over all switching functions $\eta : V \to \{\pm 1\}$, for signed planar and toroidal graphs and subgraphs of 3-dimensional grids. Analyzed structurally, in terms of perfect matchings in a modified dual graph, and algorithmically. The last is NP-hard, even when the grid has only 2 levels; the former are polynomial-time solvable even with weighted edges. Also, the problem of minimizing $|E_-(\Sigma^\eta)| + \sum_v \eta(v)$ for planar grids ("2-dimensional problem with external magnetic field"), which is NP-hard. (This corresponds to adding an extra vertex, positively adjacent to every vertex.)

(SG: Phys, Fr, Fr(Gen): D, Alg)

1982b Two theorems in planar graphs. Unpublished manuscript, 1982. **(SG: Fr)**

1990a On some applications of the Chinese Postman Problem. In: B. Korte, L. Lovász, H.J. Prömel, and A. Schrijver, eds., *Paths, Flows and VLSI-Layout*, pp. 1–16. Algorithms and Combinatorics, Vol. 9. Springer-Verlag, Berlin, 1990. MR 92b:90139. Zbl. 732.90086.

Section 2: "Spin glasses." **(SG: Phys, Fr: Exp)**

Section 5: "Max cut in graphs not contractible to K_5," pp. 12–13.

(sg: fr: Exp)

1990b Planar multicommodity flows, max cut, and the Chinese Postman problem. In: William Cook and Paul D. Seymour, eds., *Polyhedral Combinatorics* (Proc. Workshop, 1989), pp. 189–202. DIMACS Ser. Discrete Math. Theoret. Computer Sci., Vol. 1. Amer. Math. Soc. and Assoc. Comput. Mach., Providence, R.I., 1990. MR 92g:05165. Zbl. 747.05067.

Negative cutsets, where signs come from a network with real-valued capacities. Dual in the plane to negative polygons. See §2. **(SG: D: B, Alg)**

Francisco Barahona and Adolfo Casari

1988a On the magnetisation of the ground states in two-dimensional Ising spin glasses. *Comput. Phys. Comm.* 49 (1988), 417–421. MR 89d:82004. Zbl. 814.90132.

(SG: Fr: Alg)

Francisco Barahona, Martin Grötschel, and Ali Ridha Mahjoub

1985a Facets of the bipartite subgraph polytope. *Math. Oper. Res.* 10 (1985), 340–358. MR 87a:05123a. Zbl. 578.05056.

The polytope $P_B(\Gamma)$ is the convex hull in $\mathbb{R}^E$ of incidence vectors of bipartite edge sets. Various types of and techniques for generating facet-defining inequalities, thus partially extending the description of $P_B(\Gamma)$ from the weakly bipartite case (Grötschel and Pulleyblank (1981a)) in which all facets are due to edge and odd-polygon constraints. [Some can be described best via signed graphs; see Poljak and Turzík (1987a).] [A brief expository treatment of the polytope appears in Poljak and Tuza (1995a).] **(sg: p: fr: G)**

Francisco Barahona and Enzo Maccioni

1982a On the exact ground states of three-dimensional Ising spin glasses. *J. Phys. A: Math. Gen.* 15 (1982), L611–L615. MR 83k:82044.

Discusses a 3-dimensional analog of Barahona, Maynard, Rammal, and Uhry (1982a). (Here there may not always be a combinatorial LP optimum; hence LP may not completely solve the problem.) **(SG: Phys, Fr, Alg)**

Francisco Barahona and Ali Ridha Mahjoub

1986a On the cut polytope. *Math. Programming* 36 (1986), 157–173. MR 88d:05049. Zbl. 616.90058.

Call $P_{\mathrm{BS}}(\Sigma)$ the convex hull in $\mathbb{R}^E$ of incidence vectors of negation sets (or "balancing [edge] sets") in Σ. Finding a minimum-weight negation set in Σ corresponds to a maximum cut problem, whence $P_{\mathrm{BS}}(\Sigma)$ is a linear transform of the cut polytope $P_{\mathrm{C}}(|\Sigma|)$, the convex hull of cuts. Conclusions follow about facet-defining inequalities of $P_{\mathrm{BS}}(\Sigma)$. See §5: "Signed graphs". **(SG: Fr: G)**

1989a Facets of the balanced (acyclic) induced subgraph polytope. *Math. Programming Ser. B* 45 (1989), 21–33. MR 91c:05178. Zbl. 675.90071.

The "balanced induced subgraph polytope" $P_{\mathrm{BIS}}(\Sigma)$ is the convex hull in $\mathbb{R}^V$ of incidence vectors of vertex sets that induce balanced subgraphs. Conditions are studied under which certain inequalities of form $\sum_{i\in Y} x_i \leq f(Y)$ define facets of this polytope: in particular, $f(Y) =$ max. size of balance-inducing subets of Y, $f(Y) = 1$ or 2, $f(Y) = |Y| - 1$ when $Y = V(C)$ for a negative polygon C, etc. **(SG: Fr: G, Alg)**

1994a Compositions of graphs and polyhedra. I: Balanced induced subgraphs and acyclic subgraphs. *SIAM J. Discrete Math.* 7 (1994), 344–358. MR 95i:90056. Zbl. 802.05067.

More on $P_{\mathrm{BIS}}(\Sigma)$ (see (1989a)). A balance-inducing vertex set in $\pm\Gamma =$ a stable set in Γ. [See Zaslavsky (1982b) for a different correspondence.] Thm. 2.1 is an interesting preparatory result: If $\Sigma = \Sigma_1 \cup \Sigma_2$ where $\Sigma_1 \cap \Sigma_2 \cong \pm K_k$, then $P_{\mathrm{BIS}}(\Sigma) = P_{\mathrm{BIS}}(\Sigma_1) \cap P_{\mathrm{BIS}}(\Sigma_2)$. The main result is Thm. 2.2: If Σ has a 2-separation into Σ_1 and Σ_2, the polytope is the projection of the intersection of polytopes associated with modifications of Σ_1 and Σ_2. §5: "Compositions of facets", derives the facets of $P_{\mathrm{BIS}}(\Sigma)$. **(SG: G, WG, Alg)**

F. Barahona, R. Maynard, R. Rammal, and J.P. Uhry

1982a Morphology of ground states of two-dimensional frustration model. *J. Phys. A: Math. Gen.* 15 (1982), 673–699. MR 83c:82045.

§2: "The frustration model as the Chinese postman's problem", describes how to find the frustration index $l(-\Sigma) = \min_\eta |E_-(\Sigma^\eta)|$ (over all switching functions η) of a signed planar graph by solving a Chinese postman (T-join) problem in the planar dual graph, T corresponding to the frustrated face boundaries. [This was solved independently by Katai and Iwai (1978a).] The postman problem is solved by linear programming, in which there always is a combinatorial optimum: see §3: "Solution of the frustration problem by duality: rigidity". Of particular interest are vertex pairs, esp. edges, for which $\eta(v)\eta(w)$ is the same for every "ground state" (i.e., minimizing η); these are called "rigid". §5: "Results" (of numerical experiments) has interesting discussion. [Barahona (1981a) generalizes to signed toroidal graphs.]

In the preceding one minimizes $f_0(\eta) = \sum_E \sigma(vw)\eta(v)\eta(w)$. More general problems discussed are (1) allowing positive edge weights (due to variable bond strengths); (2) minimizing $f_0(\eta) + c\sum_V \eta(v)$, with $c \neq 0$ because of an external magnetic field. Then one cannot expect the LP to have a combinatorial optimum. (**SG: Phys, Fr, Fr(Gen), Alg**)

F. Barahona and J.P. Uhry

1981a An application of combinatorial optimization to physics. *Methods Oper. Res.* 40 (1981), 221–224. Zbl. 461.90080. (**SG: Phys, Fr: Exp**)

J. Wesley Barnes

See P.A. Jensen.

Lowell Bassett, John Maybee, and James Quirk

1968a Qualitative economics and the scope of the correspondence principle. *Econometrica* 36 (1968), 544–563. MR 38 #5456. Zbl. (e: 217.26802).

Lemma 3: A square matrix with every diagonal entry negative is sign-nonsingular iff every cycle is negative in the associated signed digraph. Thm. 4: A square matrix with negative diagonal is sign-invertible iff all cycles are negative and the sign of any (open) path is determined by its endpoints. And more. (**QM: Sol, Sta: sd**)

Vladimir Batagelj

1990a [Closure of the graph value matrix.] (In Slovenian. English summary.) *Obzornik Mat. Fiz.* 37 (1990), 97–104. MR 91f:05058. Zbl. 704.05035. (**SG: A, B, Cl**)

1994a Semirings for social networks analysis. *J. Math. Sociology* 19 (1994), 53–68. Zbl. 827.92029. (**SG: A, B, Cl**)

M. Behzad and G. Chartrand

1969a Line-coloring of signed graphs. *Elem. Math.* 24 (1969), 49–52. MR 39 #5415. Zbl. 175, 503 (e: 175.50302). (**SG: LG: Cl**)

[L.] W. Beineke and F. Harary

1966a Binary matrices with equal determinant and permanent. *Studia Sci. Math. Hungar.* 1 (1966), 179–183. MR 34 #7397. Zbl. (e: 145.01505). (**SD**)

Lowell W. Beineke and Frank Harary

1978a Consistency in marked digraphs. *J. Math. Psychology* 18 (1978), 260–269. MR 80d:05026. Zbl. 398.05040.

A digraph with signed vertices is "consistent" (that is, every cycle has positive sign product) iff its vertices have a bipartition so that every arc with a positive tail lies within a set but no arc with a negative tail does so. (The reason is that a strongly connected digraph with vertex signs can be regarded as edge-signed and the bipartition criterion for balance can be applied.) A corollary: the digraphs that have consistent vertex signs are characterized. (**VS**)

1978b Consistent graphs with signed points. *Riv. Mat. Sci. Econom. Social.* 1 (1978), 81–88. MR 81h:05108. Zbl. 493.05053.

A graph with signed vertices is "consistent" if every polygon has positive sign product. Elementary results, but a characterization of consistent vertex-signed graphs is presented as an open problem. For a good solution see Hoede (1992a); Rao (1984a) had found a more complicated solution. (**VS**)

Jacques Bélair, Sue Ann Campbell, and P. van den Driessche

1996a Frustration, stability, and delay-induced oscillations in a neural network model. *SIAM J. Appl. Math.* 56 (1996), 245–255. MR 96j:92003. Zbl. 840.92003.

The signed digraph of a square matrix is "frustrated" if it has a negative cycle. Somewhat simplified: frustration is necessary for there to be oscillation caused by intraneuronal processing delay. (**SD: QM, Ref**)

A. Bellacicco and V. Tulli

1996a Cluster identification in a signed graph by eigenvalue analysis. In: *Matrices and Graphs: Theory and Applications to Economics* (full title *Proceedings of the Conferences on Matrices and Graphs: Theory and Applications to Economics*) (Brescia, 1993, 1995), pp. 233–242. World Scientific, Singapore, 1996. MR 99h:00029 (book). Zbl. 914.65146.

Signed digraphs ("spin graphs") are defined. The main concepts—"dissimilarity", "balance", and "cluster"—do not involve signs. Eigenvalues are mentioned. [This may be an announcement. There are no proofs. It is hard to be sure what is being said.] (**SD: A**)

Joachim von Below

1994a The index of a periodic graph. *Results Math.* 25 (1994), 198–223. MR 95e:05081. Zbl. 802.05054.

Here a periodic graph [of dimension m] is defined as a connected graph $\Gamma = \tilde{\Psi}$ where Ψ is a finite $\mathbb{Z}^m$-gain graph with gains contained in $\{\mathbf{0}, \mathbf{b}_i, \mathbf{b}_i - \mathbf{b}_j\}$. ($\mathbf{b}_1, \ldots, \mathbf{b}_m$ are the unit basis vectors of $\mathbb{Z}^m$.) Let us call such a Ψ a small-gain base graph for Γ. Any $\tilde{\Phi}$, where Φ is a finite $\mathbb{Z}^m$-gain graph, has a small-gain base graph Ψ; thus this definition is equivalent to that of Collatz (1978a). The "index" $I(\Gamma)$, analogous to the largest eigenvalue of a finite graph, is the spectral radius of $A(||\Psi||)$ (here written $A(\Gamma, N)$) for any small-gain base graph of Γ. The paper contains basic theory and the lower bound $L_m = \inf\{I(\Gamma) : \Gamma \text{ is } m\text{-dimensional}\}$, where $1 = L_1$, $\sqrt{9/2} = L_2 \leq L_3 \leq \ldots$. (**GG(Cov): A**)

Edward A. Bender and E. Rodney Canfield

1983a Enumeration of connected invariant graphs. *J. Combin. Theory Ser. B* 34 (1983), 268–278. MR 85b:05099. Zbl. 532.05036.

§3: "Self-dual signed graphs," gives the number of n-vertex graphs that are signed, vertex-signed, or both; connected or not; self-isomorphic by reversing edge and/or vertex signs or not, for all $n \leq 12$. Some of this appeared in Harary, Palmer, Robinson, and Schwenk (1977a). (**SG, VS: E**)

Riccardo Benedetti

1998a A combinatorial approach to combings and framings of 3-manifolds. In: A. Balog, G.O.H. Katona, A. Recski, and D. Sa'sz, eds., *European Congress of Mathematics* (Budapest, 1996), Vol. I, pp. 52–63. Progress in Math., Vol. 168. Birkhäuser, Basel, 1998. MR * Zbl. 905.57018.

§8, "Spin manifolds", hints at a use for decorated signed graphs in the structure theory of spin 3-manifolds. (**sg: Appl: Exp**)

Curtis Bennett and Bruce E. Sagan

1995a A generalization of semimodular supersolvable lattices. *J. Combin. Theory Ser. A* 72 (1995), 209–231. MR 96i:05180. Zbl. 831.06003.

To illustrate the generalization, most of the article calculates the chromatic polynomial of $\pm K_n^{(k)}$ (called $\mathcal{DB}_{n,k}$; this has half edges at k vertices), builds an "atom decision tree" for $k = 0$, and describes and counts the bases of $G(\pm K_n^{(k)})$ (called $\mathcal{D}_n$) that contain no broken circuits. (**SG: M, N, col**)

M.K. Bennett, Kenneth P. Bogart, and Joseph E. Bonin

1994a The geometry of Dowling lattices. *Adv. Math.* 103 (1994), 131–161. MR 95b:05050. Zbl. 814.51003. **(gg: M, G)**

Moussa Benoumhani

1996a On Whitney numbers of Dowling lattices. *Discrete Math.* 159 (1996), 13–33. MR 98a:06005. Zbl. 861.05004. **(gg: M: N)**

1997a On some numbers related to Whitney numbers of Dowling lattices. *Adv. Appl. Math.* 19 (1997), 106–116. MR 98f:05004. Zbl. 876.05001.

Generating polynomials and infinite generating series for multiples of Whitney numbers of the second kind, analogous to usual treatments of Stirling numbers. **(gg: M: N)**

1999a Log-concavity of Whitney numbers of Dowling lattices. *Adv. Appl. Math.* 22 (1999), 186–189.

Logarithmic concavity of Whitney numbers of the second kind is deduced by proving that their generating polynomial has only real zeros. [Cf. Dur (1986a).] **(gg: M: N)**

C. Benzaken

See also P.L. Hammer.

C. Benzaken, S.C. Boyd, P.L. Hammer, and B. Simeone

1983a Adjoints of pure bidirected graphs. Proc. Fourteenth Southeastern Conf. on Combinatorics, Graph Theory and Computing (Boca Raton, Fla., 1983). *Congressus Numer.* 39 (1983), 123–144. MR 85e:05077. Zbl. 537.05024. **(sg: O: LG)**

Cl. Benzaken, P.L. Hammer, and B. Simeone

1980a Some remarks on conflict graphs of quadratic pseudo-boolean functions. In: L. Collatz, G. Meinardus, and W. Wetterling, eds., *Konstruktive Methoden der finiten nichtlinearen Optimierung* (Tagung, Oberwolfach, 1980), pp. 9–30. Internat. Ser. of Numerical Math., 55. Birkhäuser, Basel, 1980. MR 83e:90096. Zbl. 455.90063. **(p: fr)(sg: O: LG)**

C. Benzaken, P.L. Hammer, and D. de Werra

1985a Threshold characterization of graphs with Dilworth number two. *J. Graph Theory* 9 (1985), 245–267. MR 87d:05135. Zbl. 583.05048. **(SG: B)**

Claude Berge and A. Ghouila-Houri

1962a *Programmes, jeu et reseaux de transport.* Dunod, Paris, 1962. MR 33 #1137. Zbl. (e: 111.17302).

2^e partie, Ch. IV, **S**2: "Les reseaux de transport avec multiplicateurs." Pp. 223–229. **(GN: i)**

1965a *Programming, Games and Transportation Networks.* Methuen, London; Wiley, New York, 1965. MR 33 #7114.

English edition of (1962a).

Part II, 10.2: "The transportation network with multipliers." Pp. 221–227. **(GN: i)**

1967a *Programme, Spiele, Transportnetze.* B.G. Teubner Verlagsgesellschaft, Leipzig, 1967, 1969. MR 36 #1195. Zbl. (e: 183.23905, 194.19803).

German edition(s) of (1962a). **(GN: i)**

Joseph Berger, Bernard P. Cohen, J. Laurie Snell, and Morris Zelditch, Jr.

1962a *Types of Formalization in Small Group Research.* Houghton Mifflin, Boston, 1962.

See Ch. 2: "Explicational models." **(PsS)(SG: B)(Ref)**

Abraham Berman and B. David Saunders

1981a Matrices with zero line sums and maximal rank. *Linear Algebra Appl.* 40 (1981), 229–235. MR 82i:15029. Zbl. 478.15013. **(QM, sd: o)**

Gora Bhaumik

See P.A. Jensen.

V.N. Bhave

See E. Sampathkumar.

I. Bieche, R. Maynard, R. Rammal, and J.P. Uhry

1980a On the ground states of the frustration model of a spin glass by a matching method of graph theory. *J. Phys. A: Math. Gen.* 13 (1980), 2553–2576. MR 81g:82037. **(SG: Phys, Fr, Alg)**

Dan Bienstock

1991a On the complexity of testing for odd holes and induced odd paths. *Discrete Math.* 90 (1991), 85–92. MR 92m:68040a. Zbl. 753.05046. Corrigendum. *ibid.* 102 (1992), 109. MR 92m.68040b. Zbl. 760.05080.

Given a graph. Problem 1: Is there an odd hole on a particular vertex? Problem 2: Is there an odd induced path joining two specified vertices? Problem 3: Is every pair of vertices joined by an odd-length induced path? All three problems are NP-complete. [Obviously, one can replace the graph by a signed graph and "odd length" by "negative" and the problems remain NP-complete.] **(P: Polygons, Paths: Alg)**

Norman Biggs

1974a *Algebraic Graph Theory.* Cambridge Math. Tracts, No. 67. Cambridge Univ. Press, London, 1974. MR 50 #151. Zbl. 284.05101.

Ch. 19: "The covering graph construction." Especially see Exercise 19A: "Double coverings." These define what we might call the canonical covering graphs of gain graphs. **(SG, GG: Cov, Aut, b)**

1993a *Algebraic Graph Theory.* Second edn. Cambridge Math. Library, Cambridge Univ. Press, Cambridge, Eng., 1993. MR 95h:05105. Zbl. 797.05032.

As in (1974a), but Exercise 19A has become Additional Result 19a. **(SG, GG: Cov, Aut, b)**

1997a International finance. In: Lowell W. Beineke and Robin J. Wilson, eds., *Graph Connections: Relationships between Graph Theory and other Areas of Mathematics*, Ch. 17, pp. 261–279. The Clarendon Press, Oxford, 1997.

A model of currency exchange rates in which no cyclic arbitrage is possible, hence the rates are given by a potential function. [That is, the exchange-rate gain graph is balanced, with the natural consequences.] Assuming cash exchange without accumulation in any currency, exchange rates are determined. [See also Ellerman (1984a). **(GG, gn: B: Exp)**

Robert E. Bixby

1981a Hidden structure in linear programs. In: Harvey J. Greenberg and John S. Maybee, eds., *Computer-Assisted Analysis and Model Simplification* (Proc. Sympos., Boulder, Col., 1980), pp. 327–360; discussion, pp. 397–404. Academic Press, New York, 1981. MR 82g:00016 (book). Zbl. 495.93001 (book). **(GN)**

Anders Björner and Bruce E. Sagan

1996a Subspace arrangements of type B_n and D_n. *J. Algebraic Combin.* 5 (1996), 291–314. MR 97g:52028. Zbl. 864.57031.

They study lattices $\Pi_{n,k,h}$ (for $0 < h \leq k \leq n$) consisting of all spanning subgraphs of $\pm K_n^\circ$ that have at most one nontrivial component K, for which K is complete and $|V(K)| \geq k$ if K is balanced, K is induced and $|V(K)| \geq h$ if K is unbalanced (also a generalization). Characteristic polynomial, homotopy and homology of the order complex, cohomology of the real complement. **(SG: G, M(Gen): N, col)**

Anders Björner, Michel Las Vergnas, Bernd Sturmfels, Neil White, and Günter M. Ziegler

1993a *Oriented Matroids.* Encyclop. Math. App., Vol. 46. Cambridge University Press, Cambridge, Eng., 1993. MR 95e:52023. Zbl. 773.52001.

The adjacency graph of bases of an oriented matroid is signed, using circuit signatures, to make the "signed basis graph". See §3.5, "Basis orientations and chirotopes", pp. 132–3. **(M: SG)**

Andreas Blass

1995a Quasi-varieties, congruences, and generalized Dowling lattices. *J. Algebraic Combin.* 4 (1995), 277–294. MR 96i:06012. Zbl. 857.08002. Errata. *Ibid.* 5 (1996), 167.

Treats the generalized Dowling lattices of Hanlon (1991a) as congruence lattices of certain quasi-varieties, in order to calculate characteristic polynomials and generalizations. **(M(gg): Gen: N)**

Andreas Blass and Frank Harary

1982a Deletion versus alteration in finite structures. *J. Combin. Inform. System Sci.* 7 (1982), 139–142. MR 84d:05087. Zbl. 506.05038.

The theorem that deletion index = negation index of a signed graph (Harary (1959b)) is shown to be a special case of a very general phenomenon involving hereditary classes of "partial choice functions". Another special case: deletion index = alteration index of a gain graph [an immediate corollary of Harary, Lindstöm, and Zetterström (1982a), Thm. 2]. **(SG, GG: B, Fr)**

Andreas Blass and Bruce Sagan

1997a Möbius functions of lattices. *Adv. Math.* 127 (1997), 94–123. MR 98c:06001. Zbl. 970.32977.

§3: "Non-crossing B_n and D_n". Lattices of noncrossing signed partial partitions. Atoms of the lattices are defined as edge fibers of the signed covering graph of $\pm K_n^\circ$, thus corresponding to edges of $\pm K_n^\circ$. [The "half edges" are perhaps best regarded as negative loops.] The lattices studied, called $NCB_n, NCD_n, NCBD_n(S)$, consist of the noncrossing members of the Dowling and near-Dowling lattices of the sign group, i.e., $\operatorname{Lat} G(\pm K_n^{(T)})$ for $T = [n], \emptyset, [n]\backslash S$, respectively. **(SG: G, N, cov)**

1998a Characteristic and Ehrhart polynomials. *J. Algebraic Combin.* 7 (1998), 115–126. MR 99c:05204. Zbl. 899.05003.

Signed-graph chromatic polynomials are recast geometrically by observing that the number of k-colorings equals the number of points of $\{-k, -k+1, \ldots, k-1, k\}^n$ that lie in none of the edge hyperplanes of the signed graph. The interesting part is that this generalizes to subspace arrangements of signed graphs and, somewhat *ad hoc*, to the hyperplane arrangements of the exceptional root systems. [See also Zaslavsky (20xxi). For applications see articles of Sagan and Zhang.] **(SG, Gen: M(Gen), G: col, N)**

T.B. Boffey

1982a *Graph theory in Oper. Research.* Macmillan, London, 1982. Zbl. 509.90053.

Ch. 10: "Network flow: extensions." 10.1(g): "Flows with gains," pp. 224–226. 10.3: "The simplex method applied to network problems," subsection "Generalised networks," pp. 246–250. **(GN: m(bases): Exp)**

Kenneth P. Bogart

See M.K. Bennett, J.E. Bonin, and J.R. Weeks.

Ethan D. Bolker

1977a Bracing grids of cubes. *Environment and Planning B* 4 (1977), 157–172. **(EC)**

1979a Bracing rectangular frameworks. II. *SIAM J. Appl. Math.* 36 (1979), 491–503. MR 81j:73066b. Zbl. 416.70010. **(EC, SG)**

Bela Bollobás

1978a *Extremal Graph Theory.* L.M.S. Monographs, Vol. 11. Academic Press, London, 1978. MR 80a:05120. Zbl. 419.05031.

A rich source of problems: find interesting generalizations to signed graphs of questions involving even or odd polygons, or bipartite graphs or subgraphs. **(p: X)**

§3.2, Thm. 2.2, is Lovász's (1965a) characterization of the graphs having no two vertex-disjoint polygons. **(GG: Polygons)**

§6.6, Problem 47, is the theorem on all-negative vertex elimination number from Bollobás, Erdős, Simonovits, and Szemerédi (1978a). **(p: Fr)**

B. Bollobás, P. Erdös, M. Simonovits, and E. Szemerédi

1978a Extremal graphs without large forbidden subgraphs. In: B. Bollobás, ed., *Advances in Graph Theory* (Proc. Cambridge Combin. Conf., 1977), pp. 29–41. Ann. Discrete Math., Vol. 3. North-Holland, Amsterdam, 1978. MR 80a:05119. Zbl. 375.05034.

Thm. 9 asymptotically estimates upper bounds on frustration index and vertex elimination number for all-negative signed graphs with fixed negative girth. [Sharpened by Komlós (1997a).] **(p: Fr)**

J.A. Bondy and L. Lovász

1981a Cycles through specified vertices of a graph. *Combinatorica* 1 (1981), 117–140. MR 82k:05073. Zbl. 492.05049.

If Γ is k-connected [and not bipartite], then any k $[k-1]$ vertices lie on an even [odd] polygon. [*Problem.* Generalize to signed graphs, this being the all-negative case.] **(sg: b)**

J.A. Bondy and M. Simonovits

1974a Cycles of even length in graphs. *J. Combin. Theory Ser. B* 16 (1974), 97–105. MR 49 #4851. Zbl. 283.05108.

If a graph has enough edges, it has even polygons of all moderately small lengths. [*Problem* 1. Generalize to positive polygons in signed graphs, this being the antibalanced (all-negative) case. For instance, *Problem* 2. If an unbalanced signed simple graph has positive girth $\geq l$ (i.e., no balanced polygon of length $< l$), what is its maximum size? Are the extremal examples antibalanced? Balanced?] **(p: b(Polygons), X)**

Joseph E. Bonin

See also M.K. Bennett.

1993a Automorphism groups of higher-weight Dowling geometries. *J. Combin. Theory Ser. B* 58 (1993), 161–173. MR 94k:51005. Zbl. 733.05027, (789.05017).

A weight- k higher Dowling geometry of rank n, $Q_{n,k}(\mathrm{GF}(q)^\times)$, is the union of all coordinate k-flats of $\mathrm{PG}(n-1,q)$: i.e., all flats spanned by k elements of a fixed basis. If $k>2$, the automorphism groups are those of $\mathrm{PG}(n-1,q)$ for $q>2$ and are symmetric groups if $q=2$. **(gg: Gen: M)**

1993b Modular elements of higher-weight Dowling lattices. *Discrete Math.* 119 (1993), 3–11. MR 94h:05018. Zbl. 808.06012.

See definition in (1993a). For $k>2$ the only nontrivial modular flats are the projective coordinate k-flats and their subflats. This gives some information about the characteristic polynomials [which, however, are still only partially known]. [Kung (1996a), §6, has further results.] **(gg: Gen: M, N)**

1995a Automorphisms of Dowling lattices and related geometries. *Combin. Probab. Comput.* 4 (1995), 1–9. MR 96e:05039. Zbl. 950.37335.

The automorphisms of a Dowling geometry of a nontrivial group are the compositions of a coordinate permutation, switching, and a group automorphism. A similar result holds, with two exceptions, if some or all coordinate points are deleted. **(gg: M: Autom)**

1996a Open problem 6. A problem on Dowling lattices. In: Joseph E. Bonin, James G. Oxley, and Brigitte Servatius, eds., *Matroid Theory* (Proc., Seattle, 1995), pp. 417–418. Contemp. Math., Vol. 197. Amer. Math. Soc., Providence, R.I., 1996.

Problem 6.1. If a finite matroid embeds in the Dowling geometry of a group, does it embed in the Dowling geometry of some finite group? [The answer may be "no" (Squier and Zaslavsky, unwritten and possibly unrecoverable).] **(gg: M)**

Joseph E. Bonin and Kenneth P. Bogart

1991a A geometric characterization of Dowling lattices. *J. Combin. Theory Ser. A* 56 (1991), 195–202. MR 92b:05019. Zbl. 723.05033. **(gg: M)**

Joseph E. Bonin and Joseph P.S. Kung

1994a Every group is the automorphism group of a rank-3 matroid. *Geom. Dedicata* 50 (1994), 243–246. MR 95m:20005. Zbl. 808.05029. **(gg: M: Aut)**

Joseph E. Bonin and William P. Miller

20xxa Characterizing geometries by numerical invariants. Submitted

Dowling geometries are characterized amongst all simple matroids by numerical properties of large flats of ranks ≤ 7 (Thm. 3.4); amongst all matroids by their Tutte polynomials. **(gg: M)**

Joseph E. Bonin and Hongxun Qin

20xxa Size functions of subgeometry-closed classes of representable combinatorial geometries. Submitted

Extremal matroid theory. The Dowling geometry $Q_3(\mathrm{GF}(3)^\times)$ appears as an exceptional extremal matroid in Thm. 2.10. The extremal subset of $\mathrm{PG}(n-1,q)$ not containing the higher-weight Dowling geometry $Q_{m,m-1}(\mathrm{GF}(q)^\times)$ (see Bonin 1993a) is found in Thm. 2.14. **(GG, Gen: M: X, N)**

C. Paul Bonnington and Charles H.C. Little

1995a *The Foundations of Topological Graph Theory.* Springer, New York, 1995. MR 97e:05090. Zbl. 950.48477.

Signed-graph imbedding: see §2.3, §2.6 (esp. Thm. 2.4), pp. 44–48 (for the colorful 3-gem approach to crosscaps), §3.3, and Ch. 4 (esp. Thms. 4.5, 4.6). **(sg: T, b)**

E. Boros, Y. Crama, and P.L. Hammer

1992a Chvàtal cuts and odd cycle inequalities in quadratic 0—1 optimization. *SIAM J. Discrete Math.* 5 (1992), 163–177. MR 93a:90043. Zbl. 761.90069.

§4: "Odd cycles [i.e., negative polygons] in signed graphs." Main problem: Find a minimum-weight deletion set in a signed graph with positively weighted edges. Related problems: A polygon-covering formulation whose constraints correspond to negative polygons. A dual polygon-packing problem. **(SG: Fr, G, Alg)**

Endre Boros and Peter L. Hammer

1991a The max-cut problem and quadratic 0—1 optimization; polyhedral aspects, relaxations and bounds. *Ann. Oper. Res.* 33 (1991), 151–180. MR 92j:90049. Zbl. 741.90077.

Includes finding a minimum-weight deletion set (as in Boros, Crama, and Hammer (1991a)). **(SG, WG: Fr: G, Alg)**

André Bouchet

1982a Constructions of covering triangulations with folds. *J. Graph Theory* 6 (1982), 57–74. MR 83b:05057. Zbl. 488.05032. **(sg: O, Appl)**

1983a Nowhere-zero integral flows on a bidirected graph. *J. Combin. Theory Ser. B* 34 (1983), 279–292. MR 85d:05109. Zbl. 518.05058.

Introduces nowhere-zero flows on signed graphs. A connected, coloop-free signed graph has a nowhere-zero integral flow with maximum weight ≤ 216. The value 216 cannot be replaced by 5, but Bouchet conjectures that it can be replaced by 6. [See Khelladi (1987a) for some progress on this.] A topological application is outlined. [The bidirection is inessential; it is a device to keep track of the flow.] **(SG: M, O, Flows, Appl)**

Jean-Marie Bourjolly

1988a An extension of the König-Egerváry property to node-weighted bidirected graphs. *Math. Programming* 41 (1988), 375–384. MR 90c:05161. Zbl. 653.90083.

[See Sewell (1996a).] **(sg: O, GG: Alg)**

J.-M. Bourjolly, P.L. Hammer, and B. Simeone

1984a Node-weighted graphs having the König-Egerváry property. Mathematical Programming at Oberwolfach II (Oberwolfach, 1983). *Math. Programming Stud.* 22 (1984), 44–63. MR 86d:05099. Zbl. 558.05054. **(p: o)**

Jean-Marie Bourjolly and William R. Pulleyblank

1989a König-Egerváry graphs, 2-bicritical graphs and fractional matchings. *Discrete Appl. Math.* 24 (1989), 63–82. MR 90m:05069. Zbl. 684.05036.

[It is hard to escape the feeling that we are dealing with all-negative signed graphs and that something here will generalize to other signed graphs. Especially see Theorem 5.1. Consult the references for related work.] **(P; Ref)**

John Paul Boyd

1969a The algebra of group kinship. *J. Math. Psychology* 6 (1967), 139–167. Reprinted in: Samuel Leinhardt, ed., *Social Networks: A Developing Paradigm*, pp. 319–346. Academic Press, New York, 1977. Zbl. (e: 172.45501). Erratum. *J. Math. Psychology* 9 (1972), 339. Zbl. 242.92010. **(SG: B)**

S.C. Boyd
See C. Benzaken.

A.J. Bray, M.A. Moore, and P. Reed
1978a Vanishing of the Edwards-Anderson order parameter in two- and three-dimensional Ising spin glasses. *J. Phys. C: Solid State Phys.* 11 (1978), 1187–1202.
(**Phys: SG: Fr**)

Floor Brouwer and Peter Nijkamp
1983a Qualitative structure analysis of complex systems. In: P. Nijkamp, H. Leitner, and N. Wrigley, eds., *Measuring the Unmeasurable*, pp. 509–530. Martinus Nijhoff, The Hague, 1983. (**QM, SD: Sol, Sta: Exp**)

Edward M. Brown and Robert Messer
1979a The classification of two-dimensional manifolds. Trans. Amer. Math. Soc. 255 (1979), 377–402. MR 80j:57007. Zbl. 391.57010, (414.57003).
Their "signed graph" we might call a type of Eulerian partially bidirected graph. That is, some edge ends are oriented (hence "partially bidirected"), and every vertex has even degree and at each vertex equally many edge ends point in and out ("Eulerian"). More specially, at each vertex all or none of the edge ends are oriented. (**sg: o: gen: Appl**)

Gerald G. Brown and Richard D. McBride
1984a Solving generalized networks. *Management Sci.* 30 (1984), 1497–1523. Zbl. 554.-90032. (**GN: M(bases)**)

Kenneth S. Brown and Persi Diaconis
1998a Random walks and hyperplane arrangements. *Ann. Probab.* 26 (1998), 1813–1854.
The real hyperplane arrangement representing $-K_n$ is studied in §3D. It leads to a random walk on threshold graphs. (**p: G**)

Thomas A. Brown
See also F.S. Roberts.

T.A. Brown, F.S. Roberts, and J. Spencer
1972a Pulse processes on signed digraphs: a tool for analyzing energy demand. Rep. R-926-NSF, Rand Corp., Santa Monica, Cal., March, 1972. (**SDw**)

Thomas A. Brown and Joel H. Spencer
1971a Minimization of ± 1 matrices under line shifts. *Colloq. Math.* 23 (1971), 165–171. MR 46 #7059. Zbl. 222.05016.
Asymptotic estimates of $l(K_{r,s})$, the maximum frustration index of signatures of $K_{r,s}$. Improved by Gordon and Witsenhausen (1972a). Also, exact values stated for $r \leq 4$ [extended by Solé and Zaslavsky (1994a)]. (**sg: Fr**)

William G. Brown, ed.
1980a *Reviews in Graph Theory.* 4 vols. American Math. Soc., Providence, R.I., 1980. Zbl. 538.05001.
See esp.: §208: "Signed graphs ($+$ or $-$ on each edge), balance" (undirected and directed), Vol. 1, pp. 569–571. (**SG, SD**)

Richard A. Brualdi and Herbert J. Ryser
1991a *Combinatorial Matrix Theory.* Encycl. Math. Appl., Vol. 39. Cambridge University Press, Cambridge, Eng., 1991. MR 93a:05087. Zbl. 746.05002.
See §7.5. (**QM: Sol, SD, b**)(**Exp, Ref**)

Richard A. Brualdi and Bryan L. Shader

1995a *Matrices of Sign-Solvable Linear Systems.* Cambridge Tracts in Math., Vol. 116. Cambridge University Press, Cambridge, Eng., 1995. MR 97k:15001. Zbl. 833.-15002.

Innumerable results and references on signed digraphs are contained herein. (**QM, SD: Sol, Sta**)(**Exp, Ref, Alg**)

Michael Brundage

1996a From the even-cycle mystery to the L-matrix problem and beyond. M.S. thesis, Dept. of Mathematics, Univ. of Washington, Seattle, 1996. WorldWideWeb URL (10/97) http://www.math.washington.edu/~brundage/evcy/

A concise expository survey. Ch. 1: "Even cycles in directed graphs". Ch. 2: "L-matrices and sign-solvability", esp. sect. "Signed digraphs". Ch. 3: "Beyond", esp. sect. "Balanced labellings" (vertices labelled from $\{0,+1,-1\}$ so that from each vertex labelled $\epsilon \neq 0$ there is an arc to a vertex labelled $-\epsilon$) and sect. "Pfaffian orientations". (**SD, P: Polygons, Sol, Alg, VS: Exp, Ref**)

Fred Buckley, Lynne L. Doty, and Frank Harary

1988a On graphs with signed inverses. *Networks* 18 (1988), 151–157. MR 89i:05222. Zbl. 646.05061.

"Signed invertible graph" [i.e., sign-invertible graph] = graph Γ such that $A(\Gamma)^{-1} = A(\Sigma)$ for some signed graph Σ. Finds two classes of such graphs. Characterizes sign-invertible trees. [Cf. Godsil (1985a) and, for a different notion, Greenberg, Lundgren, and Maybee (1984b).] (**SG: A**)

James R. Burns and Wayland H. Winstead

1982a Input and output redundancy. *IEEE Trans. Systems Man Cybernetics* SMC-12, No. 6 (1982), 785–793.

§IV: "The computation of contradictory redundancy." Summarized in modified notation: In a signed graph, define $w^{\epsilon}_{ij}(r)$ = number of walks of length r and sign ϵ from v_i to v_j. Define an adjacency matrix A by $a_{ij} = w^{+}_{ij}(1) + w^{-}_{ij}(1)\theta$, where θ is an indeterminate whose square is 1. Then $w^{+}_{ij}(r) + w^{-}_{ij}(r)\theta = (A^r)_{ij}$ for all $r \geq 1$. [We should regard this computation as taking place in the group ring of the sign group. The generalization to arbitrary gain graphs and digraphs is obvious.] Other sections also discuss signed digraphs [but have little mathematical content]. (**SD, gd: A, Paths**)

F.C. Bussemaker, P.J. Cameron, J.J. Seidel, and S.V. Tsaranov

1991a Tables of signed graphs. EUT Report 91-WSK-01. Dept. of Math. and Computing Sci., Eindhoven Univ. of Technology, Eindhoven, 1991. MR 92g:05001. (**SG: Sw**)

F.C. Bussemaker, D.M. Cvetković, and J.J. Seidel

1976a Graphs related to exceptional root systems. T.H.-Report 76-WSK-05, 91 pp. Dept. of Math., Technological Univ. Eindhoven, Eindhoven, The Netherlands, 1976. Zbl. 338.05116.

The 187 simple graphs with eigenvalues ≥ -2 that are not (negatives of) reduced line graphs of signed graphs are found, with computer aid. By Cameron, Goethals, Seidel, and Shult (1976a), all are represented by root systems E_d, $d = 6, 7, 8$. Most interesting is Thm. 2: each such graph is Seidel-switching equivalent to a line graph of a graph. [*Problem.* Explain this within signed graph theory.] (**LG: p: A**)

1978a Graphs related to exceptional root systems. In: A. Hajnal and Vera T. Sós, eds., *Combinatorics* (Proc. Fifth Hungar. Colloq., Keszthely, 1976), Vol. 1, pp. 185–191. Colloq. Math. Soc. János Bolyai, 18. North-Holland, Amsterdam, 1978. MR 80g:05049. Zbl. 392.05055.

Announces the results of (1976a). **(LG: p: A)**

F.C. Bussemaker, R.A. Mathon, and J.J. Seidel

1979a Tables of two-graphs. TH-Report 79-WSK-05. Dept. of Math., Technological Univ. Eindhoven, Eindhoven, The Netherlands, 1979. Zbl. 439.05032. **(TG)**

1981a Tables of two-graphs. In: S.B. Rao, ed., *Combinatorics and Graph Theory* (Proc. Sympos., Calcutta, 1980), pp. 70–112. Lecture Notes in Math., 885. Springer-Verlag, Berlin, 1981. MR 84b:05055. Zbl. 482.05024.

"The most important tables from" (1979a). **(TG)**

Leishen Cai and Baruch Schieber

1997a A linear-time algorithm for computing the intersection of all odd cycles in a graph, *Discrete Appl. Math.* 73 (1997), 27–34. MR 97g:05149. Zbl. 867.05066.

By the negative-subdivision trick (subdividing each positive edge into two negative ones), the algorithm will find the intersection of all negative polygons of a signed graph. **(P, sg: Fr: Alg)**

Peter J. Cameron

See also F.C. Bussemaker.

1977a Automorphisms and cohomology of switching classes. *J. Combin. Theory Ser. B* 22 (1977), 297–298. MR 58 #16382. Zbl. 331.05113, (344.05128).

The first step towards (1977b), Thm. 3.1. **(TG: Aut)**

†1977b Cohomological aspects of two-graphs. *Math. Z.* 157 (1977), 101–119. MR 58 #21779. Zbl. 353.20004, (359.20004).

Introducing the cohomological theory of two-graphs. A two-graph τ is a 2-coboundary in the complex of GF(2)-cochains on $E(K_n)$. [The 1-cochains are the signed complete graphs, equivalently the graphs that are their negative subgraphs. Cf. D.E. Taylor (1977a).] Write Z_i, Z^i, B^i for the i-cycle, i-cocycle, and i-coboundary spaces. Switching a signed complete graph means adding a 1-cocycle to it; a switching class of signed complete graphs is viewed as a coset of Z^1 and is equivalent to a two-graph.

Take a group $\mathfrak{G}$ of automorphisms of τ. Special cohomology elements $\gamma \in H^1(\mathfrak{G}, B^1)$ and $\beta \in H^2(\mathfrak{G}, \tilde{B}^0)$ (where $\tilde{B}^0 = \{0, V(K_n)\}$, the reduced 0-coboundary group) are defined. Thm. 3.1: $\gamma = 0$ iff $\mathfrak{G}$ fixes a graph in τ. Thm. 5.1: $\beta = 0$ iff $\mathfrak{G}$ can be realized as an automorphism group of the canonical double covering graph of τ (viewing τ as a switching class of signed complete graphs). Conditions are explored for the vanishing of γ (related to Harries and Liebeck (1978a)) and β.

Z^1 is the annihilator of $Z_1 =$ the space of even-degree simple graphs; the theorems of Mallows and Sloane (1975a) follow immediately. More generally: Lemma 8.2: Z^i is the annihilator of Z_i. Thm. 8.3. The numbers of isomorphism types of i-cycles and i-cocycles are equal, for $i = 1, \ldots, n-2$. §8 concludes with discussion of possible generalizations, e.g., to oriented two-graphs (replacing GF(2) by GF(3)*) and double coverings of complete digraphs (Thms. 8.6, 8.7). [A full ternary analog is developed in Cheng and Wells (1986a).] **(TG: Sw, Aut, E. G)**

1979a Cohomological aspects of 2-graphs. II. In: C.T.C. Wall, ed., *Homological Group Theory* (Proc. Sympos., Durham, 1977), Ch. 11, pp. 241–244. London Math. Soc. Lecture Note Ser. 36. Cambridge Univ. Press, Cambridge, 1979. MR 81a:05061. Zbl. 461.20001.

Exposition of parts of (1977b) with a simplified proof of the connection between β and γ. (**TG: Aut, E, G, Exp**)

1980a A note on generalized line graphs. *J. Graph Theory* 4 (1980), 243–245. MR 81j:05089. Zbl. 403.05048, (427.05039).

[For generalized line graphs see Zaslavsky (1984c).] If two generalized line graphs are isomorphic, their underlying graphs and cocktail-party attachments are isomorphic, with small exceptions related to exceptional isomorphisms and automorphisms of root systems. The proof, along the lines of Cameron, Goethals, Seidel, and Shult (1976a), employs the canonical vector representation of the underlying signed graph. (**sg: LG: Aut, G**)

1994a Two-graphs and trees. Graph Theory and Applications (Proc., Hakone, 1990). *Discrete Math.* 127 (1994), 63–74. MR 95f:05027. Zbl. 802.05042.

Let T be a tree. Construction 1 (simplifying Seidel and Tsaranov (1990a)): Take all triples of edges such that none separates the other two. This defines a two-graph on $E(T)$ [whose underlying signed complete graph is described by Tsaranov (1992a)]. Construction 2: Choose $X \subseteq V(T)$. Take all triples of end vertices of T whose minimal connecting subtree has its trivalent vertex in X. The two-graphs $(V, \mathcal{T})$ that arise from these constructions are characterized by forbidden substructures, namely, the two-graphs of (1) C_5 and C_6; (2) C_5. Also, trees that yield identical two-graphs are characterized. (**TG**)

1995a Counting two-graphs related to trees. *Electronic J. Combin.* 2 (1995), Research Paper 4. MR 95j:05112. Zbl. 810.05031.

Counting two-graphs of the types constructed in (1994a). (**TG: E**)

P.J. Cameron, J.M. Goethals, J.J. Seidel, and E.E. Shult

††1976a Line graphs, root systems, and elliptic geometry. *J. Algebra* 43 (1976), 305–327. MR 56 #182. Zbl. 337.05142. Reprinted in Seidel (1991a), pp. 208–230.

The essential idea is that graphs with least eigenvalue ≥ -2 are represented by the angles of root systems. It follows that line graphs are so represented. [Similarly, signed graphs with largest eigenvalue ≤ 2 are represented by the inner products of root systems, as in Vijayakumar *et al.* These include the line graphs of signed graphs as in Zaslavsky (1984c), since simply signed graphs are represented by B_n or C_n with a few exceptions . The representation of ordinary graphs by all-negative signed graphs is motivated in Zaslavsky (1984c).] (**LG: sg: A, G, Sw**)

P.J. Cameron, J.J. Seidel, and S.V. Tsaranov

1994a Signed graphs, root lattices, and Coxeter groups. *J. Algebra* 164 (1994), 173–209. MR 95f:20063. Zbl. 802.05043.

A generalized Coxeter group $\mathrm{Cox}(\Sigma)$ and a Tsaranov group $\mathrm{Ts}(\Sigma)$ are defined via Coxeter relations and an extra relation for each negative polygon in Σ. They generalize Coxeter groups of tree Coxeter graphs and the Tsaranov groups of a two-graph ($|\Sigma| = K_n$; see Seidel and Tsaranov (1990a)). A new operation of "local switching" is introduced, which changes the edge set of Σ but preserves the associated groups.

§2, “Signed graphs”, proves some well-known properties of switching and reviews interesting data from Bussemaker, Cameron, Seidel, and Tsaranov (1991a). §3, “Root lattices and Weyl groups”: The “intersection matrix” $2I + A(\Sigma)$ is a hyperbolic Gram matrix of a basis of $\mathbb{R}^n$ whose vectors form only angles $\pi/2, \pi/3, 2\pi/3$. To these vectors are associated the lattice $L(\Sigma)$ of their integral linear combinations and the Weyl group $W(\Sigma)$ generated by reflecting along the vectors. W is finite iff $2I + A(\Sigma)$ is positive definite (Thm. 3.1). *Problem* 3.6. Determine which Σ have this property. §4 introduces local switching to partially solve *Problem* 4.1: Which signed graphs generate the same lattice? Results and some experimental data are reported. All-negative signed graphs play a special role. §6, “Coxeter groups”: The relationship between the Coxeter and Weyl groups of Σ. $\mathrm{Cox}(\Sigma)$ is $\mathrm{Cox}(|\Sigma|)$ with additional relations for antinegative (i.e., negative in $-\Sigma$) induced polygons. §7: “Signed complete graphs”. §8: “Tsaranov groups” of signed K_n’s §9: “Two-graphs arising from trees” (as in Seidel and Tsaranov (1990a)).

Dictionary: “(Γ, f)” $= \Sigma = (\Gamma, \sigma)$. “Fundamental signing” = all-negative signing, giving the antibalanced switching class. “The balance” of a cycle (i.e., polygon) = its sign $\sigma(C)$; “the parity” $= \sigma(-C)$ where $-C = C$ with all signs negated. “Even” = positive and “odd” = negative (referring to “parity”). “The balance” of Σ = the partition of all polygons into positive and negative classes $\mathcal{C}^+$ and $\mathcal{C}^-$; this is the bias on $|\Sigma|$ due to the signing and should not be confused with the customary meaning of “balance”, i.e., all polygons are positive.

[A more natural definition of the intersection matrix would be $2I - A$. Then signs would be negative to those in the paper. The need for “parity” would be obviated, ordinary graphs would correspond to all-positive signings (and those would be “fundamental”), and the extra Coxeter relations would pertain to negative induced polygons.] **(SG: A, G, Sw(Gen), lg)**

P.J. Cameron and Albert L. Wells, Jr.

1986a Signatures and signed switching classes. *J. Combin. Theory Ser. B* 40 (1986), 344–361. MR 87m:05115. Zbl. 591.05061. **(SG: TG: Gen)**

Sue Ann Campbell

See J. Bélair.

E. Rodney Canfield

See E.A. Bender.

D.-S. Cao

See R. Simion.

Dorwin Cartwright

See also T.C. Gleason; Harary, Norman, and Cartwright (1965a, etc.)

Dorwin Cartwright and Terry C. Gleason

1966a The number of paths and cycles in a digraph. *Psychometrika* 31 (1966), 179–199. MR 33 #5377. Zbl. (e: 143.43702). **(SD: A, Paths)**

Dorwin Cartwright and Frank Harary

1956a Structural balance: a generalization of Heider’s theory. *Psychological Rev.* 63 (1956), 277–293. Reprinted in: Dorwin Cartwright and Alvin Zander, eds., *Group Dynamics: Research and Theory*, Second Edition, pp. 705–726. Harper and Row, New York, 1960. Also reprinted in: Samuel Leinhardt, ed., *Social Networks: A Developing Paradigm*, pp. 9–25. Academic Press, New York, 1977.

Expounds Harary (1953a, 1955a) with sociological discussion. Proposes to measure imbalance by the proportion of balanced polygons (the "degree of balance") or polygons of length $\leq k$ (("degree of k-balance"). **(PsS, SG: B, Fr)**

1968a On the coloring of signed graphs. *Elem. Math.* 23 (1968), 85–89. MR 38 #2053. Zbl. 155, 317 (e: 155.31703). **(SG: Cl)**

1970a Ambivalence and indifference in generalizations of structural balance. *Behavioral Sci.* 15 (1970), 497–513. **(SD, B)**

1977a A graph theoretic approach to the investigation of system-environment relationships. *J. Math. Sociology* 5 (1977), 87–111. MR 56 #2477. Zbl. 336.92026. **(SD: Cl)**

1979a Balance and clusterability: an overview. In: Paul W. Holland and Samuel Leinhardt, eds., *Perspectives on Social Network Research* (Proc. Sympos., Dartmouth Coll., Hanover, N.H., 1975), Ch. 3, pp. 25–50. Academic Press, New York, 1979. **(SG, SD, VS: B, Fr, Cl, A: Exp)**

Adolfo Casari

See F. Barahona.

Paul A. Catlin

1979a Hajós' graph-coloring conjecture: variations and counterexamples. *J. Combin. Theory Ser. B* 26 (1979), 268–274. MR 81g:05057. Zbl. 385.05033, 395.05033.

Thm. 2: If Γ is 4-chromatic, $[-\Gamma]$ contains a subdivision of $[-K_4]$ (an "odd-K_4"). [*Question.* Can this possibly be a signed-graph theorem? For instance, should it be interpreted as concerning the 0-free (signed) chromatic number of $-\Gamma$?] **(p: col)**

Seth Chaiken

1982a A combinatorial proof of the all minors matrix tree theorem. *SIAM J. Algebraic Discrete Methods* 3 (1982), 319–329. MR 83h:05062. **(SD, SG, GG: A, I)**

1996a Oriented matroid pairs, theory and an electrical application. In: Joseph E. Bonin, James G. Oxley, and Brigitte Servatius, eds., *Matroid Theory* (Proc., Seattle, 1995), pp. 313–331. Contemp. Math., Vol. 197. Amer. Math. Soc., Providence, R.I., 1996. MR 97e:05058.

Connects a problem on common covectors of two subspaces of $\mathbb{R}^m$, and more generally of a pair of oriented matroids, to the problem of sign-solvability of a matrix and the even-cycle problem for signed digraphs. **(Sol, sd: P, Alg)**

1996b Open problem 5. A problem about common covectors and bases in oriented matroid pairs. In: Joseph E. Bonin, James G. Oxley, and Brigitte Servatius, eds., *Matroid Theory* (Proc., Seattle, 1995), pp. 415–417. Contemp. Math., Vol. 197. Amer. Math. Soc., Providence, R.I., 1996.

Possible generalizations to oriented matroids of sign-nonsingularity of a matrix. **(Sol, SD: P)**

Vijaya Chandru, Collette R. Coullard, and Donald K. Wagner

1985a On the complexity of recognizing a class of generalized networks. *Oper. Res. Letters* 4 (1985), 75–78. MR 87a:90144. Zbl. 565.90078.

Determining whether a gain graph with real multiplicative gains has a balanced polygon, i.e., is not contrabalanced, is NP-hard. So is determining whether a real matrix is projectively equivalent to the incidence matrix of a contrabalanced real gain graph. **(GN, Bic: I, Alg)**

Chung-Chien Chang and Cheng-Ching Yu

1990a On-line fault diagnosis using the signed directed graph. *Industrial and Engineering Chem. Res.* 29 (1990), 1290–1299.

Modifies the method of Iri, Aoki, O'Shima, and Matsuyama (1979a) of constructing the diagnostic signed digraph, e.g. by considering transient and steady-state situations. **(SD: Appl, Ref)**

Gerard J. Chang

See J.-H. Yan.

A. Charnes, M. Kirby, and W. Raike

1966a Chance-constrained generalized networks. *Oper. Res.* 14 (1966), 1113–1120. Zbl. (e: 152.18302). **(GN)**

A. Charnes and W.M. Raike

1966a One-pass algorithms for some generalized network problems. *Oper. Res.* 14 (1966), 914–924. Zbl. (e: 149.38106). **(GN: I)**

Gary Chartrand

See also M. Behzad.

1977a *Graphs as Mathematical Models.* Prindle, Weber and Schmidt, Boston, 1977. MR 58 #9947. Zbl. 384.05029. **(SG: B, Cl)**

Gary Chartrand, Heather Gavlas, Frank Harary, and Michelle Schultz

1994a On signed degrees in signed graphs. *Czechoslovak Math. J.* 44 (1994), 677–690. MR 95g:05084. Zbl. 837.05110.

Net degree sequences (i.e., $d^+ - d^-$; called "signed degree sequences") of signed simple graphs. A Havel–Hakimi-type reduction formula, but with an indeterminate length parameter [improved in Yan, Lih, Kuo, and Chang (1997a)]; a determinate specialization to complete graphs. A necessary condition for a sequence to be a net degree sequence. Examples: paths, stars, double stars. [Continued in Yan, Lih, Kuo, and Chang (1997a).]

[This is a special case of weighted degree sequences of K_n with integer edge weights chosen from a fixed interval of integers. In this case the interval is $[-1, +1]$. There is a theory of such sequences; however, it seems not to yield the exact results obtained here.] **(SGw: N)**

[One can interpret net degrees as the net indegrees ($d^{\text{in}} - d^{\text{out}}$) of certain bidirected graphs. Change the positive (negative) edges to extroverted (resp., introverted). Then we have the net indegree sequence of an oriented $-\Gamma$. *Problem* 1. Generalize this paper and Yan, Lih, Kuo, and Chang (1997a) to all bidirected (simple, or simply signed) graphs, especially K_n's. *Problem* 2. Find an Erdős–Gallai-type characterization of net degree sequences of signed simple graphs. *Problem* 3. Characterize the separated signed degree sequences of signed simple graphs, where the separated signed degree is $(d^+(v), d^-(v))$. *Problem* 4. Generalize Problem 3 to edge k-colorings of K_n.] **(SG: O: N)**

Gary Chartrand, Frank Harary, Hector Hevia, and Kathleen A. McKeon

1992a On signed graphs with prescribed positive and negative graphs. *Vishwa Internat. J. Graph Theory* 1 (1992), 9–18. MR 93m:05095.

What is the smallest order of an edge-disjoint union of two (isomorphism types of) simple graphs, Γ and Γ'? Bounds, constructions, and special cases. (The union is called a signed graph with Γ and Γ' as its positive and negative subgraphs.) Thm. 13: If Γ' is bipartite (i.e., the union is balanced) with color classes V_1' and V_2', the minimum order $= \min(|V_1'|, |V_2'|) + \max(|V|, |V_1'|, |V_2'|)$. (**wg**)(**SG: B**)

Guy Chaty

1988a On signed digraphs with all cycles negative. *Discrete Appl. Math.* 20 (1988), 83–85. MR 89d:05148. Zbl. 647.05028.

Clarifies the structure of "free cyclic" digraphs and shows they include strong "upper" digraphs (see Harary, Lundgren, and Maybee (1985a)). (**SD: Str**)

P.D. Chawathe and G.R. Vijayakumar

1990a A characterization of signed graphs represented by root system D_∞. *European J. Combin.* 11 (1990), 523–533. MR 91k:05071. Zbl. 764.05090. (**SG: G**)

Jianer Chen, Jonathan L. Gross, and Robert G. Rieper

1994a Overlap matrices and total imbedding distributions. *Discrete Math.* 128 (1994), 73–94. MR 95f:05031. Zbl. 798.05017. (**SG: T, Sw**)

Ying Cheng

1986a Switching classes of directed graphs and H-equivalent matrices. *Discrete Math.* 61 (1986), 27–40. MR 88a:05075. Zbl. 609.05039.

This article studies what are described as $\mathbb{Z}_4$-gain graphs Φ with underlying simple graph Γ. [However, see below.] They are regarded as digraphs D, the gains being determined by D as follows: $\varphi(u,v) = 1$ or 2 if (u,v) is an arc, 2 or 3 if (v,u) is an arc. [N.B. Γ is not uniquely determined by D.] Cheng's "switching" is gain-graph switching but only by switching functions $\eta : V \to \{0,2\}$; I will call this "semiswitching". His "isomorphisms" are vertex permutations that are automorphisms of Γ; I will call them "Γ-isomorphisms". The objects of study are equivalence classes under semiswitching (semiswitching classes) or semiswitching and Γ-isomorphism (semiswitching Γ-isomorphism classes). Prop. 3.1 concerns adjacency of vertex orbits of a Γ-isomorphism that preserves a semiswitching class (call it a Γ-automorphism of the class). Thm. 4.3 gives the number of semiswitching Γ-isomorphism classes. Thm. 5.2 characterizes those Γ-automorphisms of a semiswitching class that fix an element of the class; Thm. 5.3 characterizes the Γ-isomorphisms g that fix an element of every g-invariant semiswitching class.

[Likely the right viewpoint, as is hinted in §6, is that the edge labels are not $\mathbb{Z}_4$-gains but weights from the set $\{\pm1, \pm2, \ldots, \pm k\}$ with $k = 2$. Then semiswitching is ordinary signed switching, and so forth. However, I forbear to reinterpret everything here.]

In §6, $\mathbb{Z}_4$ is replaced by $\mathbb{Z}_{2k}$ [but this should be $\{\pm1, \pm2, \ldots, \pm k\}$]; semiswitching functions take values $0, k$ only. Generalizations of Sects. 3, 4 are sketched and are applied to find the number of H-equivalent matrices of given size with entries $\pm1, \pm1, \ldots, \pm k$. ($H$- [or Hadamard] equivalence means permuting rows and columns and scaling by -1.)

(**sg, wg, GG: Sw, Aut, E**)

Ying Cheng and Albert L. Wells, Jr.

1984a Automorphisms of two-digraphs. (Summary). Proc. Fifteenth Southeastern Conf. on Combin., Graph Theory and Computing (Baton Rouge, 1984). *Congressus Numer.* 45 (1984), 335–336. MR 86c:05004c (volume).

A two-digraph is a switching class of $\mathbb{Z}_3$-gain graphs based on K_n.

(**gg, SD: Sw, Aut**)

†1986a Switching classes of directed graphs. *J. Combin. Theory Ser. B* 40 (1986), 169–186. MR 87g:05104. Zbl. 565.05034, (579.05027).

This exceptionally interesting paper treats a digraph as a ternary gain graph Φ (i.e., with gains in $\mathrm{GF}(3)^+$) based on K_n. A theory of switching classes and triple covering graphs, analogous to that of signed complete graphs (and of two-graphs) is developed. The approach, analogous to that in Cameron (1977b), employs cohomology. The basic results are those of general gain-graph theory specialized to the ternary gain group and graph K_n.

The main results concern a switching class $[\Phi]$ of digraphs and an automorphism group $\mathfrak{A}$ of $[\Phi]$. §3, "The first invariant": Thm. 3.2 characterizes, by a cohomological obstruction γ, the pairs $([\Phi], \mathfrak{A})$ such that some digraph in $[\Phi]$ is fixed. Thm. 3.5 is an [interestingly] more detailed result for cyclic $\mathfrak{A}$. §4: "Triple covers and the second invariant". Digraph triple covers of the complete digraph are considered. Those that correspond to gain covering graphs of ternary gain graphs Φ are characterized ("cyclic triple covers", pp. 178–180). Automorphisms of Φ and its triple covering $\tilde{\Phi}$ are compared. Given $([\Phi], \mathfrak{A})$, Thm. 4.4 finds the cohomological obstruction β to lifting $\mathfrak{A}$ to $\tilde{\Phi}$. Thm. 4.7 establishes an equivalence between γ and β in the case of cyclic $\mathfrak{A}$.

§5: "Enumeration". Thm. 5.1 gives the number of isomorphism types of switching classes on n vertices, based on the method of Wells (1984a) for signed graphs. §6: "The fixed signing property". Thm. 6.1 characterizes the permutations of $V(K_n)$ that fix a gain graph in every invariant switching class, based on the method of Wells (1984a).

Dictionary: "Alternating function" on $X \times X$ = $\mathrm{GF}(3)^+$-valued gain function on K_X. (**gg, SD: Sw, Aut, E, Cov**)

Hyeong-ah Choi, Kazuo Nakajima, and Chong S. Rim

1989a Graph bipartization and via minimization. *SIAM J. Discrete Math.* 2 (1989), 38–47. MR 89m:90132. Zbl. 677.68036.

Vertex biparticity (the fewest vertices to delete to get a bipartite graph) is compared to edge biparticity (for cubic graphs) and studied algorithmically. (**p: Fr**)

Debashish Chowdhury

1986a *Spin Glasses and Other Frustrated Systems.* Princeton Univ. Press, Princeton, and World Scientific, Singapore, 1986.

Includes brief survey of how physicists look upon frustration. See esp. §1.3, "An elementary introduction to frustration", where the signed square lattice graph illustrates balance vs. imbalance; Ch. 20, "Frustration, gauge invariance, defects and SG [spin glasses]", discussing planar duality (see e.g. Barahona (1982a), "gauge theories", where gains are in the orthogonal or unitary group (and switching is called "gauge transformation" by physicists), and functions of interest to physicists; Addendum to Ch. 10, pp. 378–379, mentioning results on when the proportion of negative bonds is fixed and on gauge theories. (**Phys: SG, GG, VS, Fr: Exp, Ref**)

San Yan Chu

See S.-L. Lee.

V. Chvátal
See J. Akiyama.

F.W. Clarke, A.D. Thomas, and D.A. Waller
1980a Embeddings of covering projections of graphs. *J. Combin. Theory Ser. B* 28 (1980), 10–17. MR 81f:05066. Zbl. 351.05126, (416.05069). **(gg: T)**

Bernard P. Cohen
See J. Berger.

Edith Cohen and Nimrod Megiddo
1989a Strongly polynomial-time and NC algorithms for detecting cycles in dynamic graphs. In: *Proceedings of the Twenty First Annual ACM Symposium on Theory of Computing* (Seattle, 1989), pp. 523–534.
Partial version of (1993a). **(GD: B: Alg)**

1991a Recognizing properties of periodic graphs. In: Peter Gritzmann and Bernd Sturmfels, eds., *Applied geometry and Discrete Mathematics: The Victor Klee Festschrift*, pp. 135–146. DIMACS Ser. Discrete Math. Theoret. Computer Sci., Vol. 4. Amer. Math. Soc., Providence, R.I., and Assoc. Computing Mach., 1991. MR 92g:05166. Zbl. 753.05047.
Given: a gain graph Φ with gains in $\mathbb{Z}^d$ (a "static graph"). Found: algorithms for (1) connected components and (2) bipartiteness of the covering graph $\tilde{\Phi}$ (the "periodic graph") and, (3) given costs on the edges of Φ, for a minimum-average-cost spanning tree in the covering graph. Many references to related work. **(GG(Cov): Alg, Ref)**

1992a New algorithms for generalized network flows. In: D. Dolev, Z. Galil, and M. Rodeh, eds., *Theory of Computing and Systems* (Proc., Haifa, 1992), pp. 103–114. Lect. Notes in Computer Sci., Vol. 601. Springer-Verlag, Berlin, 1992. MR 94b:68023 (book).
Preliminary version of (1994a), differing only slightly.
(GN: Alg)(sg: O: Alg)

1993a Strongly polynomial-time and NC algorithms for detecting cycles in periodic graphs. *J. Assoc. Comput. Mach.* 40 (1993), 791–830. MR 96h:05182. Zbl. 782.68053.
Looking for a closed walk ("cycle") with gain 0 in a gain digraph with (additive) gains in $\mathbb{Q}^d$. [Cf. Kodialam and Orlin (1991a).] **(GD: B: Alg)**

1994a New algorithms for generalized network flows. *Math. Programming* 64 (1994), 325–336. MR 95k:90111. Zbl. 816.90057.
Maximize the fraction of demand satisfied by a flow on a network with gains. Positive real gains in §3. Bidirected networks with positive gains in §4; these are more general than networks with arbitrary non-zero real gains.
(GN: Alg)(sg: O: Alg)

1994b Improved algorithms for linear inequalities with two variables per inequality. *SIAM J. Comput.* 23 (1994), 1313–1347. MR 95i:90040. Zbl. 833.90094.
(GN(I): D: Alg)

Charles J. Colbourn and Derek G. Corneil
1980a On deciding switching equivalence of graphs. *Discrete Appl. Math.* 2 (1980), 181–184. MR 81k:05090. Zbl. 438.05054.
Deciding switching equivalence of graphs is polynomial-time equivalent to graph isomorphism. **(TG: Alg)**

L. Collatz

1978a Spektren periodischer Graphen. *Resultate Math.* 1 (1978), 42–53. MR 80b:05042. Zbl. 402.05054.

Introducing periodic graphs: these are connected canonical covering graphs $\Gamma = \tilde{\Phi}$ of finite $\mathbb{Z}^d$-gain graphs Φ. The "spectrum" of Γ is the set of all eigenvalues of $A(||\Phi||)$ for all possible Φ. The spectrum, while infinite, is contained in the interval $[-r, r]$ where r is the largest eigenvalue of each $A(||\Phi||)$ [the "index" of von Below (1994a)]. The inspiration is tilings. **(GG(Cov): A)**

Barry E. Collins and Bertram H. Raven

1968a Group structure: attraction, coalitions, communication, and power. In: Gardner Lindzey and Elliot Aronson, eds., *The Handbook of Social Psychology*, Second Edition, Vol. 4, Ch. 30, pp. 102–204. Addison-Wesley, Reading, Mass., 1968.

"Graph theory and structural balance," pp. 106–109. **(PsS: SG: Exp, Ref)**

Ph. Combe and H. Nencka

1995a Non-frustrated signed graphs. In: J. Bertrand *et al.*, eds., *Modern Group Theoretical Methods in Physics* (Proc. Conf. in Honour of Guy Rideau, Paris, 1995), pp. 105–113. Math. Phys. Stud., Vol. 18. Kluwer, Dordrecht, 1995. MR 96j:05105.

Σ is balanced iff a fundamental system of polygons is balanced [as is well known; see *i.a.* Popescu (1979a), Zaslavsky (1981b)]. An algorithm [incredibly complicated, compared to the obvious method of tracing a spanning tree] to determine all vertex signings of Σ that switch it to all positive. Has several physics references. **(SG: B, Fr, Alg, Ref)**

F.G. Commoner

1973a A sufficient condition for a matrix to be totally unimodular. *Networks* 3 (1973), 351–365. MR 49 #331. Zbl. 352.05012. **(SD: B)**

Michele Conforti, Gérard Cornuéjols, Ajai Kapoor, and Kristina Vušković

1994a Recognizing balanced $0, \pm 1$ matrices. In: *Proceedings of the 5th Annual ACM-SIAM Symposium on Discrete Algorithms* (Arlington, Va., 1994), pp. 103–111. Assoc. for Computing Machinery, New York, 1994. MR 95e:05022. Zbl. 867.05014. **(SG: B)**

1995a A mickey-mouse decomposition theorem. In: Egon Balas and Jens Clausen, eds., *Integer Programming and Combinatorial Optimization* (4th Internat. IPCO COnf., Copenhagen, 1995, Proc.), pp. 321–328. Lecture Notes in Computer Sci., Vol. 920. Springer, Berlin, 1995. MR 96i:05139. Zbl. 875.90002 (book).

The structure of graphs that are signable to be "without odd holes": that is, so that each triangle is negative and each chordless polygon of length greater than 3 is positive. Proof based on Truemper (1982a). **(SG: B, Str)**

1997a Universally signable graphs. *Combinatorica* 17 (1997), 67–77. MR 98g:05134. Zbl. 980.00112.

Γ is "universally signable" if it can be signed so as to make every triangle negative and the holes independently positive or negative at will. Such graphs are characterized by a decomposition theorem which leads to a polynomial-time recognition algorithm. **(SG: B, Str)**

1999a Even and odd holes in cap-free graphs. *J. Graph Theory* 30 (1999), 289–308. **(SG: B)**

20xxa Triangle-free graphs that are signable without even holes. Submitted **(SG: B)**

20xxb Even-hole-free graphs. Part I. Decomposition Theorem. Submitted **(SG: B)**

20xxc Even-hole-free graphs. Part II. Recognition algorithm. Submitted **(SG: B)**

Michele Conforti, Gérard Cornuéjols, and Kristina Vuškoviić

1999a Balanced cycles and holes in bipartite graphs. *Discrete Math.* 199 (1999), 27–33. MR 99j:05119. **(SG: B)**

Michele Conforti and Ajai Kapoor

1998a A theorem of Truemper. In: Robert E. Bixby, E. Andrew Boyd, and Roger Z. Ríos-Mercado, eds., *Integer Programming and Combinatorial Optimization* (6th Internat. IPCO Conf., Houston, 1998, Proc.), pp. 53–68. Lecture Notes in Computer Sci., Vol. 1412. Springer, Berlin, 1998. Zbl. 907.90269

A new proof of Truemper's theorem on prescribed hole signs; discussion of applications. **(SG: B)**

Derek G. Corneil

See C.J. Colbourn and Seidel (1991a).

Gérard Cornuéjols

See also M. Conforti.

20xxa *Combinatorial Optimization: Packing and Covering.* In preparation.

The topic is linear optimization over a clutter, esp. a "binary clutter", which is the class of negative circuits of a signed binary matroid. The class $\mathcal{C}_-(\Sigma)$ is an important example (see Seymour 1977a), as is its blocker $b\mathcal{C}_-(\Sigma)$ [which is the class of minimal balancing edge sets; hence the frustration index $l(\Sigma) =$ minimum size of a member of the blocker.

Ch. 5: "Graphs without odd-K_5 minors", i.e., signed graphs without $-K_5$ as a minor. Some esp. interesting results: Thm. 5.0.7 (special case of Seymour (1977a), Main Thm.): The clutter of negative polygons of Σ has the "Max-Flow Min-Cut Property" (Seymour's "Mengerian" property) iff Σ has no $-K_4$ minor. Conjecture 5.1.11 is Seymour's (1981a) beautiful conjecture (his "weak MFMC" is here called "ideal"). §5.2 reports the partial result of Guenin (1998b). (See also §8.4.)

Def. 6.2.6 defines a signed graph "$G(A)$" of a $0, \pm 1$-matrix A, whose transposed incidence matrix is a submatrix of A. §6.3.3: "Perfect $0, \pm 1$-matrices, bidirected graphs and conjectures of Johnson and Padberg" (1982a), associates a bidirected graph with a system of 2-variable pseudoboolean inequalities; reports on Sewell (1997a) (*q.v.*).

§8.4: "On ideal binary clutters", reports on Cornuéjols and Guenin (20xxa), Guenin (1998a), and Novick and Sebö (1995a) (*qq.v.*).

(S(M), SG: M, G, I(Gen), O: Exp, Ref, Exr)

Gérard Cornuéjols and Bertrand Guenin

20xxa On ideal binary clutters and a conjecture of Seymour. In preparation

A partial proof of Seymour's (1981a) conjecture. Main Thm.: A binary clutter is ideal if it has as a minor none of the circuit clutter of F_7, $\mathcal{C}_-(-K_5)$ or its blocker, or $\mathcal{C}_-(-K_4)$ or its blocker. Important are the lift and extended lift matroids, $L(M, \sigma)$ and $L_0(M, \sigma)$, defined as in signed graph theory. [See Cornuéjols (20xxa), §8.4.] **(S(M), SG: M, G)**

S. Cosares

See L. Adler.

Collette R. Coullard

See also V. Chandru.

Collette R. Coullard, John G. del Greco, and Donald K. Wagner

††1991a Representations of bicircular matroids. *Discrete Appl. Math.* 32 (1991), 223–240. MR 92i:05072. Zbl. 755.05025.

§4: §4.1 describes 4 fairly simple types of "legitimate" graph operation that preserve the bicircular matroid. Thm. 4.11 is a converse: if Γ_1 and Γ_2 have the same connected bicircular matroid, then either they are related by a sequence of legitimate operations, or they belong to a small class of exceptions, all having order ≤ 4, whose bicircular matroid isomorphisms are also described. This completes the isomorphism theorem of Wagner (1985a). §5: If finitely many graphs are related by a sequence of legitimate operations (so their bicircular matroids are isomorphic), then they have contrabalanced real gains whose incidence matrices are row equivalent. These results are also found by a different approach in Shull *et al.* (1989a, 20xxa).

(**Bic: Str, I**)

1993a Recognizing a class of bicircular matroids. *Discrete Appl. Math.* 43 (1993), 197–215. MR 94i:05021. Zbl. 777.05036. (**Bic: Alg**)

1993b Uncovering generalized-network structure in matrices. *Discrete Appl. Math.* 46 (1993), 191–220. MR 95c:68179. Zbl. 784.05044. (**GN: Bic: I, Alg**)

Yves Crama

See also E. Boros.

1989a Recognition problems for special classes of polynomials in 0–1 variables. *Math. Programming* A44 (1989), 139–155. MR 90f:90091. Zbl. 674.90069.

Balance and switching are used to study pseudo-Boolean functions. (Sects. 2.2 and 4.) (**SG: B, Sw**)

Yves Crama and Peter L. Hammer

1989a Recognition of quadratic graphs and adjoints of bidirected graphs. Combinatorial Math.: Proc. Third Internat. Conf. *Ann. New York Acad. Sci.* 555 (1989), 140–149. MR 91d:05044. Zbl. 744.05060.

"Adjoint" = unoriented positive part of the line graph of a bidirected graph. "Quadratic graph" = graph that is an adjoint. Recognition of adjoints of bidirected simple graphs is NP-complete. (**sg: O: LG: Alg**)

Yves Crama, Peter L. Hammer, and Toshihide Ibaraki

1986a Strong unimodularity for matrices and hypergraphs. *Discrete Appl. Math.* 15 (1986), 221–239. MR 88a:05105. Zbl. 647.05042.

§7: Signed hypergraphs, with a surprising generalization of balance.

(**S(Hyp): B**)

Y. Crama, M. Loebl, and S. Poljak

1992a A decomposition of strongly unimodular matrices into incidence matrices of digraphs. *Discrete Math.* 102 (1992), 143–147. MR 93g:05097. Zbl. 776.05071.

(**SG**)

William H. Cunningham

See J. Aráoz.

Dragoš M. Cvetković

See also F.C. Bussemaker and M. Doob.

1978a The main part of the spectrum, divisors and switching of graphs. *Publ. Inst. Math (Beograd) (N.S.)* 23 (37) (1978), 31–38. MR 80h:05045. Zbl. 423.05028.

1995a Star partitions and the graph isomorphism problem. *Linear Algebra Appl.* 39 (1995), 109–132. MR 97b:05105. Zbl. 831.05043.

Pp. 128–130 discuss switching-equivalent graphs. Some of the theory is invariant, hence applicable to two-graphs. [*Question.* How can this be generalized to signed graphs and their switching classes?] **(TG: A)**

Dragos M. Cvetković, Michael Doob, Ivan Gutman, and Aleksandar Torgašev

1988a *Recent Results in the Theory of Graph Spectra.* Ann. Discrete Math., 36. North-Holland, Amsterdam, 1988. MR 89d:05130. Zbl. 634.05034.

Signed graphs on pp. 44-45. All-negative signatures are implicated in the infinite-graph eigenvalue theorem of Torgašev (1982a), Thm. 6.29 of this book. **(SG, p: A: Exp, Ref)**

Dragoš M. Cvetković, Michael Doob, and Horst Sachs

1980a *Spectra of Graphs: Theory and Application.* VEB Deutscher Verlag der Wissenschaften, Berlin, 1980. Copublished as: Pure and Appl. Math., Vol. 87. Academic Press, New York-London, 1980. MR 81i:05054. Zbl. 458.05042.

§4.6: Signed digraphs with multiple edges are employed to analyze the characteristic polynomial of a digraph. (Signed) switching, too. Pp. 187–188: Exercises involving Seidel switching and the Seidel adjacency matrix. Thm. 6.11 (Doob (1973a)): The even-cycle matroid determines the eigenvaluicity of -2. §7.3: "Equiangular lines and two-graphs."

(SD, p, TG: Sw, A, G: Exp, Exr, Ref)

1995a *Spectra of Graphs: Theory and Applications.* Third edn. Johann Ambrosius Barth, Heidelberg, 1995. MR 96b:05108. Zbl. 824.05046.

Appendices update the second, slightly corrected edn. of (1980a), beyond the updating in Cvetković, Doob, Gutman, and Torgašev (1988a). App. B.3, p. 381: mentions work of Vijayakumar (q.v.). P. 422: Pseudo-inverse graphs ($A(\Gamma)^{-1} = A(\Sigma)$ for some balanced Σ; $|\Sigma|$ is the "pseudo-inverse" of Γ).

(SD, p, TG: A, Sw, G, B: Exp, Exr, Ref)

Dragoš Cvetković, Michael Doob, and Slobodan Simić

1980a Some results on generalized line graphs. *C. R. Math. Rep. Acad. Sci. Canada* 2 (1980), 147–150. MR 81f:05136. Zbl. 434.05057.

Abstract of (1981a). **(sg: LG, A(LG), Aut(LG))**

1981a Generalized line graphs. *J. Graph Theory* 5 (1981), 385–399. MR 82k:05091. Zbl. 475.05061. **(sg: LG, A(LG), Aut(LG))**

Dragoš M. Cvetković and Slobodan K. Simić

1978a Graphs which are switching equivalent to their line graphs. *Publ. Inst. Math. (Beograd) (N.S.)* 23 (37) (1978), 39–51. MR 80c:05108. Zbl. 423.05035. **(sw: LG)**

E. Damiani, O. D'Antona, and F. Regonati

1994a Whitney numbers of some geometric lattices. *J. Combin. Theory Ser. A* 65 (1994), 11–25. MR 95e:06019. Zbl. 793.05037.

Dowling lattices are an example. **(gg: M: N)**

O. D'Antona

See E. Damiani.

George B. Dantzig

1963a *Linear Programming and Extensions.* Princeton Univ. Press, Princeton, N.J., 1963. MR 34 #1073. Zbl. (e: 108.33103).

Chapter 21: "The weighted distribution problem." 21-2: "Linear graph structure of the basis." **(GN: M(Bases))**

Prabir Das and S.B. Rao

1983a Alternating eulerian trails with prescribed degrees in two edge-colored complete graphs. *Discrete Math.* 43 (1983), 9–20. MR 84k:05069. Zbl. 494.05020.

Given an all-negative bidirected K_n and a positive integer $f_i = 2g_i$ for each vertex v_i. There is a connected subgraph having in-degree and out-degree $= g_i$ at v_i iff there is a g-factor of introverted and one of extroverted edges and the degrees satisfy a complicated degree condition. Generalizes Thm. 1 of Bánkfalvi and Bánkfalvi (1968a). [See Bang-Jensen and Gutin (1997a) for how to convert an edge 2-coloring to an orientation of an all-negative graph and for further developments on alternating walks.] **(p: o)**

James A. Davis

1963a Structural balance, mechanical solidarity, and interpersonal relations. *Amer. J. Sociology* 68 (1963), 444–463. Reprinted with minor changes in: Joseph Berger, Morris Zelditch, Jr., and Bo Anderson, eds., *Sociological Theories in Progress, Vol. One*, Ch. 4, pp. 74–101. Houghton Mifflin, Boston, 1966. Also reprinted in: Samuel Leinhardt, ed., *Social Networks: A Developing Paradigm*, pp. 199–217. Academic Press, New York, 1977. **(PsS: SG, WG: Exp)**

1967a Clustering and structural balance in graphs. *Human Relations* 20 (1967), 181–187. Reprinted in: Samuel Leinhardt, ed., *Social Networks: A Developing Paradigm*, pp. 27–33. Academic Press, New York, 1977.

James A. Davis and Samuel Leinhardt

1972a The structure of positive interpersonal relations in small groups. In: Joseph Berger, Morris Zelditch, Jr., and Bo Anderson, eds., *Sociological Theories in Progress, Vol. Two*, Ch. 10, pp. 218–251. Houghton Mifflin, Boston, 1972.

Analysis of a sociological theory incorporating structural balance in relation to both randomly generated and observational data. **(PsS: SG)**

A.C. Day, R.B. Mallion, and M.J. Rigby

1983a On the use of Riemannian surfaces in the graph-theoretical representation of Möbius systems. In: R.B. King, ed., *Chemical Applications of Topology and Graph Theory* (Proc. Sympos., Athens, Ga., 1983), pp. 272–284. Stud. Physical Theoret. Chem., 28. Elsevier, Amsterdam, 1983. MR 85h:05039.

A clumsy but intriguing way of representing some signed (or more generally, $\mathbb{Z}_n$-weighted) graphs: via 2-page (or, n-page) looseleaf book embedding (all vertices are on the spine and each edge is in a single page), with an edge in page k weighted by the "sheet parity index" $\alpha_k = (-1)^k$ (or, $e^{2\pi ik/n}$). (Described in the [unnecessary] terminology of an n-sheeted Riemann surface.) [A $\mathbb{Z}_n$-weighted) graph has such a representation iff the subgraph of edges with each weight is outerplanar.]

A variation to get switching classes of signed polygons: replace α_k by the "connectivity parity index" $\alpha_k^{\sigma_k}$ where $\sigma_k =$ number of edges in page k. [The variation is valid only for polygons.] [*Questions* vaguely suggested by these procedures: Which signed graphs can be switched so that the edges of each sign form an outerplanar graph? Also, the same for gain graphs. And there are many similar questions: for instance, the same ones with "outerplanar" replaced by "planar."]

(SG: sw, A, T, Chem: Exp, Ref)(WG: A, T: Exp, Ref)

Anne Delandtsheer

1995a Dimensional linear spaces. In: F. Buekenhout, ed., *Handbook of Incidence Geometry: Buildings and Foundations*, Ch. 6, pp. 193–294. North-Holland, Amsterdam,1995. MR 96k:51012. Zbl. 950.23458.

"Dimensional linear space" (DLS) = simple matroid. §2.7: "Dowling lattices," from Dowling (1973b). §6.7: "Subgeometry-closed and hereditary classes of DLS's," from Kahn and Kung (1982a). In §2.6, the "Enough modular hyperplanes theorem" from Kahn and Kung (1986a). (**GG: M: Exp**)

John G. del Greco

See also C.R. Coullard.

1992a Characterizing bias matroids. *Discrete Math.* 103 (1992), 153–159. MR 93m:05050. Zbl. 753.05021.

How to decide, given a matroid M and a biased graph Ω, whether $M = G(\Omega)$. (**GG: M**)

B. Derrida, Y. Pomeau, G. Toulouse, and J. Vannimenus

1979a Fully frustrated simple cubic lattices and the overblocking effect. *J. Physique* 40 (1979), 617–626. (**SG: Phys, Fr**)

1980a Fully frustrated simple cubic lattices and phase transitions. *J. Physique* 41 (1980), 213–221. MR 80m:82020. (**Phys: SG**)

Michel Marie Deza and Monique Laurent

1997a *Geometry of Cuts and Metrics.* Algorithms and Combinatorics, Vol. 15. Springer, Berlin, 1997. MR 98g:52001. Zbl. 885.52001.

A main object of interest is the cut polytope, which is the bipartite subgraph polytope (see Barahona, Grötschel, and Mahjoub (1985a)) of K_n, i.e. the balanced subgraph polytope (Poljak and Turzík (1987a)) of $-K_n$. §4.5, "An application to statistical physics", briefly discusses the spin glass application. §26.3, "The switching operation", discusses graph switching and its generalization to sets. §30.3, "Circulant inequalities", mentions Poljak and Turzík (1987a, 1992a). No explicit mention of signed graphs. (**p: fr: G: Exp**)

Persi Diaconis

See K.S. Brown.

V. Di Giorgio

1974a 2-modules dans un graphe: equilibre et coequilibre d'un bigraphe—application taxonomique. *Bull. Math. Soc. Sci. Math. R. S. Roumanie (N.S.)* 18 (66) (1974), 81–102 (1975). MR 57 #16124. Zbl. 324.05127. (**SG: B**)

Yvo M.I. Dirickx and M.R. Rao

1974a Networks with gains in discrete dynamic programming. *Management Sci.* 20 (1974), No. 11 (July, 1974), 1428–1431. MR 50 #12279. Zbl. 303.90052. (**GN: M(bases)**)

Michael Doob

See also D.M. Cvetković.

1970a A geometric interpretation of the least eigenvalue of a line graph. In: *Proc. Second Chapel Hill Conference on Combinatorial Mathematics and Its Applications* (1970), pp. 126–135. Univ. of North Carolina at Chapel Hill, Chapel Hill, N.C., 1970. MR 42 #2959. Zbl. 209, 554 (e: 209.55403).

A readable, tutorial introduction to (1973a) (without matroids). (**ec: LG, I, A(LG)**)

1973a An interrelation between line graphs, eigenvalues, and matroids. *J. Combin. Theory Ser. B* 15 (1973), 40–50. MR 55 #12573. Zbl. 245.05125, (257.05132).

Along with Simões-Pereira (1973a), introduces to the literature the even-cycle matroid $G(-\Gamma)$ [previously invented by Tutte, unpublished]. The multiplicity of -2 as an eigenvalue (in characteristic 0) equals the number of independent even polygons $= n - \operatorname{rk} G(-\Gamma)$. In characteristic p there is a similar theorem, but the pertinent matroid is $G(\Gamma)$ if $p = 2$ and, when $p|n$, the matroid has rank 1 greater than otherwise [a fact that mystifies me]. **(EC: LG, I, A(LG))**

1974a Generalizations of magic graphs. *J. Combin. Theory Ser. B* 17 (1974), 205–217. MR 51 #274. Zbl. 271.05128, (287.05124). **(ec: I)**

1974b On the construction of magic graphs. In: F. Hoffman *et al.*, eds., *Proceedings of the Fifth Southeastern Conference on Combinatorics, Graph Theory and Computing* (Boca Raton, 1974), pp. 361–374. Utilitas Math. Publ. Inc., Winnipeg, Man., 1974. MR 53 #13039. Zbl. 325.05123. **(ec: I)**

1978a Characterizations of regular magic graphs. *J. Combin. Theory Ser. B* 25 (1978), 94–104. MR 58 #21840. Zbl. 384.05054. **(ec: I)**

Michael Doob and Dragoš Cvetković

1979a On spectral characterizations and embeddings of graphs. *Linear Algebra Appl.* 27 (1979), 17–26. MR 81d:05050. Zbl. 417.05025. **(sg: LG, A(LG))**

Patrick Doreian, Roman Kapuscinski, David Krackhardt, and Janusz Szczypula

1996a A brief history of balance through time. *J. Math. Sociology* 21 (1996), 113–131. Reprinted in Patrick Doreian and Frans N. Stokman, eds., *Evolution of Social Networks*, pp. 129–147. Gordon and Breach, Australia, Amsterdam, etc., 1997.

§2.3: "A method for group balance". Describes the negation-minimal index of clusterability (generalized imbalance) from Doreian and Mrvar (1996a). **(SG: B, Cl: Fr(Gen): Exp)**

§3.3: "Results for group balance". Describes results from analysis of data on a small (social) group, in terms of frustration index l and a clusterability index $\min_{k>2} 2P_{k,.5}$ (slightly different from the index in Doreian and Mrvar (1996a)), finding both measures (but more so the latter) decreasing with time. **(PsS: B, Cl: Fr(Gen))**

Patrick Doreian and Andrej Mrvar

1996a A partitioning approach to structural balance. *Social Networks* 18 (1996), 149–168.

They propose indices for clusterability that generalize the frustration index. Fix $k \geq 2$ and $\alpha \in [0,1]$. For a partition π of V into k parts, they define $P(\pi) := \alpha n_- + (1-\alpha)n_+$, where $n_+ := |E_+\langle\pi\rangle| =$ number of positive edges between parts and $n_- := |E_-{:}\pi| =$ number of negative edges within parts. The first proposed measure is $\min P(\pi)$, minimized over k-partitions. [Call this $P_{k,\alpha}$.] A second suggestion is the "negation-minimal index of generalized imbalance [i.e., of clusterability]", the smallest number of edges whose negation (equivalently, deletion) makes Σ clusterable; it $= \min_k 2P_{k,.5}$. [Note that $P(\pi)$ effectively generalizes the Potts Hamiltonian as given by Welsh (1993a). *Question.* Does $P(\pi)$ fit into an interesting generalized Potts model?] [$P(\pi)$ also resembles the Potts Hamiltonian in

Fischer and Hertz (1991a) (q.v. for a related research question).]

They employ a local optimization algorithm to evaluate $P_{k,\alpha}$ and find an optimal partition: random descent from partition to neighboring partition, where π and π' are neighbors if they differ by transfer of one vertex or exchange of two vertices between two parts. This was found to work well if repeated many times. [A minimizing partition into at most k parts is equivalent to a ground state of the k-spin Potts model in the form given by Welsh (1993a), but not quite of that in Fischer and Hertz (1991a).]

Terminology: $P(\pi)$ is called the "criterion function" [more explicitly, one might call it the 'clusterability (adjusted by α)' of π]; clusterability is "k-balance" or "generalized balance". The partition's parts are "plus-sets". Signed digraphs are employed in the notation but direction is ignored.

(**SD: sg: B, Cl: Fr(Gen), Alg, PsS**)

1996b Structural balance and partitioning signed graphs. In: A. Ferligoj and A. Kramberger, eds., *Developments in Data Analysis*, pp. 195–208. Metodološki zvezki, Vol. 12. FDV, Ljubljana, Slovenia, 1996.

Similar to (1996a). Some lesser theoretical detail; some new examples. The k-clusterability index $P_{k,\alpha}$ (see (1996a)) is compared for different values of k, seeking the minimum. [But for which value(s) of α is not stated.] Interesting observation: optimal values of k were small. It is said that positive edges between parts are far more acceptable socially than negative edges within parts [thus, in the criterion function α should be rather near 1].

(**SD: sg: B, Cl: Fr(Gen), Alg, PsS**)

W. Dörfler

1977a Double covers of graphs and hypergraphs. In: *Beitrage zur Graphentheorie und deren Anwendungen* (Proc. Internat. Colloq., Oberhof, D.D.R., 1977), pp. 67–79. Technische Hochschule, Ilmenau, 1977. MR 82c:05074. Zbl. 405.05055.

(**SG: Cov, LG**)(**SD, S(Hyp): Cov**)

1978a Double covers of hypergraphs and their properties. *Ars Combinatoria* 6 (1978), 293–313. MR 82d:05085. Zbl. 423.050532. (**S(Hyp): Cov, LG**)

Lynne L. Doty

See F. Buckley.

Peter Doubilet

1971a Dowling lattices and their multiplicative functions. In: *Möbius Algebras* (Proc. Conf., Waterloo, Ont., 1971), pp. 187–192. Univ. of Waterloo, Ont., 1971, reprinted 1975. MR 50 #9605. Zbl. 385.05008. (**GG: M**)

Peter Doubilet, Gian-Carlo Rota, and Richard Stanley

1972a On the foundations of combinatorial theory (VI): The idea of generating function. In: *Proceedings of the Sixth Berkeley Symposium on Mathematical Statistics and Probability* (Berkeley, Calif., 1970/71), Vol. II: *Probability Theory*, pp. 267–318. Univ. of California Press, Berkeley, Calif., 1972. MR 53 #7796. Zbl. 267.05002. Reprinted in: Gian-Carlo Rota, *Finite Operator Calculus*, pp. 83–134. Academic Press, New York, 1975. MR 52 #119. Zbl. 328.05007. Reprinted again in: Joseph P.S. Kung, ed., *Gian-Carlo Rota on Combinatorics: Introductory Papers and Commentaries*, pp. 148–199. Birkhäuser, Boston, 1995. MR 99b:01027. Zbl. 841.01031.

Section 5.3: Brief gain-graphic treatment of Dowling lattices. (**GG: M**)

T.A. Dowling

1971a Codes, packings, and the critical problem. In: *Atti del Convegno di Geometria Combinatoria e sue Applicazioni (Perugia, 1970)*, pp. 209–224. Ist. Mat., Univ. di Perugia, Perugia, Italy, 1971. MR 49 #2438. Zbl. 231.05029.

Pp. 221–223: The first intimations of Dowling lattices/geometries/matroids, as in (1973a, 1973b), and their higher-weight relatives (see Bonin 1993a). **(gg, Gen: M)**

1973a A q-analog of the partition lattice. Ch. 11 in: J. N. Srivastava *et al.*, eds., *A Survey of Combinatorial Theory* (Proc. Internat. Sympos., Ft. Collins, Colo., 1971), pp. 101–115. North-Holland, Amsterdam, 1973. MR 51 #2954. Zbl. 259.05023.

Linear-algebraic progenitor of (1973b). Treats the Dowling lattice of group $\mathrm{GF}(q)^\times$ as naturally embedded in $\mathrm{PG}^{n-1}(q)$. Interesting is p. 105, Remark: One might generalize some results to any ambient (simple) matroid. **(gg: M, N, GG)**

††1973b A class of geometric lattices based on finite groups. *J. Combin. Theory Ser. B* 14 (1973), 61–86. MR 46 #7066. Zbl. 247.05019. Erratum. *Ibid.* 15 (1973), 211. MR 47 #8369. Zbl. 264.05022.

Introduces the Dowling lattices of a group, treated as lattices of group-labelled partial partitions. Equivalent to the bias matroid of complete $\mathfrak{G}$-gain graph $\mathfrak{G}K_n^\bullet$. [The gain-graphic approach was known to Dowling (1973a, p. 109) but first published in Doubilet, Rota, and Stanley (1972a).] Isomorphism, vector representation, Whitney numbers and characteristic polynomial. [The first and still fundamental paper.] **(gg: M, N)**

Pauline van den Driessche

See van den Driessche (under 'V').

J.M. Drouffe

See R. Balian.

Richard A. Duke, Paul Erdös, and Vojtěch Rödl

1992a Cycle-connected graphs. *Discrete Math.* 108 (1992), 261–278. MR 94a:05106. Zbl. 776.05057.

All graphs are simple. This is one of four related papers that prove extremal results concerning subgraphs of $-\Gamma$ within which every two edges belong to a balanced polygon of length at most $2k$, for all or particular k. Typical theorem: Let $F_l(n,m) =$ the largest number $m' = m'(n,m)$ such that every $-\Gamma$ with $|V| = n$ and $|E| \geq m$ has a subgraph Σ' with $|E'| = m'$ in which every two edges belong to a balanced polygon of length at most l. For $m = m(n) \geq n^{3/2}$, there is a constant $c_3 > 0$ such that $F_l(n,m) \leq c_3 m^2 n^{-2}$ for all l. (§2, (2).) [*Problem.* Extend these extremal results in an interesting way to arbitrary signed simple graphs, or to simply signed graphs (no repeated edges with the same sign). (Merely allowing positive edges in addition to negative ones just makes the problem easier. Something more is required.)] **(p: b(Polygons): X)**

Arne Dür

1986a *Möbius Functions, Incidence Algebras and Power Series Representations.* Lecture Notes in Math., Vol. 1202. Springer-Verlag, Berlin, 1986. MR 88m:05005. Zbl. 592.05006.

Dowling lattices are an example of a categorial approach to incidence-algebra techniques in Ch. IV, §7. Computed are the characteristic polynomial and second kind of Whitney numbers. Binomial concavity, hence unimodality of

the latter [cf. Stonesifer (1975a)] is proved by showing that a suitable generating polynomial has only distinct, negative roots [cf. Benoumhani (1999a)]. **(gg: M: N)**

Paul H. Edelman and Victor Reiner

1994a Free hyperplane arrangements between A_{n-1} and B_n. *Math. Z.* 215 (1994), 347–365. MR 95b:52021. Zbl. 793.05122.

Characterizes all $\Sigma \supseteq +K_n$ whose bias matroid $G(\Sigma)$ is supersolvable, free, or inductively free. Essentially, iff the negative links form a threshold graph. [Continued in Bailey (20xxa). Generalized in part to arbitrary gain groups in Zaslavsky (20xxh).] **(sg: M, G, col)**

1996a Free arrangments and rhombic tilings. *Discrete Computat. Geom.* 15 (1996), 307–340. MR 97f:52019. Zbl. 853.52013. Erratum. *Discrete Computat. Geom.* 17 (1997), 359. MR 97k:52013. Zbl. 853.52013.

Paul H. Edelman and Michael Saks

1979a Group labelings of graphs. *J. Graph Theory* 3 (1979), 135–140. MR 80j:05071. Zbl. 411.05059.

Given Γ and abelian group $\mathfrak{A}$. Vertex and edge labellings $\lambda : V \to \mathfrak{A}$ and $\eta : E \to \mathfrak{A}$ are "compatible" if $\lambda(v) = \sum_e \eta(e)$ for every vertex v, the sum taken over all edges incident with v. λ is "admissible" if it is compatible with some η. Admissible vertex labellings are characterized (differently for bipartite and nonbipartite graphs) and the number of edge labelings compatible with a given vertex labelling is computed. [Dual in a sense to Gimbel (1988a).] **(WG, VS: B(D), E)**

Jack Edmonds

See also J. Aráoz and E.L. Lawler (1976a).

1965a Paths, trees, and flowers. *Canad. J. Math.* 17 (1965), 449–467. MR 31 #2165. Zbl. 132, 209 (e: 132.20903).

Followed up by much work, e.g., Witzgall and Zahn (1965a); see Ahuja, Magnanti, and Orlin (1993a) for some references. **(p: o: i, Alg)**

1965b Maximum matching and a polyhedron with 0, 1-vertices. *J. Res. Nat. Bur. Standards (U.S.A.) Sect. B* 69B (1965), 125–130. MR 32 #1012. Zbl. (e: 141.21802).

Alludes to the polyhedron of Edmonds and Johnson (1970a). **(p: o: I, G)**

Jack Edmonds and Ellis L. Johnson

††1970a Matching: a well-solved class of integral linear programs. In: Richard Guy *et al.*, eds., *Combinatorial Structures and Their Applications* (Proc. Calgary Internat. Conf., Calgary, 1969), pp. 89–92. Gordon and Breach, New York, 1970. MR 42 #2799. Zbl. 258.90032.

Introduces "bidirected graphs". A "matching problem" is an integer linear program with nonnegative and possibly bounded variables and otherwise only equality constraints, whose coefficient matrix is the incidence matrix of a bidirected graph. No proofs. [See Aráoz, Cunningham, Edmonds, and Green-Krótki (1983a) for further work.] **(sg: O: I, Alg, G)**

Richard Ehrenborg and Margaret A. Readdy

1998a On valuations, the characteristic polynomial, and complex subspace arrangements. *Advances Math.* 134 (1998), 32–42. MR 98m:52018. Zbl. 906.52004.

An abstract additive approach to the characteristic polynomial, applied in particular to "divisor Dowling arrangements" of hyperplanes and certain

interpolating arrangements. Let $\Phi = \mathfrak{G}_1 K_1 \cup \cdots \cup \mathfrak{G}_n K_n$, where $V(K_i) = \{v_1, \ldots, v_i\}$ and $\mathfrak{G}_1 \geq \cdots \geq \mathfrak{G}_n$ is a chain of subgroups of a gain group $\mathfrak{G} = \mathfrak{G}_1$. When $\mathfrak{G}$ is finite cyclic, the complex hyperplane representation of $\Phi^\bullet$ is a "divisor Dowling arrangement". [Its polynomial equals the chromatic polynomial of $\Phi^\bullet$, which is easily computed via gain-graph coloring without the restriction to cyclic gain group. The same appears to be true for the other arrangements treated herein.] **(gg: M: G, N)**

1999a On flag vectors, the Dowling lattice, and braid arrangements. *Discrete Computat. Geom.* 21 (1999), 389–403.

The Dowling lattice is that of a finite cyclic group $\mathbb{Z}_k$. Thm. 4.9 is a recursive formula for its flag h-vector (in the form of the **ab**-index). Thm. 5.2 is a similar formula for the $\mathbf{c}, 2\mathbf{d}$-index of the face lattices of the real root system arrangements A_n and B_n, whose intersection lattices are the Dowling lattices of $\mathbb{Z}_1$ and $\mathbb{Z}_2$. §6 presents a combinatorial description of the face lattice of B_n [which it is interesting to compare with that in Zaslavsky (1991b)]. **(gg: M: G, N)**

A. Ehrenfeucht, T. Harju, and G. Rozenberg

1996a Group based graph transformations and hierarchical representations of graphs. In: J. Cuny, H. Ehrig, G. Engels and G. Rozenberg, eds., *Graph Grammars and Their Application to Computer Science* (5th Internat. Workshop, Williamsburg, Va., 1994), pp. 502–520. Lecture Notes in Computer Science, Vol. 1073. Springer-Verlag, Berlin, 1996. MR 97h:68097.

The "heierarchical structure" of a switching class of skew gain graphs based on K_n. **(gg: K: Sw)**

1997a 2-Structures—A framework for decomposition and transformation of graphs. In: Grzegorz Rozenberg, ed., *Handbook of Graph Grammars and Computing by Graph Transformation. Vol. 1: Foundations*, Ch. 6, pp. 401–478. World Scientific, Singapore, 1997. MR 99b:68006 (book). Zbl. 908.68095 (book).

A tutorial (with some new proofs). §6.7: "Dynamic labeled 2-structures". §6.8: "Dynamic ℓ2-structures with variable domains". §6.9: "Quotients and plane trees". §6.10: "Invariants". **(gg: sw: Exp, Ref)**

1997b Invariants of inversive 2-structures on groups of labels. *Math. Structures Computer Sci.* 7 (1997), 303–327. MR 98g:20089. Zbl. 882.05119.

Given a gain graph $(K_n, \varphi, \mathfrak{G})$, a word w in the oriented edges of K_n has a gain $\varphi(w)$; call this $\psi_w(\varphi)$. A "free invariant" is a ψ_w that is an invariant of switching classes. Thm.: There is a number $d = d(K_n, \mathfrak{G})$ such that the group of free invariants is generated by ψ_w with $w = z_1^d \cdots z_k^d u_1 \cdots u_l$ where w_i are triangular cycles (directed!) and u_i are commutators. [The whole paper applies *mutatis mutandis* to arbitrary graphs, the triangular cycles being replaced by any set of cycles containing a fundamental system.] Dictionary: "Inversive 2-structure" = gain graph based on K_n. **(gg: K: Sw, N)**

Andrzej Ehrenfeucht and Grzegorz Rozenberg

1993a An introduction to dynamic labeled 2-structures. In: Andrzej M. Borzyszkowski and Stefan Sokołowski, eds., *Mathematical Foundations of Computer Science 1993* (Proc., 18th Internat. Sympos., MFCS '93, Gdańsk, 1993), pp. 156–173. Lecture Notes in Computer Sci., Vol. 711. Springer-Verlag, Berlin, 1993. MR 95j:68126.

Extended summary of (1994a). **(GG(Gen): K: Sw, Str)**

1994a Dynamic labeled 2-structures. *Math. Structures Comput. Sci.* 4 (1994), 433–455.

MR 96j:68144. Zbl. 829.68099.

They prove that a complicated definition of "reversible dynamic labeled 2-structure" G amounts to a complete graph with a set, closed under switching, of twisted gains in a gain group Δ. The twist is a gain-group automorphism α such that $\lambda(e;x,y) = [\alpha\lambda(e;y,x)]^{-1}$, λ being the gain function. Dictionary: their "domain" $D =$ vertex set, "labeling function" λ (or equivalently, g) = gain function, "alphabet" = gain group, "involution" $\delta = \alpha\circ$ inversion, "δ-selector" $\hat{S} =$ switching function, "transformation induced by $\hat{S}$" = switching by $\hat{S}$; a "single axiom" d.l. 2-structure consists of a single switching class.

Further, they investigate "clans" of G. Given g (i.e., λ), deleting identity-gain edges leaves isolated vertices ("horizons") and forms connected components, any union of which is a "clan" of g. A clan of G is any clan of any $g \in G$. (**GG(Gen): K: Sw, Str**)

1994b Dynamic labeled 2-structures with variable domains. In: J. Karhumäki, H. Maurer, and G. Rozenberg, eds., it Results and Trends in Theoretical Computer Science (Proc., Colloq. in Honor of Arto Alomaa, Graz, 1994), pp. 97–123. Lecture Notes in Computer Science, Vol. 812. Springer-Verlag, Berlin, 1994. MR 95m:68128.

Combinations and decompositions of complete graphs with twisted gains. (**GG(Gen): K: Str, Sw**)

Kurt Eisemann

1964a The generalized stepping stone method for the machine loading model. *Management Sci.* 11 (1964/65), No. 1 (Sept., 1964), 154–176. Zbl. 136, 139 (e: 136.13901). (**GN: I, M(bases)**)

Joyce Elam, Fred Glover, and Darwin Klingman

1979a A strongly convergent primal simplex algorithm for generalized networks. *Math. Oper. Res.* 4 (1979), 39–59. MR 81g:90049. Zbl. 422.90081. (**GN: M(bases), I**)

David P. Ellerman

1984a Arbitrage theory: A mathematical introduction. *SIAM Rev.* 26 (1984), 241–261. MR 85g:90024. Zbl. 534.90014. (**GG: B, I, Flows: Appl, Ref**)

M.N. Ellingham

1991a Vertex-switching, isomorphism, and pseudosimilarity. *J. Graph Theory* 15 (1991), 563–572. MR 92g:05136. Zbl. 802.05057.

Main theorem (§2) characterizes, given two signings of K_n (where n may be infinite) and a vertex set S, when switching S makes the signings isomorphic. [*Problem* 1. Generalize to other underlying graphs. *Problem* 2. Prove an analog for bidirected K_n's.] A corollary (§3) characterizes when vertices u, v of $\Sigma = (K_n, \sigma)$ satisfy $\Sigma^{\{u\}} \cong \Sigma^{\{v\}}$ and discusses when in addition no automorphism of Σ moves u to v. All is done in terms of Seidel (graph) switching (here called "vertex-switching") of unsigned simple graphs. (**k: sw, TG**)

1996a Vertex-switching reconstruction and folded cubes. *J. Combin. Theory Ser. B* 66 (1966), 361–364. MR 96i:05120. Zbl. 856.05071.

Deepens the folded-cube theory of Ellingham and Royle (1992a). Nicely generalizing Stanley (1985a), the number of subgraphs of a signed K_n that are isomorphic to a fixed signed K_m is reconstructible from the s-vertex switching deck if the Krawtchouk polynomial $K_s^n(x)$ has no even zeros between 0 and m. (Closely related to Krasikov and Roditty (1992a), Theorems 5 and 6.) Remark 4: balance equations (Krasikov and Roditty (1987a)) and Krawtchouk polynomials both reflect properties of folded cubes. All is done in terms of Seidel switching of unsigned simple graphs. [It seems clear that the folded cube appears because it corresponds to the effect of switchings on signatures of K_n (or any connected graph), since switching by X and X^c have the same effect. For the bidirected case (Problem 2 under Stanley (1985a)), the unfolded cube should play a similar role. *Question.* When treating a general underlying graph Γ, will a polynomial influenced by $\operatorname{Aut}\Gamma$ replace the Krawtchouk polynomial?] **(k: sw, TG)**

M.N. Ellingham and Gordon F. Royle

1992a Vertex-switching reconstruction of subgraph numbers and triangle-free graphs. *J. Combin. Theory Ser. B* 54 (1992), 167–177. MR 93d:05112. Zbl. 695.05053 (748.05071).

Reconstruction of induced subgraph numbers of a signed K_n from the s-vertex switching deck, dependent on linear transformation and thence Krawtchouk polynomials as in Stanley (1985a). The role of those polynomials is further developed. Done in terms of Seidel switching of unsigned simple graphs, with the advantage of reconstructing arbitrary subgraph numbers as well. A gap is noted in Krasikov and Roditty (1987a), proof of Lemma 2.5. [Methods and results are closely related to Krasikov (1988a) and Krasikov and Roditty (1987a, 1992a).] **(k: sw, TG)**

Gernot M. Engel and Hans Schneider

1973a Cyclic and diagonal products on a matrix. *Linear Algebra Appl.* 7 (1973), 301–335. MR 48 #2160. Zbl. 289.15006. **(gg: Sw)**

1975a Diagonal similarity and equivalence for matrices over groups with 0. *Czechoslovak Math. J.* 25 (100) (1975), 389–403. MR 53 #477. Zbl. 329.15007. **(gg: Sw)**

1980a Matrices diagonally similar to a symmetric matrix. *Linear Algebra Appl.* 29 (1980), 131–138. MR 81k:15017. Zbl. 432.15014. **(gg: Sw)**

R.C. Entringer

1985a A short proof of Rubin's block theorem. In: B.R. Alspach and C.D. Godsil, eds., *Cycles in Graphs*, pp. 367–368. Ann. Discrete Math., Vol. 27. North-Holland Math. Stud., Vol. 115. North-Holland, Amsterdam, 1985. MR 87f:05144. Zbl. 576.05037.

See Erdős, Rubin, and Taylor (1980a). **(p: b)**

H. Era

See J. Akiyama.

Pál Erdős [sometimes, Paul Erdös]

See also B. Bollobás and R.A. Duke.

1996a On some of my favourite theorems. In: D. Miklós, V.T. Sós and T. Szőnyi, eds., *Combinatorics, Paul Erdős is Eighty* (Papers from the Internat. Conf. on Combinatorics, Keszthely, 1993), Vol. 2, pp. 97–132. Bolyai Soc. Math. Studies, 2. János Bolyai Mathematical Society, Budapest, 1996. MR 97g:00002. Zbl. 837.00020 (book).

P. 119 mentions the theorem of Duke, Erdős, and Rödl (1991a) on even polygons.

Pp. 120–121 mention (amongst similar problems) a theorem of Erdős and Hajnal (source not stated): Every all-negative signed graph with chromatic number $\aleph_1$ contains every finite bipartite graph [i.e., every finite, balanced, all-negative signed graph]. [*Problem.* Find generalizations to signed graphs. For instance: *Conjecture.* Every signed graph with chromatic number $\aleph_1$, that does not become antibalanced upon deletion of any finite vertex set, contains every finite, balanced signed graph up to switching equivalence.]

[The MR review: "this is one of the best collections of problems that Erdos has published."] **(p: b: Exp, Ref)**

P. Erdös, R.J. Faudree, A. Gyárfás, and R.H. Schelp

1991a Odd cycles in graphs of given minimum degree. In: Y. Alavi, G. Chartrand, O.R. Oellermann, and A.J. Schenk, eds., *Graph Theory, Combinatorics, and Applications* (Proc. Sixth Quadren. Internat. Conf. Theory Appl. Graphs, Kalamazoo, Mich., 1988), Vol. 1, pp. 407–418. Wiley, New York, 1991. MR 93d:05085. Zbl. 840.05050.

A large, nonbipartite, 2-connected graph with large minimum degree contains a polygon of given odd length or is one of a single type of exceptional graph. [*Question.* Can this be generalized to negative polygons in unbalanced signed graphs?] **(p, sg: Polygons, X)**

P. Erdős, E. Győri, and M. Simonovits

1992a How many edges should be deleted to make a triangle-free graph bipartite? In: G. Halász, L. Lovász, D. Miklós, and T. Szönyi, eds., *Sets, Graphs and Numbers* (Proc., Budapest, 1991), pp. 239–263. Colloq. Math. Soc. János Bolyai, Vol. 60. János Bolyai Math. Soc., Budapest, and North-Holland, Amsterdam, 1992. MR 94b:05104. Zbl. 785.05052.

Assume $|\Sigma|$ simple of order n and $\not\supseteq$ a fixed graph Δ. Results on frustration index l of antibalanced Σ if Δ is 3-chromatic, esp. C_3. Thm.: If $|E| > n^2/5 - o(n^2)$, then $l(\Sigma) < n^2/25 - o(n^2)$. *Conjecture* (Erdős): For $\Delta = C_3$ the hypothesis on $|E|$ is unnecessary. [*Question* 1(a). Is the answer different when Σ need not be antibalanced? *Question* 2(a). Exclude a fixed signed graph whose signed chromatic number $= 1$. *Question* 3(a). In particular, exclude $-K_3$. *Question* 4(a). Exclude $-K_l$. *Question* 5(a). Exclude an unbalanced C_l. *Questions* 1–5(b). Even if $l(\Sigma)$ cannot be estimated, is there always an extremal graph that is antibalanced—as when no graph is excluded, by Petersdorf (1966a)?] **(p: X)**

Paul Erdös, Arthur L. Rubin, and Herbert Taylor

1980a Choosability in graphs. In: *Proceedings of the West Coast Conference on Combinatorics, Graph Theory and Computing* (Arcata, Calif., 1979), pp. 125–157. Congressus Numer., XXVI. Utilitas Math. Publ. Inc., Winnipeg, Man., 1980. MR 82f:05038. Zbl. 469.05032.

Rubin's block theorem (Thm. R, p. 136): a block graph, not complete or an odd polygon, contains an induced even polygon with at most one chord. [See also Entringer (1985a).] [*Question.* Does this generalize to signed graphs, Rubin's block theorem being the antibalanced case? Rubin's 2-choosability theorem, p. 132, is also tantalizingly reminiscent of antibalanced graphs, but in reverse.] **(p: Str, b)**

Cloyd L. Ezell

1979a Observations on the construction of covers using permutation voltage assignments. *Discrete Math.* 28 (1979), 7–20. MR 81a:05040. Zbl. 413.05005. **(GG: T, Cov, sw)**

Arthur M. Farley and Andrzey Proskurowski

1981a Computing the line index of balance of signed outerplanar graphs. Proc. Twelfth

Southeastern Conf. on Combinatorics, Graph Theory and Computing (Baton Rouge, 1981), Vol. I. *Congressus Numer.* 32 (1981), 323–332. MR 83m:68119. Zbl. 489.68065.

Calculating frustration index is NP-complete, since it is more general than max-cut. However, for signed outerplanar graphs with bounded size of bounded faces, it is solvable in linear time. [It is quickly solvable for signed planar graphs. See Katai and Iwai (1978a), Barahona (1981a, 1982a), and more.] (**SG: Fr**)

M. Farzan

1978a Automorphisms of double covers of a graph. In: *Problemes Combinatoires et Theorie des Graphes* (Colloq. Internat., Orsay, 1976), pp. 137–138. Colloques Internat. du CNRS, 260. Editions du C.N.R.S., Paris, 1978. MR 81a:05063. Zbl. 413.05064.

A "double cover of a graph" means the double cover of a signing of a simple graph. (**sg: Cov, Aut**)

R.J. Faudree

See P. Erdős.

Katherine Faust

See S. Wasserman.

N.T. Feather

1971a Organization and discrepancy in cognitive structures. *Psychological Rev.* 78 (1971), 355–379.

A suggestion for defining balance in weighted digraphs: pp. 367–369. (**PsS: B: Exp**)(**WD: B**)

Lori Fern, Gary Gordon, Jason Leasure, and Sharon Pronchik

20xxa Matroid automorphisms and symmetry groups. Submitted.

Consider a subgroup W of the hyperoctahedral group O_n that is generated by reflections. Let $M(W)$ be the vector matroid of the vectors corresponding to reflections in W. The possible direct factors of any automorphism group of $M(W)$ are S_k, O_k, and O_k^+. The proof is stricly combinatorial, via signed graphs. (**SG: Aut, G**)

Miroslav Fiedler

1957a Uber qualitative Winkeleigenschaften der Simplexe. *Czechoslovak Math. J.* 7 (82) (1957), 463–478. MR 20 #1252. Zbl. 93, 336 (e: 093.33602). (**SG: G**)

1957b Einige Satze aus der metrischen Geometrie der Simplexe in euklidischen Raumen. *Schr. Forschungsinst. Math.* 1 (1957), 157. MR 19, 303. Zbl. 89, 167 (e: 089.16706). (**SG: G**)

1961a Uber die qualitative Lage des Mittelpunktes der ungeschriebenen Hyperkugel im n-Simplex. *Comment. Math. Univ. Carolin.* 2, No. 1 (1961), 1–51. Zbl. 101, 132 (e: 101.13205). (**SG: G**)

1964a Some applications of the theory of graphs in matrix theory and geometry. In: *Theory of Graphs and Its Applications* (Proc. Sympos., Smolenice, 1963), pp. 37–41. Publ. House Czechoslovak Acad. Sci., Prague, 1964. MR 30 #5294. Zbl. (e: 163.45605). (**SG: G**)

1967a Graphs and linear algebra. In: *Theory of Graphs: International Symposium* (Rome, 1966), pp. 131–134. Gordon and Breach, New York; Dunod, Paris, 1967. MR 36 #6313. Zbl. 263.05124. (**SG: G**)

1969a Signed distance graphs. *J. Combin. Theory* 7 (1969), 136–149. MR 39 #4034. Zbl. 181, 260 (e: 181.26001). **(SG: G)**

1970a Poznámka o distancnich grafech [A remark on distance graphs] (in Czech). In: *Matematika (geometrie a teorie grafu)* [Mathematics (Geometry and Graph Theory)], pp. 85–88. Univ. Karlova, Prague, 1970. MR 43 #3143. Zbl. 215.50203. **(SG: G)**

1975a Eigenvectors of acyclic matrices. *Czechoslovak Math. J.* 25 (100) (1975), 607–618. MR 52 #8151. Zbl. 325.15014. **(sg: Trees: A)**

1985a Signed bigraphs of monotone matrices. In: Horst Sachs, ed., *Graphs, Hypergraphs and Applications* (Proc. Internat. Conf., Eyba, 1984), pp. 36–40. Teubner-Texte zur Math., B. 73. B.G. Teubner, Leipzig, 1985. MR 87m:05121. Zbl. 626.05023. **(SG: A: Exp)**

Miroslav Fiedler and Vlastimil Ptak

1967a Diagonally dominant matrices. *Czechoslovak Math. J.* 17 (92) (1967), 420–433. MR 35 #6704. Zbl. (e: 178.03402). **(GG: Sw, b)**

1969a Cyclic products and an inequality for determinants. *Czechoslovak Math. J.* 19 (94) (1969), 428–451. MR 40 #1409. Zbl. 281.15014. **(gg: Sw)**

Joseph Fiksel

1980a Dynamic evolution in societal networks. *J. Math. Sociology* 7 (1980), 27–46. MR 81g:92023(q.v.). Zbl. 434.92022. **(SG: Cl, VS)**

Steven D. Fischer

1993a Signed Poset Homology and q-Analog Möbius Functions. Ph.D. thesis, Univ. of Michigan, 1993.

§1.2: "Signed posets". Definition of signed poset: a positively closed subset of the root system B_n whose intersection with its negative is empty. (Following Reiner (1990).) Equivalent to a partial ordering of $\pm[n]$ in which negation is a self-duality and each dual pair of elements is comparable. [This is really a special type of signed poset. The latter restriction does not hold in general.]

Relevant contents: Ch. 2: "Cohen-Macaulay signed posets", §2.2: "EL-labelings of posets and signed posets", and shellability. Ch. 3: "Euler characteristics", and a fixed-point theorem. §5.1: "The homology of the signed posets S_Π" (a particular example). App. A: "Open problems", several concerning signed posets.

[Partially summarized by Hanlon (1996a).] **(S: sg, o, G, N)**

K.H. Fischer and J.A. Hertz

1991a *Spin Glasses.* Cambridge Studies in Magnetism: 1. Cambridge Univ. Press, Cambridge, Eng., 1991. MR 93m:82019.

§2.5, "Frustration", discusses the spin glass Ising model (essentially, signed graphs) in square and cubical lattices, including the "Mattis model" (a switching of all positive signs), as well as a vector analog, the "XY" model (planar spins) and (p. 46) even a general gain-graph model with switching-invariant Hamiltonian. From the point of view of physics (mainly theoretical physics). **(Phys: SG: Fr, Sw: Exp, Ref)**

§3.7: "The Potts glass". The Hamiltonian (without edge weights) is $H = -\frac{1}{2}\sum \sigma(e_{ij})(k\delta(s_i, s_j) - 1)$. [It is not clear that the authors intend to permit negative edges. If they are allowed, H is rather like Doreian and Mrvar's (1996a) $P(\pi)$. *Question.* Is there a worthwhile generalized signed and

weighted Potts model with Hamiltonian that specializes both to this form of H and to P?] [Also cf. Welsh (1993a) on the Ashkin–Teller–Potts model.] (**Phys: sg, cl: Exp**)

P.C. Fishburn and N.J.A. Sloane

1989a The solution to Berlekamp's switching game. *Discrete Math.* 74 (1989), 263–290. MR 90e:90151. Zbl. 664.94024.

The maximum frustration index of a signed $K_{t,t}$, which equals the covering radius of the Gale–Berlekamp code, is evaluated for $t \leq 10$, thereby extending results of Brown and Spencer (1971a). See Table 1. (**sg: Fr**)

Claude Flament

1958a L'etude mathematique des structures psycho-sociales. *L'Annee Psychologique* 58 (1958), 119–131.

Signed graphs are treated on pp. 126–129. (**SG: B, PsS: Exp**)

1963a *Applications of Graph Theory to Group Structure.* Prentice-Hall, Englewood Cliffs, N.J., 1963. MR 28 #1014. Zbl. 141, 363 (e: 141.36301).

English edition of (1965a). Ch. 3: "Balancing processes." (**SG: K: B, Alg**)

1965a *Theorie des graphes et structures sociales.* Math. et sci. de l'homme, Vol. 2. Mouton and Gauthier-Villars, Paris, 1965. MR 36 #5018. Zbl. 169, 266 (e: 169.26603).

Ch. III: "Processus d'equilibration." (**SG: K: B, Alg**)

1970a Equilibre d'un graphe, quelques resultats algebriques. *Math. Sci. Humaines*, No. 30 (1970), 5–10. MR 43 #4704. Zbl. 222.05124.

1979a Independent generalizations of balance. In: Paul W. Holland and Samuel Leinhardt, eds., *Perspectives on Social Network Research* (Proc. Sympos., Dartmouth Coll., Hanover, N.H., 1975), Chapter 10, pp. 187–200. Academic Press, New York, 1979. (**SG: B, PsS**)

C.M. Fortuin and P.W. Kasteleyn

1972a On the random cluster model. I. Introduction and relation to other models. *Physica* 57 (1972), 536–564. MR 50 #12107.

Most of the paper recasts classical physical and other models (percolation, ferromagnetic Ising, Potts, graph coloring, linear resistance) in a common form that is generalized in §7, "Random cluster model". The "cluster (generating) polynomial" $Z(\Gamma; p, \kappa)$, where $p \in \mathbb{R}^E$ and $\kappa \in \mathbb{R}$, is a 1-variable specialization of the general parametrized dichromatic polynomial. In the notation of Zaslavsky (1992b) it equals $Q_\Gamma(q, p; \kappa, 1)$, where $q_e = 1 - p_e$. Thus it partially anticipates the general polynomials of Przytycka and Przytycki (1988a), Traldi (1989a), and Zaslavsky (1992b) that were based on Kauffman's (1989a) sign-colored Tutte polynomial. A spanning-tree expansion is given only for the resistance model. A feature [that seems not to have been taken up by subsequent workers] is the differentiation relation (7.7) connecting $\partial \ln Z/\partial q_e$ with [I think!] the expectation that the endpoints of e are disconnected in a subgraph. [Grimmett (1994a) summarizes subsequent work in the probabilistic direction.] (**sgc: Gen: N, Phys**)

J.-C. Fournier

1979a *Introduction à la notion de matroïde (géométrie combinatoire).* Publ. Math. d'Orsay, [No.] 79-03. Univ. Paris-Sud, Dép. Math., Orsay, 1979. MR 81a:05027. Zbl. 424.05018.

[Ch.] 3, [Sect.] 12: “Matroïdes de Dowling” (p. 52). States definition by partial $\mathfrak{G}$-partitions and the linear representability theorem. (**gg: M: Exp**)

Aviezri S. Fraenkel and Peter L. Hammer

1984a Pseudo-Boolean functions and their graphs. In: *Convexity and graph theory* (Jerusalem, 1981), pp. 137–146. North-Holland Math. Stud., 87. North-Holland, Amsterdam, 1984. MR 87b:90147. Zbl. 557.94019. (**sh: lg**)

András Frank

1996a A survey on T-joins, T-cuts, and conservative weightings. In: D. Miklós, V.T. Sós, and T. Szőnyi, eds., *Combinatorics, Paul Erdős is Eighty*, Vol. 2, pp. 213–252. Bolyai Soc. Math. Stud., 2. János Bolyai Math. Soc., Budapest, 1996. MR 97c:05115. Zbl. 846.05062.

A “conservative ±1-weighting” of G is an edge labelling by $+1$'s and -1's so that in every polygon the sum of edge weights is nonnegative. It is a tool in several theorems. [Related: Ageev, Kostochka, and Szigeti (1995a), Sebö (1990a).] (**SGw: Str, Alg: Exp, Ref**)

Howard Frank and Ivan T. Frisch

1971a *Communication, Transmission, and Transportation Networks.* Addison-Wesley, Reading, Mass., 1971. MR 49 #12063. Zbl. 281.94012.

§6.12: “Graphs with gains,” pp. 277–288. (**GN: Exp**)

Ove Frank and Frank Harary

1979a Balance in stochastic signed graphs. *Social Networks* 2 (1979/80), 155–163. MR 81e:05116.

The model: an edge is present with probability α and positive with probability p. The expected value is computed for two kinds of measures of imbalance: the number of balanced triangles (whose variance is also given), and the number of induced subgraphs of order 3 having specified numbers of positive and negative edges. (**SG: Rand, Fr**)

Ivan T. Frisch

See H. Frank.

Toshio Fujisawa

1963a Maximal flow in a lossy network. In: J.B. Cruz, Jr., and John C. Hofer, eds., *Proceedings, First Annual Allerton Conference on Circuit and System Theory* (Monticello, Ill., 1963), pp. 385–393. Dept. of Electrical Eng. and Coordinated Sci. Lab., Univ. of Illinois, Urbana, Ill., [1963]. (**GN: M(bases)**)

Satoru Fujishige

See K. Ando.

David Gale

See also A. J. Hoffman.

David Gale and A.J. Hoffman

1982a Two remarks on the Mendelsohn-Dulmage theorem. In: Eric Mendelsohn, ed., *Algebraic and Geometric Combinatorics*, pp. 171–177. North-Holland Math. Stud., 65. Ann. Discrete Math., 15. North-Holland, Amsterdam, 1982. MR 85m:05054. Zbl. 501.05049. (**sg: I, B**)

Marianne L. Gardner [Marianne Lepp]

See R. Shull.

Michael Gargano and Louis V. Quintas

1985a A digraph generalization of balanced signed graphs. *Congressus Numerantium* 48 (1985), 133–143. MR 87m:05095. Zbl. 622.05027.

Characterizes balance in abelian gain graphs. [See Harary, Lindström, and Zetterström (1982a).] Very simple results on existence, for a given graph, of balanced nowhere-zero gains from a given abelian group. [Elementary, if one notes that such gains exist iff the graph is $|G|$-colorable, G being the gain group]. Comparison with the approach of Sampathkumar and Bhave (1973a). Dictionary: "Symmetric G-weighted digraph" = gain graph with gains in the (abelian) group G. "Weight" = gain. "Non-trivial" (of the gain function) = nowhere zero. (**GG: B**)

Michael L. Gargano, John W. Kennedy, and Louis V. Quintas

1998a Group weighted balanced digraphs and their duals. Proc. Twenty-ninth South-eastern Internat. Conf. on Combinatorics, Graph Theory and Computing (Boca Raton, Fla., 1998). *Congressus Numer.* 131 (1998), 161–167. MR 99j:05080.

An abelian gain graph Φ is cobalanced (here called "cut-balanced") if the sum of gains on the edges of each coherently oriented cutset is 0. [This generalizes Kabell (1985a).] Given Φ with $\|\Phi\|$ embedded in a surface, the surface dual graph is given gains by a right-rotation rule, thus forming a surface dual Φ^* of Φ. [This appears to require that the surface be orientable. Note that cobalance generalizes to nonabelian gains on orientably embedded graphs, since the order of multiplication for the gain product on a cutset is given by the embedding.] Thm. 3.2: For a plane embedding of Φ, Φ is cobalanced iff Φ^* is balanced. Thm. 3.4 restates as criteria for cobalance of Φ the standard criteria for balance of Φ^*, as in Gargano and Quintas (1985a). More interesting are "well-balanced" graphs, which are both balanced and cobalanced. *Problem.* Characterize them. Dictionary (also see Gargano and Quintas 1985a): Balance is called "cycle balance". (**GG: B(D)**)

Gilles Gastou and Ellis L. Johnson

1986a Binary group and Chinese postman polyhedra. *Math. Programming* 34 (1986), 1–33. MR 88e:90060. Zbl. 589.52004.

§10 introduces the co-postman and "odd circuit" problems, treated more thoroughly in Johnson and Mosterts (1987a) (q.v). "Odd" edges and circuits are precisely negative edges and polygons in an edge signing. The "odd circuit matrix" represents $L(\Sigma)$ (p. 30). The "odd circuit problem" is to find a shortest negative polygon; a simple algorithm uses the signed covering graph (pp. 30–31). The "Fulkerson property" may be related to planarity and K_5 minors [which suggests comparison with Barahona (1990a), §5].
(**SG: Fr(Gen), I, M(Bases), cov, Alg**)

Heather Gavlas

See G. Chartrand.

Joseph Genin and John S. Maybee

1974a Mechanical vibration trees. *J. Math. Anal. Appl.* 45 (1974), 746–763. MR 49 #4351. Zbl. 272.70015. (**QM: A, SG**)

A.M.H. Gerards

1988a Homomorphisms of graphs into odd cycles. *J. Graph Theory* 12 (1988), 73–83. MR 89h:05045. Zbl. 691.05013.

If an antibalanced, unbalanced signed graph has no homomorphism into its shortest negative polygon, then it contains a subdivision of $-K_4$ or of a loose $\pm C_3$ (here called an "odd K_4" and an "odd K_3^2"). (A loose $\pm C_n$ consists of n negative digons in circular order, each adjacent pair joined either at a common vertex or by a link.) [*Question.* Do the theorem and proof carry over to any unbalanced signed graph?] Other results about antibalanced signed graphs are corollaries. Several interesting results about signed graphs are lemmas. **(P, SG: Str)**

1989a A min-max relation for stable sets in graphs with no odd-K_4. *J. Combin. Theory Ser. B* 47 (1989), 330–348. MR 91c:05143. Zbl. 691.05021.

Let Σ be antibalanced and without isolated vertices and contain no subdivision of $-K_4$. Then max. stable set size = min. cost of a cover by edges and negative polygons. Also, min. vertex-cover size = max. profit of a packing of edges and negative polygons. Also, weighted analogs. [*Question.* Do the theorem and proof extend to any Σ?] **(p, sg: Str)**

1989b A short proof of Tutte's characterization of totally unimodular matrices. *Linear Algebra Appl.* 114 (115) (1989), 207–212. MR 90b:05033. Zbl. 676.05028.

The proof of Lemma 3 uses a signed graph. **(SG: B)**

††1990a *Graphs and polyhedra: Binary spaces and cutting planes.* CWI Tract, 73. Centrum voor Wiskunde en Informatica, Amsterdam, 1990. MR 92f:52027. Zbl. 727.90044.

(Very incomplete annotation.) Thm.: Given Σ, the set $\{x \in \mathbb{R}^n : d_1 \leq x \leq d_2, b_1 \leq I(\Sigma)^{\mathrm{T}} x \leq b_2\}$ has Chvatal rank ≤ 1 for all integral vectors d_1, d_2, b_1, b_2, iff Σ contains no subdivided $-K_4$. **(SG: I, G, B, Str)**

1992a On shortest T-joins and packing T-cuts. *J. Combin. Theory Ser. B* 55 (1992), 73–82. MR 93d:05093. Zbl. 810.05056. **(SG: Str)**

1994a An orientation theorem for graphs. *J. Combin. Theory Ser. B* 62 (1994), 199–212. MR 96d:05051. Zbl. 807.05020. **(p, sg: M, O)**

1995a On Tutte's characterization of graphic matroids—a graphic proof. *J. Graph Theory* 20 (1995), 351–359. MR 96h:05038. Zbl. 836.05017.

Signed graphs used to prove Tutte's theorem. The signed-graph matroid employed is the extended lift matroid ("extended even cycle matroid"). The main theorem (Thm. 2): Let Σ be a signed graph with no $-K_4$, $\pm K_3$, $-Pr_3$, or Σ_4 link minor; then Σ can be converted by Whitney 2-isomorphism operations ("breaking" = splitting a component in two at a cut vertex, "glueing" = reverse, "switching" = twisting across a vertex 2-separation) to a signed graph that has a balancing vertex ("blocknode"). Here Σ_4 consists of $+K_4$ with a 2-edge matching doubled by negative edges and one other edge made negative.

More translation: His "Σ" is our E_-. "Even, odd" = positive, negative (for edges and polygons). "Bipartite" = balanced; "almost bipartite" = has a balancing vertex. **(SG: M, Str, I)**

A.M.H. Gerards and M. Laurent

1995A A characterization of box $\frac{1}{d}$-integral binary clutters. *J. Combin. Theory Ser. B* 65 (1995), 186–207. MR 96k:90052. Zbl. 835.05017.

Thm. 5.1: The collection of negative polygons of Σ is box $\frac{1}{d}$-integral for some/any integer $d \geq 2$ iff it does not contain $-K_4$ as a link minor. **(SG: Polygons, G)**

A.M.H. Gerards, L. Lovász, A. Schrijver, P.D. Seymour, and K. Truemper

†1990a Manuscript in preparation, 1990.

Extension of Gerards and Schrijver (1986b). [Same comments apply. The proliferating authorship may prevent this major contribution from ever being

published—though one hopes not! See Seymour (1995a) for description of two main theorems.] (**SG: Str, M, T**)

A.M.H. Gerards and A. Schrijver

1986b Signed graph – regular matroids – grafts. Research Memorandum, Faculteit der Economische Wetenschappen, Tilburg Univ., 1986.

Essential, major theorems. The (extended) lift matroid of a signed graph is one of the objects studied. Some of this material is published in Gerards (1990a). This paper is in the process of becoming Gerards, Lovász, *et al.* (1990a). (**SG: Str, M**)

1986a Matrices with the Edmonds-Johnson property. *Combinatorica* 6 (1986), 365–379. MR 88g:05087. Zbl. (565.90048), 641.05039.

A subsidiary result: If $-\Gamma$ contains no subdivided $-K_4$, then Γ is t-perfect. (**sg: P: G, Str**)

A.M.H. Gerards and F.B. Shepherd

1998a Strong orientations without even directed circuits. *Discrete Math.* 188 (1998), 111–125. MR 99i:05091.

1998b The graphs with all subgraphs t-perfect. *SIAM J. Discrete Math.* 11 (1998), 524–545. Zbl. 980.38493

Extension of Gerards (1989a). An "odd-K_4 is a graph whose all-negative signing is a subdivided $-K_4$. A "bad-K_4" is an odd-K_4 which does not consist of exactly two undivided K_4 edges that are nonadjacent while the other edges are replaced by even paths. Thm. 1: A graph that contains no bad-K_4 as a subgraph is t-perfect. Thm. 2 characterizes the graphs that are subdivisions of 3-connected graphs and contain an odd-K_4 but no bad-K_4. [The fact that 'badness' is not strictly a parity property weighs against the possibility that Gerards (1989a) extends well to signed graphs.] (**p, sg: Str, Alg**)

Anna Maria Ghirlanda

See L. Muracchini.

A. Ghouila-Houri

See C. Berge.

Rick Giles

1982a Optimum matching forests. I: Special weights. II: General weights. III: Facets of matching forest polyhedra. *Math. Programming* 22 (1982), 1–11, 12–38, 39–51. MR 82m:05075a,b,c. Zbl. 468.90053, 468.90054, 468.90055. (**sg: o**)

Mukhtiar Kaur Gill [Mukti Acharya]

See also B.D. Acharya.

1981a A graph theoretical recurrence formula for computing the characteristic polynomial of a matrix. In: S.B. Rao, ed., *Combinatorics and Graph Theory* (Proc. Sympos., Calcutta, 1980), pp. 261–265. Lecture Notes in Math., 885. Springer-Verlag, Berlin, 1981. MR 83f:05047. Zbl. 479.05030. (**SG: A**)

1981b A note concerning Acharya's conjecture on a spectral measure of structural balance in a social system. In: S.B. Rao, ed., *Combinatorics and Graph Theory* (Proc. Sympos., Calcutta, 1980), pp. 266–271. Lecture Notes in Math., 885. Springer-Verlag, Berlin, 1981. MR 84d:05121. Zbl. 476.05073. (**SG: B, A**)

1982a Contributions to Some Topics in Graph Theory and Its Applications. Ph.D. thesis, Dept. of Mathematics, Indian Institute of Technology, Bombay, 1982.

Most of the results herein have been published separately. See Gill (1981a, 1981b), Gill and Patwardhan (1981a, 1983a, 1986a). (**SG, SD: B, LG, A**)

M.K. Gill and B.D. Acharya

1980a A recurrence formula for computing the characteristic polynomial of a sigraph. *J. Combin. Inform. System Sci.* 5 (1980), 68–72. MR 81m:05097. Zbl. 448.05048. (**SG: A**)

1980b A new property of two dimensional Sperner systems. *Bull. Calcutta Math. Soc.* 72 (1980), 165–168. MR 83m:05121. Zbl. 531.05058. (**SG: B, G**)

M.K. Gill and G.A. Patwardhan

1981a A characterization of sigraphs which are switching equivalent to their line sigraphs. *J. Math. Phys. Sci.* 15 (1981), 567–571. MR 84h:05106. Zbl. 488.05054. (**SG: LG**)

1982a A characterization of sigraphs which are switching equivalent to their iterated line sigraphs. *J. Combin. Inform. System. Sci.* 7 (1982), 287–296. MR 86a:05103. Zbl. 538.05060. (**SG: LG**)

1986a Switching invariant two-path signed graphs. *Discrete Math.* 61 (1986), 189–196. MR 87j:05138. Zbl. 594.05059. (**SG, Sw**)

John Gimbel

1988a Abelian group labels on graphs. *Ars Combinatoria* 25 (1988), 87–92. MR 89k:-05046. Zbl. 655.05034.

The topic is "induced" edge labellings, that is, $w(e_{uv}) = f(u)f(v)$ for some $f : V \to \mathfrak{A}$. The number of f that induce a given induced labelling, the number of induced labellings, and a characterization of induced labellings. All involve the 2-torsion subgroup of $\mathfrak{A}$, unless Γ is bipartite. The inspiration is dualizing magic graphs. [Somewhat dual to Edelman and Saks (1979a).] (**p: i**)(**VS(Gen): E**)

Terry C. Gleason

See also D. Cartwright.

Terry C. Gleason and Dorwin Cartwright

1967a A note on a matrix criterion for unique colorability of a signed graph. *Psychometrika* 32 (1967), 291–296. MR 35 #989. Zbl. 184, 492 (e: 184.49202). (**SG: Cl, A**)

Fred Glover

See also J. Elam.

F. Glover, J. Hultz, D. Klingman, and J. Stutz

1978a Generalized networks: A fundamental computer-based planning tool. *Management Sci.* 24 (1978), 1209–1220. (**GN: Alg, M(bases): Exp, Ref**)

Fred Glover and D. Klingman

1973a On the equivalence of some generalized network problems to pure network problems. *Math. Programming* 4 (1973), 269–278. MR 47 #6393. Zbl. 259.90012. (**GN: B, I**)

1973b A note on computational simplifications in solving generalized transportation problems. *Transportation Sci.* 7 (1973), 351–361. MR 54 #6502. (**GN: M(bases), g**)

Fred Glover, Darwin Klingman, and Nancy V. Phillips

1992a *Network Models in Optimization and Their Applications in Practice.* Wiley-Interscience, New York, 1992.

Textbook. See especially Ch. 5: "Generalized networks." (**GN: Alg: Exp**)

F. Glover, D. Klingman, and J. Stutz

1973a Extensions of the augmented predecessor index method to generalized network problems. *Transportation Sci.* 7 (1973), 377–384. (**GN: M(bases), m**)

C.D. Godsil

1985a Inverses of trees. *Combinatorica* 5 (1985), 33–39. MR 86k:05084. Zbl. 578.05049.

If T is a tree with a perfect matching, then $A(T)^{-1} = A(\Sigma)$ where Σ is balanced and $|\Sigma| \supseteq \Gamma$. *Question.* When does $|\Sigma| = \Gamma$? [Solved by Simion and Cao (1989a).] [Cf. Buckley, Doty, and Harary (1984a) and, for a different notion, Greenberg, Lundgren, and Maybee (1984b).] (**sg: A, B**)

J.M. Goethals

See also P.J. Cameron.

Jay R. Goldman and Louis H. Kauffman

1993a Knots, tangles, and electrical networks. *Adv. Appl. Math.* 14 (1993), 267–306. MR 94m:57013. Zbl. 806.57002. Reprinted in Louis H. Kauffman, *Knots and Physics*, 2nd edn., pp. 684–723. Ser. Knots Everything, Vol. 1. World Scientific, Singapore, 1993. MR 95i:57010. Zbl. 868.57001.

The parametrized Tutte polynomial [as in Zaslavsky (1992b) et al.] of an $\mathbb{R}^*$-weighted graph is used to define a two-terminal "conductance". Interpreting weights as crossing signs in a planar link diagram with two blocked regions yields invariants of tunnel links. [Also see Kauffman (1997a).]

(**SGw: Gen: N, Knot, Phys**)

Richard Z. Goldstein and Edward C. Turner

1979a Applications of topological graph theory to group theory. *Math. Z.* 165 (1979), 1–10. MR 80g:20050. Zbl. 377.20027, (387.20034). (**SG: T**)

Harry F. Gollub

1974a The subject-verb-object approach to social cognition. *Psychological Rev.* 81 (1974), 286–321. (**PsS: vs**)

Martin Charles Golumbic

1979a A generalization of Dirac's theorem on triangulated graphs. In: Allan Gewirtz and Louis V. Quintas, eds., Second International Conference on Combinatorial Mathematics (New York, 1978). *Ann. New York Acad. Sci.* 319 (1979), 242–246. MR 81c:05077. Zbl. 479.05055.

Further results on chordal bipartite graphs. Their properties imply standard properties of ordinary chordal graphs. [See (1980a) for more.] (The "only if" portion of Thm. 4 is false, according to (1980a), p. 267.) (**sg: b, cov**)

1980a *Algorithmic Graph Theory and Perfect Graphs.* Academic Press, New York, 1980. MR 81e:68081. Zbl. 541.05054.

§12.3: "Perfect elimination bipartite graphs," and §12.4: "Chordal bipartite graphs," expound perfect elimination and chordality for bipartite graphs from Golumbic and Goss (1978a) and Golumbic (1979a). In particular, Cor. 12.11: A bipartite graph is chordal bipartite iff every induced subgraph has perfect edge elimination scheme. [*Problem.* Guided by these results, find a signed-graph generalization of chordality that corresponds to supersolvability and perfect vertex elimination (cf. Zaslavsky (20xxh)).] (**sg: b, cov**)

Martin Charles Golumbic and Clinton F. Goss

1978a Perfect elimination and chordal bipartite graphs. *J. Graph Theory* 2 (1978), 155–163. MR 80d:05037. Zbl. 411.05060.

A perfect edge elimination scheme is a bipartite analog of a perfect vertex elimination scheme. A chordal bipartite graph is a bipartite graph in which every polygon longer than 4 edges has a chord. Analogs of properties of chordal graphs, e.g., Dirac's separator theorem, are proved. In particular, a chordal bipartite graph has a perfect edge elimination scheme. [See Golumbic (1980a) for more.] **(sg: b)**

Gary Gordon

See also L. Fern.

1997a Hyperplane arrangements, hypercubes and mixed graphs. Proc. Twenty-eighth Southeastern Internat. Conf. on Combinatorics, Graph Theory and Computing (Boca Raton, Fla., 1997). *Congressus Numer.* 126 (1997), 65–72. MR 98j:05038. Zbl. 901.05055.

An explicit bijection between the regions of the real hyperplane arrangement corresponding to $\pm K_n^\circ$ and the set of "good signed [complete] mixed graphs" $G_{\mathbf{a}}$ of order n. The latter are a notational variant of the acyclic orientations τ of $\pm K_n^\circ$ [and are therefore in bijective correspondence with the regions, by Zaslavsky (1991b), Thm. 4.4]; the dictionary is: a directed edge in $G_{\mathbf{a}}$ is an oriented positive edge in τ, while a positive or negative undirected edge in $G_{\mathbf{a}}$ is an introverted or extroverted negative edge of τ. The main result, Thm. 1, is an interesting and significant explicit description of the acyclic orientations of $\pm K_n^\circ$. Namely, one orders the vertices and directs all positive edges upward; then one steps inward randomly from both ends of the ordered vertex set, one vertex at a time, at each new vertex orienting all previously unoriented negative edges to be introverted if the vertex was approached from below, extroverted if from above in the vertex ordering. [This clearly guarantees acyclicity.] [*Problem.* Generalize to arbitrary signed graphs.]

Lemma 2, "a standard exercise", is that an orientation of $\pm K_n^\circ$ (with the loops replaced by half edges) is acyclic iff the magnitudes of its net degrees are a permutation of $\{1, 3, \ldots, 2n-1]\}$. [Similarly, an orientation of $\pm K_n^\circ$ is acyclic iff its net degree vector is a signed permutation of $\{2, 4, \ldots, 2n\}$ (Zaslavsky (1991b), p. 369, but possibly known beforehand in other terminology). Both follow easily from Zaslavsky (1991b), Cor. 5.3: an acyclic orientation has a vertex that is a source or sink.] **(SG: o: i, G)**

20xxa The answer is $2^n \cdot n!$ What's the question? *Amer. Math. Monthly* 106, No. 7 (August–September, 1999), 636–645.

§5 presents the signed-graph question: an appealing presentation of material from (1997a). **(SG: o, I, G, N: Exp)**

Y. Gordon and H.S. Witsenhausen

1972a On extensions of the Gale–Berlekamp switching problem and constants of l_p-spaces. *Israel J. Math.* 11 (1972), 216–229. MR 46 #3213. Zbl. 238.46009.

Asymptotic estimates of $l(K_{r,s})$, the maximum frustration index of signatures of $K_{r,s}$, improving the bounds of Brown and Spencer (1971a). **(sg: Fr)**

Clinton F. Goss

See M.C. Golumbic.

R.L. Graham and N.J.A. Sloane

1985a On the covering radius of codes. *IEEE Trans. Inform. Theory* IT-31 (1985), 385–401. MR 87c:94048. Zbl. 585.94012.

See Example b, p. 396 (the Gale–Berlekamp code). **(sg: Fr)**

Ante Graovac, Ivan Gutman, and Nenad Trinajstić

1977a *Topological Approach to the Chemistry of Conjugated Molecules.* Lecture Notes in Chem., 4. Springer-Verlag, Berlin-Heidelberg-New York, 1977. Zbl. 385.05032.

§2.7. "Extension of graph-theoretical considerations to Mobius systems." (**SG: A, Chem**)

A. Graovac and N. Trinajstić

1975a Mobius molecules and graphs. *Croatica Chemica Acta (Zagreb)* 47 (1975), 95–104. (**SG: A, Chem**)

1976a Graphical description of Möbius molecules. *J. Molecular Structure* 30 (1976), 416–420.

The "Möbius graph" (i.e., signed graph of a suitably twisted ring hydrocarbon) is introduced with examples of the adjacency matrix and characteristic polynomial. (**Chem: SG: A**)

John G. del Greco

See del Greco (under 'D').

F. Green

1987a More about NP-completeness in the frustration model. *OR Spektrum* 9 (1987), 161–165. MR 88m:90053. Zbl. 625.90070.

Proves polynomial time for the reduction employed in Bachas (1984a) and improves the theorem to: the frustration index decision problem on signed (3-dimensional) cubic lattice graphs with 9 layers is NP-complete. [Cf. Barahona (1982a).] (**SG: Fr: Alg**)

Jan Green-Krótki

See J. Aráoz.

Harvey J. Greenberg, J. Richard Lundgren, and John S. Maybee

1983a Rectangular matrices and signed graphs. *SIAM J. Algebraic Discrete Methods* 4 (1983), 50–61. MR 84m:05052. Zbl. 525.05045.

From a matrix B, with row set R and column set C, form the "signed bipartite graph" BG^+ with vertex set $R \cup C$ and an edge $r_i c_k$ signed $\operatorname{sgn} b_{ik}$ whenever $b_{ik} \neq 0$. The "signed row graph" RG^+ is the two-step signed graph of BG^+ on vertex set R: that is, $r_i r_j$ is an edge if $\operatorname{dist}^{BG^+}(r_i, r_j) = 2$ and its sign is the sign of any shortest $r_i r_j$-path. If some edge has ill-defined sign, RG^+ is undefined. The "signed column graph" CG^+ is similar. The paper develops simple criteria for existence and balance of these graphs and the connection to matrix properties. It examines simple special forms of B. (**QM: SG, B, Appl**)

1984a Signed graphs of netforms. Proc. Fifteenth Southeastern Conf. on Combinatorics, Graph Theory and Computing. *Congressus Numer.* 44 (1984), 105–115. MR 87c:05085. Zbl. 557.05048.

Application of (1983a, 1984b). "Netform" = incidence matrix of a positive real gain graph (neglecting a minor technicality). Thm. 1: B is a netform iff $RG^+(B)$ exists and is all negative. (Then $CG^+(B)$ also exists.) Thm. 2: If the row set partitions so that all negative elements are in some rows and all positives are in the other rows, then $RG^+(B)$ is all negative and balanced. Thm. 3: If Σ is all negative and balanced, then B exists as in Thm. 2 with $RG^+(B) = \Sigma$. [Equivalent to theorem of Hoffman and Gale (1956a).] B is an "inverse" of Σ. Thm. 4 concerns "inverting" $-\Gamma$ in a minimal way. Then B will be (essentially) the incidence matrix of $+\Gamma$.

(**SG, gg: i, B, VS, Exp, Appl**)

1984b Inverting signed graphs. *SIAM J. Algebraic Discrete Methods* 5 (1984), 216–223. MR 86d:05085. Zbl. 581.05052.

See (1983a). "Inversion" means, given a signed graph Σ_R, or Σ_R and Σ_C, finding a matrix B such that $\Sigma_R = RG^+(B)$, or $\Sigma_R = RG^+(B)$ and $\Sigma_C = CG^+(B)$. The elementary solution is in terms of coverings of Σ_R by balanced cliques. It may be desirable to minimize the size of the balanced clique cover; this difficult problem is not tackled. (**QM: SG, VS, B**)

Harvey J. Greenberg and John S. Maybee, eds.

1981a *Computer-Assisted Analysis and Model Simplification* (Proc. First Sympos., Univ. of Colorado, Boulder, Col., 1980). Academic Press, New York, 1981. MR 82g:-00016. Zbl. 495.93001.

Several articles relevant to signed (di)graphs. (**QM**)(**SD, SG: B**)

Curtis Greene and Thomas Zaslavsky

1983a On the interpretation of Whitney numbers through arrangements of hyperplanes, zonotopes, non-Radon partitions, and orientations of graphs. *Trans. Amer. Math. Soc.* 280 (1983), 97–126. MR 84k:05032. Zbl. 539.05024.

§9: "Acyclic orientations of signed graphs." Continuation of Zaslavsky (1991b), counting acyclic orientations with specified unique source; also, with edge e having specified orientation and with no termini except at the ends of e. The proof is geometric. (**SG: M, O, G, N**)

G. Grimmett

1994a The random-cluster model. In: F.P. Kelly, ed., *Probability, Statistics and Optimisation*, Ch. 3, pp. 49–63. Wiley, Chichester, 1994. MR 96d:60154. Zbl. 858.60093.

Reviews Fortuin and Kasteleyn (1972a) and subsequent developments esp. in multidimensional lattices. The viewpoint is mainly probabilistic and asymptotic. §3.7, "Historical observations," reports Kasteleyn's account of the origin of the model. (**sgc: Gen: N, Phys: Exp**)

Richard C. Grinold

1973a Calculating maximal flows in a network with positive gains. *Oper. Res.* 21 (1973), 528–541. MR 50 #3900. Zbl. 304.90043.

Objective: to find the maximum output for given input. Basic solutions correspond to bases of $G(\Phi')$, Φ' being the underlying gain graph Φ together with an unbalanced loop adjoined to the sink. Onaga (1967a) also treats this problem. (**GN: M(bases), Alg**)

Heinz Gröflin and Thomas M. Liebling

1981a Connected and alternating vectors: polyhedra and algorithms. *Math. Programming* 20 (1981), 233–244. MR 83k:90061. Zbl. 448.90035. (**sg, G**)

Jonathan L. Gross

See also J. Chen.

1974a Voltage graphs. *Discrete Math.* 9 (1974), 239–246. MR 50 #153. Zbl. 286.05106. (**GG: T, Cov**)

Jonathan L. Gross and Thomas W. Tucker

1977a Generating all graph coverings by permutation voltage assignments. *Discrete Math.* 18 (1977), 273–283. MR 57 #5803. Zbl. 375.55001. (**GG: T, Cov**)

1979a Fast computations in voltage graph theory. In: Allan Gewirtz and Louis V. Quintas, eds., Second International Conference on Combinatorial Mathematics (New

York, 1978). *Ann. New York Acad. Sci.* 319 (1979), 247–253. MR 80m:94111. Zbl. 486.05027. (**GG: T, Cov, Sw**)

1987a *Topological Graph Theory.* Wiley, New York, 1987. MR 88h:05034. Zbl. 621.05013.

Ch. 2: "Voltage graphs and covering spaces." Ch. 4: "Imbedded voltage graphs and current graphs." (**GG: T, Cov**)

§3.2.2: "Orientability." §3.2.3: "Rotation systems." §4.4.5: "Nonorientable current graphs", discusses how to deduce, from the signs on a current graph, the signs of the "derived" graph of the dual voltage graph. [The same rule gives the signs on the surface dual of any orientation-embedded signed graph.] (The sign group here is $\mathbb{Z}_2$.) (**SG: T**)

Jerrold W. Grossman and Roland Häggkvist

1983a Alternating cycles in edge-partitioned graphs. *J. Combin. Theory Ser. B* 34 (1983), 77–81. MR 84h:05044. Zbl. 491.05039, (506.05040).

They prove the special case in which B is all negative of the following generalization, which is an immediate consequence of their result. [*Theorem.* If B is a bidirected graph such that for each vertex v there is a block of B in which v is neither a source nor a sink, then B contains a coherent polygon. ("Coherent" means that at each vertex, one edge is directed inward and the other outward.)] (**p: o**)

Martin Grötschel

See also F. Barahona.

M. Grötschel, M. Jünger, and G. Reinelt

1987a Calculating exact ground states of spin glasses: a polyhedral approach. In: J.L. van Hemmen and I. Morgenstern, eds., *Heidelberg Colloquium on Glassy Dynamics* (Proc., 1986), pp. 325–353. Lect. Notes in Physics, Vol. 275. Springer-Verlag, Berlin, 1987. MR 88g:82002 (book).

§2, "The spin glass model": finding the weighted frustration index in a weighted signed graph (Σ, w), or finding a ground state in the corresponding Ising model, is equivalent to the weighted max-cut problem in $(-\Sigma, w)$. This article concerns finding the exact weighted frustration index. §3, "Complexity", describes previous results on NP-completeness and polynomial-time solvability. §4, "Exact methods", discusses previous solution methods. §5, "Polyhedral combinatorics", shows that finding weighted frustration index is a linear program on the cut polytope; also expounds related work. The remainder of the paper concerns a specific cutting-plane method suggested by the polyhedral combinatorics.

(**sg: fr(gen): Alg, G, Ref**)(**Phys, Ref: Exp**)

M. Grötschel and W.R. Pulleyblank

1981a Weakly bipartite graphs and the max-cut problem. *Oper. Res. Lett.* 1 (1981/82), 23–27. MR 83e:05048. Zbl. 478.05039, 494.90078.

Includes a polynomial-time algorithm, which they attribute to "Waterloo folklore", for shortest (more generally, min-weight) even or odd path, hence (in an obvious way) odd or even polygon. [Attributed by Thomassen (1985a) to Edmonds (unpublished). Adapts to signed graphs by the negative subdivision trick: Subdivide each positive edge of Σ into two negative edges, each with half the weight. The min-weight algorithm applied to the subdivision finds a min-weight (e.g., a shortest) negative polygon of Σ.] [This paper is very easy to understand. It is one of the best written I know.] [Weakly bipartite graphs are certain signed graphs. Further work: Barahona, Grötschel, and Mahjoub (1985a), Polyak and Tuza (1995a), and esp. Guenin (1998a, 20xxa).] **(p: Alg, G, Paths, Polygons)(sg: G)**

Bertrand Guenin

See also G. Cornuéjols.

1998a On Packing and Covering Polyhedra. Ph.D. dissertation, Grad. Sch. Industrial Engin., Carnegie-Mellon Univ., 1998. **(SG: G)(S(M): G)**

1998b A characterization of weakly bipartite graphs. In: Robert E. Bixby, E. Andrew Boyd, and Roger Z. Ríos-Mercado, eds., *Integer Programming and Combinatorial Optimization* (6th Internat. IPCO Conf., Houston, 1998, Proc.), pp. 9–22. Lecture Notes in Computer Sci., Vol. 1412. Springer, Berlin, 1998. Zbl. 909.90264.

Outline of (20xxa). **(SG: G)**

20xxa A characterization of weakly bipartite graphs. Submitted

Σ is "weakly bipartite" (Grötschel and Pulleyblank 1981a) if its clutter of negative polygons is ideal (i.e., has the "weak MFMC" property of Seymour (1977a)). Thm.: Σ is weakly bipartite iff it has no $-K_5$ minor. This proves part of Seymour's conjecture (1981a) (see Cornuéjols 20xxa). **(SG: G)**

Gregory Gutin

See also J. Bang-Jensen.

Gregory Gutin, Benjamin Sudakov, and Anders Yeo

1998a Note on alternating directed cycles. *Discrete Math.* 191 (1998), 101–107. MR 99d:05050.

Existence of a coherent polygon with alternating colors in a digraph with an edge 2-coloring is NP-complete. However, if the minimum in- and out-degrees of both colors are sufficiently large, such a cycle exists. [This problem generalizes the undirected, edge-2-colored alternating-polygon problem, which is a special case of the existence of a bidirected coherent polygon—see Bang-Jensen and Gutin (1997a). *Question.* Is this alternating cycle problem also signed-graphic?] **(p: o: Polygons: Gen)**

Ivan Gutman

See also D.M. Cvetković, A. Graovac and S.-L. Lee.

1978a Electronic properties of Möbius systems. *Z. Naturforsch.* 33a (1978), 214–216. MR 58 #8800. **(SG: A, Chem)**

1988a Topological analysis of eigenvalues of the adjacency matrices in graph theory: A difficulty with the concept of internal connectivity. *Chemical Physics Letters* 148 (1988), 93–94.

Points out an ambiguity in the definitions of Lee, Lucchese, and Chu (1987a) in the case of multiple eigenvalues. [See Lee and Gutman (1989a) for the repair.] **(VS, SGw)**

Ivan Gutman, Shyi-Long Lee, Yeung-Long Luo, and Yeong-Nan Yeh

1994a Net signs of molecular graphs: dependence of molecular structure. *Internat. J. Quantum Chem.* 49 (1994), 87–95.

How to compute the balanced signing of Γ that corresponds to eigenvalue λ_i (see Lee, Lucchese, and Chu (1987a)), without computing the eigenvector X_i. Theorem: If v_r, v_s are adjacent, then $X_{ir}X_{is} = \sum_P f(P;\lambda_i)$, where $f(P;\lambda) := \varphi(G - V(P);\lambda)/\varphi'(G;\lambda)$, $\varphi(G;\lambda)$ is the characteristic polynomial, and the sum is over all paths connecting v_r and v_s. Hence $\sigma_i(v_r v_s) = \operatorname{sgn}(X_{ir}X_{is})$ is determined. [An interesting theorem. *Questions.*

Does it generalize if one replaces Γ by a signed graph, this being the balanced (all-positive) case? In such a generalization. if any, how will σ enter in—by restricting the sum to positive paths, perhaps? What about graphs with real gains, or weights?] (**VS, SGw**)

Ivan Gutman, Shyi-Long Lee, Jeng-Horng Sheu, and Chiuping Li

1995a Predicting the nodal properties of molecular orbitals by means of signed graphs. *Bull. Inst. Chem., Academica Sinica* No. 42 (1995), 25–31.

Points out some difficulties with the method of Lee and Li (1994a). (**VS, SGw, Chem**)

Ivan Gutman, Shyi-Long Lee, and Yeong-Nan Yeh

1992a Net signs and eigenvalues of molecular graphs: some analogies. *Chemical Physics Letters* 191 (1992), 87–91.

A connected graph Γ has n eigenvalues and n corresponding balanced signings (see Lee, Lucchese, and Chu (1987a)). Let $S_1 \geq S_2 \geq \cdots \geq S_n$ be the net signs of these signings and $m = |E|$. The net signs satisfy analogs of properties of eigenvalues. (A) If $\Delta \subset \Gamma$, then $S_1(\Delta) < S_1$. (B) $S_1 = m \geq S_2 + 2$. (C, D) For bipartite Γ, $S_n = -m$. Otherwise, $S_n \geq -m+2$. From (B, C, D) we have $|S_i| \leq m-2$ for all $i \neq 1$ and, if Γ is bipartite, $i \neq n$. (E, F) If Γ is bipartite, then $S_i = -S_{n+1-i}$ and at least $a-b$ net signs equal 0, where $a \geq b$ are the numbers of vertices in the two color classes. The analogy is imperfect, since $S_1 + S_2 + \cdots + S_n \geq 0$, while equality holds for eigenvalues. [*Questions.* Some of these conclusions require Γ to be bipartite. Does that mean that they will generalize to an arbitrary balanced signed graph Σ in place of the bipartite Γ, the eigenvectors being those of Σ? Will the other results generalize with Γ replaced by any signed graph? How about real gains, or weights?] (**VS, SGw**)

A. Gyárfás

See P. Erdős.

Ervin Győri

See also P. Erdős.

Ervin Győri, Alexandr V. Kostochka, and Tomasz Łuczak

1997a Graphs without short odd cycles are nearly bipartite. *Discrete Math.* 163 (1997), 279–284. MR 97g:05203. Zbl. 871.05040.

Given all-negative Σ and positive ρ, suppose every odd polygon has length $\geq n/\rho$. Then Σ has frustration index $\leq 200\rho^2(\ln(10\rho))^2$ (best possible up to a constant factor) and vertex deletion number $\leq 15\rho\ln(10\rho)$ (best possible up to a logarithmic factor). The proof is based on an interesting, refining lemma. [*Problem.* Generalize to arbitrary Σ.] (**sg: P: Fr**)

Jurriaan Hage and Tero Harju

1998a Acyclicity of switching classes. *European J. Combin.* 19 (1998), 321–327. MR 99d:05051. Zbl. 905.05057.

Classifies the switching-equivalent pairs of forests. Thm. 2.2: In a Seidel switching class of graphs there is at most one isomorphism type of tree; and there is at most one tree, with exceptions that are completely classified. Thms. 3.1 and 4.1: In a switching class that contains a disconnected forest there are at most 3 forests (not necessarily isomorphic); the cases in which there are 2 or 3 forests are completely classified. (Almost all are trees plus isolated vertices.) [*Question.* Regarding these results as concerning the negative subgraphs of switchings of signed complete graphs, to what extent do they generalize to switchings of arbitrary signed simple graphs?] [B.D. Acharya (1981a) asked which simple graphs switch to forests, with partial results.] (**TG**)

20xxa The size of switching classes with skew gains. Submitted.

Introducing "skew gain graphs", which generalize gain graphs (see Zaslavsky (1989a)) to incorporate dynamic labelled 2-structures (see Ehrenfeucht and Rozenberg). Inversion is replaced by a gain-group antiautomorphism δ of period at most 2. Thus $\varphi(e^{-1}) = \delta(\varphi(e))$, while in switching by τ, one defines $\varphi^{\tau}(e; v, w) = \delta(\tau(v))\varphi(e; v, w)\tau(w)$. The authors find the size of a switching class [*phi*] in terms of the centralizers and/or δ-centralizers of various parts of the image of φ_T, that is, φ switched to be the identity on a spanning tree T. The exact formulas depend on whether Γ is complete, or bipartite, or general, and on the choice of T (the case where $T \cong K_{1,n-1}$ being simplest). (**GG(Gen): Sw**)

Per Hage and Frank Harary

1983a *Structural Models in Anthropology.* Cambridge Univ. Press, Cambridge, Eng., 1983. MR 86e:92002.

Signed graphs are treated in Ch. 3 and 6, marked graphs in Ch. 6. (**SG, PsS: B: Exp**)(**VS: Exp**)

Roland Häggkvist

See J.W. Grossman.

J. Hammann

See E. Vincent.

Peter L. Hammer

See also E. Balas, C. Benzaken, E. Boros, J.-M. Bourjolly, Y. Crama, and A. Fraenkel.

1974a Boolean procedures for bivalent programming. In: P.L. Hammer and G. Zoutendijk, eds., *Mathematical Programming in Theory and Practice* (Proc. NATO Adv. Study Inst., Figueira da Foz, Portugal, 1972), pp. 311–363. North-Holland, Amsterdam, and American Elsevier, New York, 1974. MR 57 #18817. Zbl. 335.90034 (book).

1977a Pseudo-Boolean remarks on balanced graphs. In: L. Collatz, G. Meinardus, and W. Wetterling, eds., *Numerische Methoden bei Optimierungsaufgaben, Band 3: Optimierung bei graphentheoretischen und ganzzahligen Problemen* (Tagung, Oberwolfach, 1976), pp. 69–78. Internat. Ser. Numer. Math., Vol. 36. Birkhäuser, Basel, 1977. MR 57 #5833. Zbl. 405.05054. (**SG: B**)

P.L. Hammer, C. Benzaken, and B. Simeone

1980a Graphes de conflit des fonctions pseudo-booleennes quadratiques. In: P. Hansen and D. de Werra, eds., *Regards sur la Theorie des Graphes* (Actes du Colloq., Cerisy, 1980), pp. 165–170. Presses Polytechniques Romandes, Lausanne, Switz., 1980. MR 82d:05054 (book).

P.L. Hammer, T. Ibaraki, and U. Peled

1980a Threshold numbers and threshold completions. In: M. Deza and I.G. Rosenberg, eds., *Combinatorics 79* (Proc. Colloq., Montreal, 1979), Part II. *Ann. Discrete Math.* 9 (1980), 103–106. MR 81k:05092. Zbl. 443.05064. (**p: o**)

1981a Threshold numbers and threshold completions. In: Pierre Hansen, ed., *Studies on Graphs and Discrete Programming* (Proc. Workshop, Brussels, 1979), pp. 125–

145. North-Holland Math. Studies, 59. Ann. Discrete Math., 11. North-Holland, Amsterdam, 1981. MR 83m:90062. Zbl. 465.00007 (book).

See description of Thm. 8.5.2 in Mahadev and Peled (1995a). **(p: o)**

P.L. Hammer and N.V.R. Mahadev

1985a Bithreshold graphs. *SIAM J. Algebraic Discrete Methods* 6 (1985), 497–506. MR 86h:05093. Zbl. 5797.05052.

See description of §8.3 of Mahadev and Peled (1995a). **(SG: B: Appl)**

P.L. Hammer, N.V.R. Mahadev, and U.N. Peled

1989a Some properties of 2-threshold graphs. *Networks* 19 (1989), 17–23. MR 89m:05096. Zbl. 671.05059.

A restricted line graph with signed edges is a proof tool. **(SG, LG)**

Peter L. Hammer and Sang Nguyen

1979a A partial order in the solution space of bivalent programs. In: Nicos Christofides, Aristide Mingozzi, Paolo Toth, and Claudio Sandi, eds., *Combinatorial Optimization*, Ch. 4, pp. 93–106. Wiley, Chichester, 1979. MR 82a:90099 (book). Zbl. 414.90063. **(sg: o)**

Phil Hanlon

1984a The characters of the wreath product group acting on the homology groups of the Dowling lattices. *J. Algebra* 91 (1984), 430–463. MR 86j:05046. Zbl. 557.20009. **(gg: M: Aut)**

1988a A combinatorial construction of posets that intertwine the independence matroids of B_n and D_n. Manuscript, 1988.

Computes the Möbius functions of posets obtained from $\operatorname{Lat} G(\pm K_n^\circ)$ by discarding those flats with unbalanced vertex set in a given lower-hereditary list. Examples include $\operatorname{Lat} G(\pm K_n^{(k)})$, the exponent denoting the addition of k negative loops. Generalized and superseded by Hanlon and Zaslavsky (1998a). **(sg: M: Gen: N)**

1991a The generalized Dowling lattices. *Trans. Amer. Math. Soc.* 325 (1991), 1–37. MR 91h:06011. Zbl. 748.05043.

The lattices are based on a rank, n, a group, and a meet sublattice of the lattice of subgroups of the group. The Dowling lattices are a special case. **(gg: M: Gen: N)**

1996a A note on the homology of signed posets. *J. Algebraic Combin.* 5 (1996), 245–250. MR 97f:05194. Zbl. 854.06004.

Partial summary of Fischer (1993a). **(S)**

Phil Hanlon and Thomas Zaslavsky

1998a Tractable partially ordered sets derived from root systems and biased graphs. *Order* 14 (1997–98), 229–257. Zbl. 990.03811

Computes the characteristic polynomials (Thm. 4.1) and hence the Möbius functions (Cor. 4.4) of posets obtained from $\operatorname{Lat} G(\Omega)$, Ω a biased graph, by discarding those flats with unbalanced vertex set in a given lower-hereditary list. Examples include $\operatorname{Lat} G(\mathfrak{G} K_n^{(k)})$ where $\mathfrak{G}$ is a finite group, the exponent denoting the addition of k unbalanced loops. The interval structure, existence of a rank function, covering pairs, and other properties of these posets are investigated. There are many open problems. **(GG: M, Gen: N, Str, Col)**

Pierre Hansen

1978a Labelling algorithms for balance in signed graphs. In: *Problèmes Combinatoires et Theorie des Graphes* (Colloq. Internat., Orsay, 1976), pp. 215–217. Colloques Internat. du CNRS, 260. Editions du C.N.R.S., Paris, 1978. MR 80m:68057. Zbl. 413.05060.

§1: Algorithm 1 labels vertices of a signed graph to detect imbalance and a negative polygon if one exists. [It is equivalent to switching a maximal forest to all positive and looking for negative edges.] §2: Algorithm 2 is the unweighted case of the algorithm of (1984a). Path balance in a signed digraph is discussed. §3: The frustration index of a signed graph is bounded below by the negative-polygon packing number, which can be crudely bounded by Alg. 1. (**SG, SD: B, Fr: Alg, sw**)

1979a Methods of nonlinear 0–1 programming. In: P.L. Hammer, E.L. Johnson, and B.H. Korte, eds., *Discrete Optimization II* (Proc., Banff and Vancouver, 1977), pp. 53–70. Ann. Discrete Math., Vol. 5. North-Holland, Amsterdam, 1979. MR 84h:90034 (book). Zbl. 426.90063.

See pp. 58–59. (**SG: B: Exp**)

1983a Recognizing sign solvable graphs. *Discrete Appl. Math.* 6 (1983), 237–241. MR 84i:68112. Zbl. 524.05048.

Improves the characterization by Maybee (1981a) of sign-solvable digraphs with an eye to more effective algorithmic recognition. Thm. 2.2. A signed digraph D is sign solvable iff its positive subdigraph is acyclic and each strongly connected component has a vertex that is the terminus of no negative, simple directed path. §3: "An algorithm for sign solvability" in time $O(|V|\,|E|)$. (**SD: Sol: Alg**)

1984a Shortest paths in signed graphs. In: A. Burkard *et al.*, eds., *Algebraic Methods in Operations Research*, pp. 201–214. North-Holland Math. Stud., 95. Ann. of Discrete Math., 19. North-Holland, Amsterdam, 1984. MR 86i:05086. Zbl. 567.05032.

Algorithm to find shortest walks of each sign from vertex x_1 to each other vertex, in a signed digraph with positive integral(?) weights (i.e., lengths) on the edges. Applied to digraphs with signed vertices and edges; N-balance in signed graphs; sign solvability. The problem for (simple) paths is discussed [which is solvable by any min-weight parity path algorithm; see the notes on Grötschel and Pulleyblank (1981a)].

(**SD, WD: Paths, VS, B, Sol: Alg**)

Pierre Hansen and Bruno Simeone

1986a Unimodular functions. *Discrete Appl. Math.* 14 (1986), 269–281. MR 88a:90138. Zbl. 597.90058.

Three types of relatively easily maximizable pseudo-Boolean function ("unimodular" and two others) are defined. For quadratic pseudo-Boolean functions f, the three types coincide; f is unimodular iff an associated signed graph is balanced (Thm. 3). Thus one can quickly recognize unimodular quadratic functions, although not unimodular functions in general. If the graph is a tree, the function can be maximized in linear time. (**SG: B, Alg**)

Frank Harary

See also L.W. Beineke, A. Blass, F. Buckley, D. Cartwright, G. Chartrand, O. Frank, and P. Hage.

††1953a On the notion of balance of a signed graph. *Michigan Math. J.* 2 (1953–1954), 143–146. Addendum, *ibid.*, preceding p. 1. MR 16, 733. Zbl. 56, 421 (e: 056.42103).

[The birth of signed graph theory. Although Thm. 3 was anticipated by König (1936a) (Thm. X.11, for finite and infinite graphs) without the terminology of signs, here is the first recognition of the crucial fact that labelling edges by elements of a group—specifically, the sign group—can lead to a general theory.] The main theorem (Thm. 3) characterizes balanced signings as those for which there is a bipartition of the vertex set such that an edge is positive iff it lies within a part [I call this a Harary bipartition]. Thm. 2: A signing of a simple [or a loop-free] graph is balanced iff, for each pair of vertices, every path joining them has the same sign. Discussion of the number of nonisomorphic signed graphs with specific numbers of vertices and positive and negative edges. **(SG: B, E)**

1955a On local balance and N-balance in signed graphs. *Michigan Math. J.* 3 (1955–1956), 37–41. MR 17, 394. Zbl. 70, 185 (e: 070.18502).

Σ is (locally) balanced at a vertex v if every polygon on v is positive; then Thm. 3′: Σ is balanced at v iff every block containing v is balanced. Σ is N-balanced if every polygon of length $\leq N$ is positive; Thm. 2 concerns characterizing N-balance. Lemma 3: For each polygon basis, Σ is balanced iff every polygon in the basis is positive. [For finite graphs this strengthens König (1936a) Thm. 13.] **(SG: B)**

1957a Structural duality. *Behavioral Sci.* 2 (1957), 255–265. MR 24B #B851.

"Antithetical duality" (pp. 260–261) introduces antibalance. Remarks on signed and vertex-signed graphs are scattered about the succeeding pages. **(SG: B, P)**

1958a On the number of bi-colored graphs. *Pacific J. Math.* 8 (1958), 743–755. MR 21 #2598. Zbl. 84, 194 (e: 084.19402).

Section 6: "Balanced signed graphs". **(SG: B: E)**

1959a Graph theoretic methods in the management sciences. *Management Sci.* 5 (1959), 387–403. MR 21 #7103. Reprinted in: Samuel Leinhardt, ed., *Social Networks: A Developing Paradigm*, pp. 371–387. Academic Press, New York, 1977.

See pp. 400–401. **(SG: B: Exp)**

1959b On the measurement of structural balance. *Behavioral Sci.* 4 (1959), 316–323. MR 22 #3696.

Proposes to measure imbalance by (i) $\beta(\Sigma)$, the proportion of balanced polygons ("degree of balance"), (ii) the frustration index ("line index") [cf. Abelson and Rosenberg (1958a)], i.e., the smallest number of edges whose deletion or equivalently (Thm. 7) negation results in balance, and (iii) the vertex elimination number: the smallest number of vertices whose deletion results in balance ("point index"). Thm. 4 is an upper bound on the minimum β of unbalanced blocks with given cyclomatic number. Thm. 5 is a lower bound on the maximum. *Conjecture*. These bounds are best possible. Thm. 6 (contributed by J. Riordan) is an asymptotic evaluation of $\beta(-K_n)$. **(SG: Fr)**

1960a A matrix criterion for structural balance. *Naval Res. Logistics Quarterly* 7, No. 2 (June, 1960), 195–199. Zbl. 91, 159 (e: 091.15904). **(SG: B: A)**

1970a Graph theory as a structural model in the social sciences. In: Bernard Harris, ed., *Graph Theory and Its Applications*, pp. 1–16. Academic Press, New York, 1970. MR 41 #8277. Zbl. 224.05129.

1979a Independent discoveries in graph theory. In: Frank Harary, ed., Topics in Graph Theory (Proc. Conf., New York, 1977). *Ann. New York Acad. Sci.* 328 (1979), 1–4. MR 81a:05001. Zbl. 465.05026.

1980a Some theorems about graphs from social sciences. In: *Proc. West Coast Conf. on Combinatorics, Graph Theory and Computing* (Arcata, Calif., 1979), pp. 41–47. Congressus Numerantium, XXVI. Utilitas Math. Publ. Inc., Winnipeg, Man., 1980. MR 81m:05118. Zbl. 442.92027. (**SG: B: History, Exp**)

1981a Structural models and graph theory. In: Harvey J. Greenberg and John S. Maybee, eds., *Computer-Assisted Analysis and Model Simplification* (Proc. Sympos., Boulder, Col., 1980), pp. 31–58. Discussion, pp. 103–111. Academic Press, New York, 1981. MR 82g:00016 (book). Zbl. 495.93001 (book).

See remarks of Bixby (p. 111). (**SG, VS, SD: B, Alg: Exp**)

1983a Consistency theory is alive and well. *Personality and Social Psychology Bull.* 9 (1983), 60–64. (**PsS**)

1985a The reconstruction conjecture for balanced signed graphs. In: B.R. Alspach and C.D. Godsil, eds., *Cycles in Graphs*, pp. 439–442. Ann. Discrete Math., Vol. 27. North-Holland Math. Stud., Vol. 115. North-Holland, Amsterdam, 1985. MR 87d:05122. Zbl. 572.05048.

Reconstruction from the multiset of vertex-deleted subgraphs. Σ_+ is reconstructible if Σ is connected and balanced and not all positive or all negative. (**SG: B**)

F. Harary and G. Gupta

1997a Dynamic graph models. *Math. Computer Modelling* 25 (1997), no. 7, pp. 79–87. MR 98b:05092. Zbl. 879.68085.

§3.9, "Signed graphs", mentions that deletion index = frustration index (Harary (1959b)). (**SG: Fr: Exp**)

Frank Harary and Jerald A. Kabell

1980a A simple algorithm to detect balance in signed graphs. *Math. Social Sci.* 1 (1980/81), 131–136. MR 81j:05098. Zbl. 497.05056. (**SG: B, Alg**)

1981a Counting balanced signed graphs using marked graphs. *Proc. Edinburgh Math. Soc.* (2) 24 (1981), 99–104. MR 83a:05072. Zbl. 476.05043. (**SG, VS: E**)

Frank Harary and Helene J. Kommel

1978a Matrix measures for transitivity and balance. *J. Math. Sociol.* 6 (1978/79), 199–210. MR 81a:05056. Zbl. 408.05028. (**SG: Fr, A**)

1979a The graphs with only self-dual signings. *Discrete Math.* 26 (1979), 235–241. MR 80h:05047. Zbl. 408.05045. (**SG, VS: Aut**)

Frank Harary and Bernt Lindström

1981a On balance in signed matroids. *J. Combin. Inform. System. Sci.* 6 (1981), 123–128. MR 83i:05024. Zbl. 474.05021.

Thm. 1: The number of balanced signings of matroid M is $\leq 2^{\mathrm{rk}(M)}$, with equality iff M is binary. Thm. 3: Minimal deletion and negation sets coincide for all signings of M iff M is binary. Thm. 5: For connected binary M, a signing is balanced iff every circuit containing a fixed point is balanced. (**S: M: B, Fr**)

Frank Harary, Bernt Lindström, and Hans-Olov Zetterström

1982a On balance in group graphs. *Networks* 12 (1982), 317–321. MR 84a:05055. Zbl. 496.05052.

Implicitly characterizes balance and balancing sets in a gain graph Φ by switching (proof of Thm. 1). [For balance, see also Acharya and Acharya (1986a), Zaslavsky (1977a) and (1989a), Lemma 5.3. For abelian gains, see also Gargano and Quintas (1985a). In retrospect we can see that the characterization of balanced gains is as the 1-coboundaries with values in a group, which for abelian groups is essentially classical.] Thm. 1: The number of balanced gain functions. Thm. 2: Any minimal deletion set is an alteration set. Thm. 3: $l(\Phi) \leq m(1 - |\mathfrak{G}|^{-1})$. Thm. 4: $l(\Sigma) \leq \frac{1}{2}(m - \frac{n-1}{2})$, with strict inequality if not all degrees are even. [Compare with Akiyama, Avis, Chvátal, and Era (1981a), Thm. 1.] (**GG, SG: sw(B), E(B), Fr**)

Frank Harary, J. Richard Lundgren, and John S. Maybee

1985a On signed digraphs with all cycles negative. *Discrete Appl. Math.* 12 (1985), 155–164. MR 87g:05108. Zbl. 586.05019.

Which digraphs D can be signed so that every cycle is negative? Three types of example. Type 1: The vertices can be numbered $1, 2, \dots, n$ so that the downward arcs are just $(2,1), (3,2), \dots, (n, n-1)$. (Strong "upper" digraphs; Thm. 2.) Type 2: No cycle is covered by the remaining cycles ("free cyclic" digraphs). This type includes arc-minimal strong digraphs. Type 3: A symmetric digraph, if and only if the underlying graph Γ is bipartite and no two points on a common polygon and in the same color class are joined by a path outside the cycle (Thm. 10; proved by signing Γ via Zaslavsky (1981b)). [Further work in Chaty (1988a).] (**SD: B, SG**)

Frank Harary, Robert Z. Norman, and Dorwin Cartwright

1965a *Structural Models: An Introduction to the Theory of Directed Graphs.* Wiley, New York, 1965. MR 32 #2345. Zbl. 139, 415 (e: 139.41503).

In Ch. 10, "Acyclic digraphs": "Gradable digraphs", pp. 275–280. That means a digraph whose vertices can be labelled by integers so that $f(w) = f(v) + 1$ for every arc (v, w). [Equivalently, the Hasse diagram of a graded poset.] [Characterized by Topp and Ulatowski (1987a).] (**GD: b, Exr**)

Ch. 13: "Balance in structures". "Criteria for balance", pp. 340–346 (cf. Harary (1953a)); local balance (Harary (1955a)). "Measures of structural balance", pp. 346–352: "degree of balance" (proportion of balanced polygons; Cartwright and Harary (1956A)); "line-index for balance" [frustration index] (Abelson and Rosenberg (1958a), Harary (1959b)).

"Limited balance", pp. 352–355. Harary (1955a); also: Adjacency matrix (nonsymmetric) $A(D, \sigma)$ of a signed digraph: entries are $0, \pm 1$. The "valency matrix" is the $R(\Sigma)$ of Abelson and Rosenberg (1958a). Thm. 13.8: Entries of $R(\Sigma)^k$ show the existence of (undirected) walks of length k of each sign between pairs of vertices. [The symbols might be treated as 0, a_+, a_-, $a_+ + a_-$ in the group ring $\mathbf{R}$ of the sign group. Then $R(\Sigma)$ is equivalent to the $\mathbf{R}$-valued adjacency matrix $A_{\mathbf{R}}(\Sigma)$. Thm. 13.8 follows upon substituting in $A_{\mathbf{R}}^k$: $0 \mapsto o$, $ma_+ \mapsto p$, $ma_- \mapsto n$, $ma_+ + m'a_- \mapsto a$, where m, m' are positive integers. $A_{\mathbf{R}}^k$ itself provides an exact count of walks of each sign. Obviously, $A_{\mathbf{R}}$ and walk-counting generalize to gain graphs.]

"Cycle-balance and path-balance", pp. 355–358: here directions of arcs are

taken into account. E.g., Thm. 13.11: Every cycle is positive iff each strong component is balanced as an undirected graph.
(SG: B, Fr, A: Exp, Exr)(SD: B, Exr)

1968a *Introduction a la théorie des graphes orientés. Modèles structuraux.* Dunod, Paris, 1968. Zbl. 176, 225 (e: 176.22501).
French edition of (1965a). **(GD: b, Exr)**
(SG: B, Fr, A: Exp, Exr)(SD: B, Exr)

Frank Harary and Edgar M. Palmer

1967a On the number of balanced signed graphs. *Bull. Math. Biophysics* 29 (1967), 759–765. Zbl. 161, 209 (e: 161.20904). **(SG: B: E)**

1973a *Graphical Enumeration.* Academic Press, New York, 1973. MR 50 #9682. Zbl. 266.05108.
Four exercises and a remark concern signed graphs, balanced signed graphs, and signed trees. Russian transl.: Kharari and Palmer (1977a). **(SG: E, B)**

1977a (As "F. Kharari and È. Palmer") *Perechislenie grafov.* "Mir", Moscow, 1977. MR 56 #5353.
Russian translation of (1973a). **(SG: E, B)**

Frank Harary, Edgar M. Palmer, Robert W. Robinson, and Allen J. Schwenk

1977a Enumeration of graphs with signed points and lines. *J. Graph Theory* 1 (1977), 295–308. MR 57 #5818. Zbl. 379.05035.
See Bender and Canfield (1983a). **(SG, VS: E)**

Frank Harary and Michael Plantholt

1983a The derived signed graph of a digraph. *Expositiones Math.* 1 (1983), 343–347. MR 86h:05056. Zbl. 525.05030. **(SG: LG, B)**

Frank Harary and Geert Prins

1959a The number of homeomorphically irreducible trees, and other species. *Acta Math.* 101 (1959), 141–162. MR 21 #653. Zbl. 84, 193 (e: 084.19304). **(SG: E)**

Frank Harary and Robert W. Robinson

1977a Exposition of the enumeration of point-line-signed graphs enjoying various dualities. In: R.C. Read and C.C. Cadogan, eds., *Proc. Second Carribean Conf. in Combinatorics and Computing* (Cave Hill, Barbados, 1977), pp. 19–33. Dept. of Math., Univ. of the West Indies, Cave Hill, Barbados, 1977. **(SG, VS: E)**

Frank Harary and Bruce Sagan

1984a Signed posets. In: *Calcutta Mathematical Society Diamond-cum-Platinum Jubilee Commemoration Volume (1908–1983)*, Part I, pp. 3–10. Calcutta Math. Soc., Calcutta, 1984. MR 87k:06003. Zbl. 588.05048.
A signed poset is a (finite) partially ordered set P whose Möbius function takes on only values in $\{0, \pm 1\}$. $S(P)$ is the signed graph with $V = P$ and $E_\epsilon = \{xy : x \leq y \ and \mu(x,y) = \epsilon 1\}$ for $\epsilon = +, -$. Some examples are chains, tree posets, and any product of signed posets. Thm. 1 characterizes P such that $|S(P)| \cong H(P)$, the Hasse diagram of P. Thm. 3 characterizes posets for which $S(P)$ is balanced. Thm. 4 gives a sufficient condition for clusterability of $S(P)$. There are many unanswered questions, most basically *Question* 1. Which signed graphs have the form $S(P)$? [See Zelinka (1988a) for a partial answer.] **(SG, S)**

Frank Harary and Marcello Truzzi

1979a The graph of the zodiac: On the persistence of the quasi-scientific paradigm of

astrology. *J. Combin. Inform. System Sci.* 4 (1979), 147–160. MR 82e:00004 (q.v.). **(SG: B)**

Katsumi Harashima

See H. Kosako.

Tero Harju

See A. Ehrenfeucht and J. Hage.

David Harries and Hans Liebeck

1978a Isomorphisms in switching classes of graphs. *J. Austral. Math. Soc. (A)* 26 (1978), 475–486. MR 80a:05109. Zbl. 411.05044.

Given $\Sigma = (K_n, \sigma)$ and an automorphism group $\mathfrak{A}$ of the switching class $[\Sigma]$, is $\mathfrak{A}$ "exposable" on $[\Sigma]$ (does it fix a representative of $[\Sigma]$)? General techniques and a solution for the dihedral group. Done in terms of Seidel switching of unsigned simple graphs. (A further development from Mallows and Sloane (1975a). [Related work in M. Liebeck (1982a) and Cameron (1977a).]) **(k: sw, TG: Aut)**

Nora Hartsfield and Gerhard Ringel

1989a Minimal quadrangulations of nonorientable surfaces. *J. Combin. Theory Ser. A* 50 (1989), 186-195. MR 90j:57003. Zbl. 665.51007.

"Cascades": see Youngs (1968b). **(sg: O: Appl)**

Kurt Hässig

1975a Theorie verallgemeinerter Flüsse und Potentiale. In: *Siebente Oberwolfach-Tagung uber Operations Research* (1974), pp. 85–98. Operations Research Verfahren, Band XXI. A. Hain, Meisenheim am Glan, 1975. MR 56 #8434. Zbl. 358.90070. **(GN: I)**

1979a *Graphentheoretische Methoden des Operations Research.* Leitfaden der angew. Math. und Mechanik, 42. B.G. Teubner, Stuttgart, 1979. MR 80f:90002. Zbl. 397.90061.

Ch. 5: "Verallgemeinerte Fluss- und Potentialdifferenzen-probleme." **(GN: I, M, B: Exp, Ref)**

Refael Hassin

1981a Generalizations of Hoffman's existence theorem for circulations. *Networks* 11 (1981), 243–254. MR 83c:90055. Zbl. 459.90026. **(GN)**

Patrick Headley

1997a On a family of hyperplane arrangements related to the affine Weyl groups. *J. Algebraic Combin.* 6 (1997), 331–338. MR 98e:52010. Zbl. 970.66199.*

The characteristic polynomials of the Shi hyperplane arrangements $\mathcal{S}(W)$ of type W for each Weyl group W, evaluated computationally. $\mathcal{S}(W)$ is obtained by splitting the reflection hyperplanes of W in two in a certain way; thus $\mathcal{S}(A_{n-1})$ splits the arrangement representing $\operatorname{Lat} G(K_n)$—more precisely, it represents $\operatorname{Lat}^{\mathrm{b}} \Phi$ where $\Phi = (K_n, \varphi_0) \cup (K_n, \varphi_1)$ (see Stanley (1996a) for notation); that of type B_n splits the arrangement representing $\operatorname{Lat} G(\pm K_n^\bullet)$, and so on. [See also Athanasiadis (1996a).] **(gg: G, M, N)**

Fritz Heider

1946a Attitudes and cognitive organization. *J. Psychology* 21 (1946), 107–112.

No mathematics, but a formative article. [See Cartwright and Harary (1956a).] **(PsS)**

1979a On balance and attribution. In: Paul W. Holland and Samuel Leinhardt, eds., *Perspectives on Social Network Research* (Proc. Sympos., Dartmouth Coll., Hanover, N.H., 1975), Ch. 2, pp. 11–23. Academic Press, New York, 1979. (**PsS**)(**SG: B**)

Richard V. Helgason

See J.L. Kennington.

I. Heller

1957a On linear systems with integral valued solutions. *Pacific J. Math.* 7 (1957), 1351–1364. MR 20 #899. Zbl. 79, 19 (e: 0779.01903).

I. Heller and C. B. Tompkins

1956a An extension of a theorem of Dantzig's. In: H. W. Kuhn and A. W. Tucker, eds., *Linear Inequalities and Related Systems*, pp. 247–252. Annals of Math. Studies, No. 38. Princeton Univ. Press, Princeton, N.J., 1956. MR 18, 459. Zbl. 72, 378 (e: 072.37804). (**sg: I, B**)

Robert L. Hemminger and Joseph B. Klerlein

1979a Line pseudodigraphs. *J. Graph Theory* 1 (1977), 365–377. MR 57 #5812. Zbl. 379.05032.

An attempt, intrinsically unsuccessful, to represent the (signed) line graph of a digraph (see Zaslavsky 20xxb) by a digraph. [Continued by Klerlein (1975a).] (**sg: LG, o**)

Robert L. Hemminger and Bohdan Zelinka

1973a Line isomorphisms on dipseudographs. *J. Combin. Theory Ser. B* 14 (1973), 105–121. MR 47 #3230. Zbl. 263.05107. (**sg: LG, o**)

J.A. Hertz

See K.H. Fischer.

Hector Hevia

See G. Chartrand.

Dorit S. Hochbaum

20xxa A framework for half integrality and 2-approximations with applications to feasible cut and minimum satisfiability. Submitted.

Slightly extends Hochbaum and Naor (1994a) and Hochbaum, Megiddo, Naor, and Tamir (1993a). (**GN: I(D): Alg**)

Dorit S. Hochbaum, Nimrod Megiddo, Joseph (Seffi) Naor, and Arie Tamir

1993a Tight bounds and 2-approximation algorithms for integer programs with two variables per inequality. *Math. Programming Ser. B* 62 (1993), 69–83. MR 94k:90050. Zbl. 802.90080.

Approximate solution of integer linear programs with real, dually gain-graphic coefficient matrix. [See Sewell (1996a).] (**GN: I(D): Alg**)

Dorit S. Hochbaum and Joseph (Seffi) Naor

1994a Simple and fast algorithms for linear and integer programs with two variables per inequality. *SIAM J. Computing* 23 (1994), 1179–1192. MR 95h:90066. Zbl. 831.90089.

Linear and integer programs with real, dually gain-graphic coefficient matrix: feasibility for linear programs, solution of integer programs when the gains are positive ("monotone inequalities"), and identification of "fat" polytopes (that contain a sphere larger than a unit hypercube). (**GN: I(D): Alg, Ref**)

Cornelis Hoede

1981a The integration of cognitive consistency theories. Memorandum nr. 353, Dept. of Appl. Math., Twente Univ. of Tech., Enschede, The Netherlands, Oct., 1981. (**PsS: Gen**)(**SG, VS: B**)

1982a Anwendungen von Graphentheoretischen Methoden und Konzepten in den Socialwissenschaften. Memorandum nr. 390, Dept. of Appl. Math., Twente Univ. of Tech., Enschede, the Netherlands, May, 1982.
Teil 4: "Kognitive Konsistenz." (**PsS: Gen: Exp**)

††1992a A characterization of consistent marked graphs. *J. Graph Theory* 16 (1992), 17–23. MR 93b:05141. Zbl. 748.05081.
Characterizes when one can sign the vertices of a graph so every polygon has positive sign product, solving the problem of Beineke and Harary (1978b). [The definitive word.] (**VS: B: Str**)

Alan J. Hoffman

See also David Gale.

1970a $-1-\sqrt{2}$? In: Richard Guy *et al.*, eds., *Combinatorial Structures and Their Applications* (Proc. Calgary Internat. Conf., 1969), pp. 173–176. Gordon and Breach, New York, 1970. Zbl. 262.05133. (**LG**)

1972a Eigenvalues and partitionings of the edges of a graph. *Linear Algebra Appl.* 5 (1972), 137–146. MR 46 #97. Zbl. 247.05125. (**p: A, Fr**)

1974a On eigenvalues of symmetric $(+1,-1)$ matrices. *Israel J. Math.* 17 (1974), 69–75. MR 50 #2202. Zbl. 281.15003.
Eigenvalues of signed complete graphs. (**k: A**)

1975a Spectral functions of graphs. In: *Proceedings of the International Congress of Mathematicians* (Vancouver, 1974), Vol. 2, pp. 461–463. Canad. Math. Congress, Montreal, 1975. MR 55 #7850. Zbl. 344.05164. (**TG, A**)

1976a On spectrally bounded signed graphs. (Abstract.) In: *Trans. Twenty-First Conference of Army Mathematicians* (White Sands, N.M., 1975), pp. 1–5. ARO Rep. 76-1. U.S. Army Research Office, Research Triangle Park, N.C., 1976. MR 58 #27648.
Abstract of (1977b). (**SG: LG**)

1977a On graphs whose least eigenvalue exceeds $-1-\sqrt{2}$. *Linear Algebra Appl.* 16 (1977), 153–165. MR 57 #9607. Zbl. 354.05048. (**LG**)

1977b On signed graphs and gramians. *Geometriae Dedicata* 6 (1977), 455–470. MR 57 #3167. Zbl. 407.05064. (**SG: LG**)

[**A. J. Hoffman and D. Gale**]

1956a Appendix [to the paper of Heller and Tompkins]. In: H. W. Kuhn and A. W. Tucker, eds., *Linear Inequalities and Related Systems*, pp. 252–254. Annals of Math. Studies., No. 38. Princeton Univ. Press, Princeton, N.J., 1956.

Alan J. Hoffman and Peter Joffe

1978a Nearest $\mathcal{S}$-matrices of given rank and the Ramsey problem for eigenvalues of bipartite $\mathcal{S}$-graphs. In: *Problèmes Combinatoires et Théorie des Graphes* (Colloq. Internat., Orsay, 1976), pp. 237–240. Colloques Internat. du CNRS, 260. Editions du C.N.R.S., Paris, 1978. MR 81b:05080. Zbl. 413.05031. (**SG: A**)

Alan J. Hoffman and Francisco Pereira

1973a On copositive matrices with -1, 0, 1 entries. *J. Combinatorial Theory Ser. A* 14 (1973), 302–309. MR 47 #5029. Zbl. 273.15019.

Franz Höfting and Egon Wanke

1993a Polynomial algorithms for minimum cost paths in periodic graphs. In: Vijaya Ramachandran *et al.*, eds., *Proceedings of the Fourth Annual ACM-SIAM Symposium on Discrete Algorithms* (Austin, Tex., 1993), pp. 493–499. Assoc. Comput. Mach., New York, and Soc. Indust. Appl. Math., Philadelphia, 1993. MR 93m:05184. Zbl. 801.68133.

Given a finite gain digraph Φ (the "static graph") with gains in $\mathbb{Z}^d$ and a rational cost for each edge, find a minimum-cost walk ("path") in its canonical covering graph $\tilde{\Phi}$ with given initial and final vertices. (**GD(Cov): Alg**)

1994a Polynomial time analysis of toroidal periodic graphs. In: Serge Abiteboul and Eli Shamir, eds., *Automata, Languages and Programming* (Proc. 21st Internat. Colloq., ICALP 94, Jerusalem, 1994), pp. 544–555. Lect. Notes Computer Sci., Vol. 820. Springer-Verlag, Berlin, 1994. MR 96c:05164.

Take a gain digraph Φ (the "static graph") with gains in $\mathbb{Z}_\alpha = \mathbb{Z}_{\alpha_1} \times \cdots \times \mathbb{Z}_{\alpha_d}$ (where $\alpha = (\alpha_1, \cdots, \alpha_d)$) and its canonical covering digraph $\tilde{\Phi}$ (the "toroidal periodic graph"). Treated algorithmically via integer linear programming and linear Diophantine equations: existence of directed paths (NP-complete, but polynomial-time if Φ is strongly connected) and number of strongly connected components of $\tilde{\Phi}$. (**GD(Cov): Alg, G**)

1995a Minimum cost paths in periodic graphs. *SIAM J. Computing* 24 (1995), 1051–1067. MR 96d:05061. Zbl. 839.05063.

Full version of (1993a). The min-cost problem is expressed as an integer linear program. Various conditions under which the problem is NP-hard, even a very restricted version without costs (Thms. 3.3, 3.5), or polynomial-time solvable (e.g.: without costs, when Φ is an undirected gain graph: Thm. 3.4; with costs, when d is fixed: Thm. 4.5). (**GD, GG(Cov): Alg, G, Ref**)

20xxa Polynomial time analysis of toroidal periodic graphs. Submitted.

Full version of (1994a). (**GD(Cov): Alg, G**)

Paul W. Holland and Samuel Leinhardt

1971a Transitivity in structural models of small groups. *Comparative Group Studies* 2 (1971), 107–124. (**PsS: SG: B**)

Paul W. Holland and Samuel Leinhardt, eds.

1979a *Perspectives on Social Network Research* (Proc. Math. Soc. Sci. Board Adv. Res. Symp. on Social Networks held at Dartmouth College, Hanover, N.H., September 18–21, 1975). Academic Press, New York, 1979. (**PsS, SG**)

John Hultz

See also F. Glover.

John Hultz and D. Klingman

1979a Solving singularly constrained generalized network problems. *Appl. Math. Optim.* 4 (1978), 103–119. MR 57 #15414. Zbl. 373.90075. (**GN: M(bases)**)

John E. Hunter

1978a Dynamic sociometry. *J. Math. Sociology* 6 (1978), 87–138. MR 58 #20631. (**SG: B, Cl**)

C.A.J. Hurkens

1989a On the existence of an integral potential in a weighted bidirected graph. *Linear Algebra Appl.* 114/115 (1989), 541–553. MR 90c:05142. Zbl. 726.05050.

Given: a bidirected graph B (with no loose or half edges or positive loops) and an integer weight b_e on each edge. Wanted: an integral vertex weighting x such that $I(B)^{\mathrm{T}}x \leq b$, where $I(B)$ is the incidence matrix. Such x exists iff (i) every coherent polygonal or handcuff walk has nonnegative total weight and (ii) each doubly odd Korach walk (a generalization of a coherent handcuff that has a cutpoint dividing it into two parts, each with odd total weight) has positive total weight. This improves a theorem of Schrijver (1991a) and is best possible. Dictionary: "path" ("cycle") = coherent (closed) walk.

(**sg: O: I**)

T. Ibaraki

See also Y. Crama and P.L. Hammer.

T. Ibaraki and U.N. Peled

1981a Sufficient conditions for graphs to have threshold number 2. In: Pierre Hansen, ed., *Studies on Graphs and Discrete Programming* (Proc. Workshop, Brussels, 1979), pp. 241–268. North-Holland Math. Studies, 59. Ann. Discrete Math., 11. North-Holland, Amsterdam, 1981. MR 84f:05056. Zbl. 479.05058. (**p: o**)

Takeo Ikai

See H. Kosako.

Yoshiko T. Ikebe and Akihisa Tamura

20xxa Perfect bidirected graphs. Submitted

A transitively closed bidirection of a simple graph is perfect iff its underlying graph is perfect. (See Johnson and Padberg (1982a) for definitions.) [Also proved by Sewell (1996a).] (**sg: O: I, G**)

Masao Iri and Katsuaki Aoki

1980a A graphical approach to the problem of locating the origin of the system failure. *J. Oper. Res. Soc. Japan* 23 (1980), 295–312. MR 82c:90041. Zbl. 447.90036.

(**SD, VS: Appl**)

Masao Iri, Katsuaki Aoki, Eiji O'Shima, and Hisayoshi Matsuyama

1976a [A graphical approach to the problem of locating the system failure.] (In Japanese.) [???] 76 (135) (1976), 63–68. (**SD, VS: Appl**)

1979a An algorithm for diagnosis of system failures in the chemical process. *Computers and Chem. Eng.* 3 (1979), 489–493 (1981).

The process is modelled by a signed digraph with some nodes v marked by $\mu(v) \in \{+, -, 0\}$. (Marks $+, -$ indicate a failure in the process.) Object: to locate the node which is origin of the failure. An oversimplified description of the algorithm: μ is extended arbitrarily to V. Arc (u, v) is discarded if $0 \neq \mu(u)\mu(v) \neq \sigma(u, v)$. If the resulting digraph has a unique initial strongly connected component S, the nodes in it are possible origins. Otherwise, this extension provides no information. (I have overlooked: special marks on "controlled" nodes; speedup by stepwise extension and testing of μ.) [This article and/or (1976a) seems to be the origin of a whole literature. See e.g. Chang and Yu (1990a), Kramer and Palowitch (1987a).]

(**SD, VS: Appl, Alg**)

C. Itzykson

See R. Balian.

P.L. Ivanescu [P.L. Hammer]
See E. Balas and P.L. Hammer.

Sousuke Iwai
See O. Katai.

François Jaeger
1992a On the Kauffman polynomial of planar matroids. In: Jaroslav Nešetřil and Miroslav Fiedler, eds., *Fourth Czechoslovak Symposium on Combinatorics, Graphs and Complexity* (Prachatice, 1990), pp. 117–127. Ann. Discrete Math., Vol. 51. North-Holland, Amsterdam, 1992. MR 94d:57016. Zbl. 763.05021.
(This is not the colored Tutte polynomial of Kauffman (1989a).) Jaeger shows that the Kauffman polynomial, originally defined for link diagrams and here transformed to an invariant of signed plane graphs, depends only on the edge signs and the polygon matroid. It can also be reformulated to be essentially independent of signs. *Problem.* Define a similar invariant for more general matroids. (**SGc, S(M): N, Knot**)

François Jaeger, Nathan Linial, Charles Payan, and Michael Tarsi
1992a Group connectivity of graphs—a nonhomogeneous analogue of nowhere-zero flow properties. *J. Combin. Theory Ser. B* 56 (1992), 165–182. MR 93h:05088. Zbl. 824.05043.
Let $\mathfrak{A}$ be abelian group. Γ is "$\mathfrak{A}$-colorable" if every $\mathfrak{A}$-gain graph on Γ has a proper group-coloring (as in Zaslavsky (1991a)). Prop. 4.2. Every simple planar graph is $\mathfrak{A}$-colorable for every abelian group $\mathfrak{A}$ of order ≥ 6. (For the same reason as the classical 6-Color Theorem.) [Improved by Lai and Zhang (20xxb).] (**GG: Col**)

John C. Jahnke
See J.O. Morrissette.

John J. Jarvis and Anthony M. Jezior
1972a Maximal flow with gains through a special network. *Oper. Res.* 20 (1972), 678–688. MR 47 #6286. Zbl. 241.90021. (**GN: M(bases)**)

Clark Jeffries
1974a Qualitative stability and digraphs in model ecosystems. *Ecology* 55 (1974), 1415–1419.
Sufficient (and necessary) conditions for sign stability in terms of negative cycles and a novel color test. Proofs are sketched or (for necessity) absent. (**SD: Sta**)

Clark Jeffries, Victor Klee, and Pauline van den Driessche
1977a When is a matrix sign stable? *Canad. J. Math.* 29 (1977), 315–326. MR 56 #5603. Zbl. 383.15005. (**SD: Sta**)

Paul A. Jensen and J. Wesley Barnes
1980a *Network Flow Programming.* Wiley, New York, 1980. MR 82f:90096. Zbl. 502.-90057. Reprinted by: Robert E. Krieger, Melbourne, Fla., 1987. MR 89a:90152.
§1.4: "The network-with-gains model." §2.8: "Networks with gains—example applications." Ch. 9: "Network manipulation algorithms for the generalized network." Ch. 10: "Generalized minimum cost flow problems." (**GN: M(bases)**)
Sec. 5.5: "Negative cycles." (**OG: M(bases)**)

1984a *Potokovoe programmirovanie.* Radio i Svyaz, Moskva, 1984. Zbl. 598.90035.
Russian translation of (1980a). (**GN: M(bases)**)(**OG: M(bases)**)

P.A. Jensen and Gora Bhaumik

1977a A flow augmentation approach to the network with gains minimum cost flow problem. *Management Sci.* 28 (1977), No. 6 (Feb., 1977), 631–643. MR 55 #14163. Zbl. 352.90024. **(GN)**

Tommy R. Jensen and Bjarne Toft

1f995a *Graph Coloring Problems.* Wiley, New York, 1995. MR 95h:05067. Zbl. 950.45277.

8.14: "t-perfect graphs." Related to all-negative Σ with no subgraph homeomorphic to $-K_4$ (no "odd-K_4"). See Gerards and Schrijver (1986a), Gerards and Shepherd (1998b).

15.9: "Square hypergraphs." Related to nonexistence of even cycles in a digraph and to sign nonsingularity. See Seymour (1974a) and Thomassen (1985a, 1986a, 1992a). **(sd: P: b, Sol: Exp)**

R.H. Jeurissen

1975a Covers, matchings and odd cycles of a graph. *Discrete Math.* 13 (1975), 251–260. MR 54 #168. Zbl. 311.05129. **(ec: b)**

1981a The incidence matrix and labellings of a graph. *J. Combin. Theory Ser. B* 30 (1981), 290–301. MR 83f:05048. Zbl. 409.05042, (457.05047). **(SG: I, EC)**

1983a Disconnected graphs with magic labellings. *Discrete Math.* 43 (1983), 47–53. MR 84c:05064. Zbl. 499.05053. **(SG: I, EC)**

1983b Pseudo-magic graphs. *Discrete Math.* 43 (1983), 207–214. MR 84g:05122. Zbl. 514.05054. **(SG: I, EC)**

1988a Magic graphs, a characterization. *European J. Combin.* 9 (1988), 363–368. MR 89f:05138. Zbl. 657.05065.

William S. Jewell

1962a Optimal flow through networks with gains. *Oper. Res.* 10 (1962), 476–499. MR 26 #2325. Zbl. (e: 109.38203). **(GN)**

Anthony M. Jezior

See J.J. Jarvis.

Samuel Jezný and Marián Trenkler

1983a Characterization of magic graphs. *Czechoslovak Math. J.* 33 (108) (1983), 435–438. MR 85c:05030. Zbl. 571.05030. **(p: I)**

Peter Joffe

See A.J. Hoffman.

Eugene C. Johnsen

1989a The micro-macro connection: Exact structure and process. In: Fred Roberts, ed., *Applications of Combinatorics and Graph Theory to the Biological and Social Sciences*, pp. 169–201. IMA Vols. Math. Appl., Vol. 17. Springer-Verlag, New York, 1989. MR 90g:92089. Zbl. 725.92026 (q.v.).

An elaborate classificatory analysis of "triads" (signed complete directed graphs of 3 vertices) vis-á-vis "macrostructures" (signed complete directed graphs) with reference to structural interactions and implications of triadic numerical restrictions on "dyads" (s.c.d.g. of 2 vertices). Connections to certain models of affect in social psychology. ["Impenetrability! That's what *I* say!" "Would you tell me, please," said Alice, "what that means?"]

(K, SD, SG: B, PsS: Exp)

Eugene C. Johnsen and H. Gilman McCann

1982a Acyclic triplets and social structure in complete signed digraphs. *Social Networks* 3 (1982), 251–272. **(SD: B, Cl)**

Charles R. Johnson and John Maybee

1991a Qualitative analysis of Schur complements. In: *Applied Geometry and Discrete Mathematics: The Victor Klee Festschrift*, pp. 359–365. DiMACS Ser. Discrete Math. Theoret. Computer Sci., Vol. 4. Amer. Math. Soc., Providence, 1991. MR 92h:15004. Zbl. 742.15009.

In square matrix A let $A[S]$ be the principal submatrix with rows and columns indexed by S. Thm. 1: Assume $A[S]$ is sign-nonsingular in standard form and $i, j \notin S$. Then the (i, j) entry of the Schur complement of $A[S]$ has sign determined by the sign pattern of A iff, in the signed digraph of A, every path $i \to j$ via S has the same sign. **(QM: SD)**

Charles R. Johnson, D.D. Olesky, Michael Tsatsomeros, and P. van den Driessche

1993a Spectra with positive elementary symmetric functions. *Linear Algebra Appl.* 180 (1993), 247–261. MR 94a:15028. Zbl. 778.15006.

Suppose the signed digraph D of an $n \times n$ matrix has longest cycle length k and all cycles of $-D$ are negative. Theorem: If $k = n - 1$, the eigenvalues lie in a domain subtending angle $< 2\pi/k$. This is known for $k = 2$ but false for $k = n - 3$. **(QM, SD)**

Ellis L. Johnson

See also J. Edmonds and G. Gastou.

1965a Programming in networks and graphs. Report ORC 65-1, Operations Research Center, Univ. of California, Berkeley, Calif., Jan. 1965.

§7: "Flows with gains." §8: "Linear programming in an undirected graph." §9: "Integer programming in an undirected graph."

(GN: I, M(bases))(ec: I, M(bases), Alg)

1966a Networks and basic solutions. *Oper. Res.* 14 (1966), 619–623. **(GN)**

Ellis L. Johnson and Sebastiano Mosterts

1987a On four problems in graph theory. *SIAM J. Algebraic Discrete Methods* 8 (1987), 163–185. MR 88d:05097. Zbl. 614.05036.

Two of the problems: Given a signed graph (edges called "even" and "odd" rather than "positive" and "negative"). The co-postman problem is to find a minimum-cost deletion set (of edges). The "odd circuit" problem is to find a minimum-cost negative polygon. The Chinese postman problem is described in a way that involves cobalance and "switching" around a polygon.

(SG: Fr(Gen), I)

Ellis L. Johnson and Manfred W. Padberg

1982a Degree-two inequalities, clique facets, and biperfect graphs. In: Achim Bachem, Martin Grotschel, and Bernhard Korte, eds., *Bonn Workshop on Combinatorial Optimization* (Fourth, 1980), pp. 169–187. North-Holland Math. Studies, 66. Ann. Discrete Math., 16. North-Holland, Amsterdam, 1982. MR 84j:05085. Zbl. 523.52009.

Geometry of the bidirected stable set polytope $P(B)$ (which generalizes the stable set polytope to bidirected graphs), defined as the convex hull of $0, 1$ solutions of $x_i + x_j \leq 1$, $-x_i - x_j \leq -1$, $x_i \leq x_j$ for extroverted, introverted, and directed edges of B. (Thus, undirected graphs correspond to extroverted bidirected graphs.) It suffices to treat transitively closed bidirections of simple graphs ([unfortunately] called "bigraphs"). [Such a bidirected graph must be balanced.] A "biclique" (S_+, S_-) is the Harary bipartition of a balanced complete subgraph (S_+, S_- are the source and sink sets of the subgraph). It is "strong" if no external vertex has an edge directed out of every vertex of S_+ and an edge directed into every vertex of S_-. Strong bicliques generate facet inequalities of the polytope. Call B perfect if these facets (and nonnegativity) determine $P(B)$. Γ is "biperfect" if every transitively closed bidirection B of Γ is perfect. Conjectures: Γ is biperfect iff it is perfect. Γ is perfect iff some transitively closed bidirection is perfect. [Both proved by Sewell (1996a) and independently by Ikebe and Tamura (20xxa). See e.g. Tamura (1997a), Conforti (20xxa) for further work.] **(sg: O: I, G, sw)**

Leif Kjær Jørgensen

1989a Some probabilistic and extremal results on subdivisions and odd subdivisions of graphs. *J. Graph Theory* 13 (1989), 75–85. MR 90d:05186. Zbl. 672.05070.

Let $\sigma_{\text{op}}(\Gamma)$, or $\sigma_{\text{odd}}(\Gamma)$, be the largest s for which $-\Gamma$ contains a subdivision of $-K_s$ (an "odd-path-K_sS"), or $[-\Gamma]$ contains an antibalanced subdivision of K_s (an "odd-K_sS"). Thm. 4: $\sigma_{\text{op}}(\Gamma), \sigma_{\text{odd}}(\Gamma) \approx \sqrt{n}$. Thms. 7, 8 (simplified): For $p = 4, 5$ and large enough $n = |V|$, $\sigma_{\text{odd}}(\Gamma) \geq p$ or Γ is a specific exceptional graph. *Conjecture* 9. The same holds for all $p \geq 4$. [*Problem.* Generalize this to signed graphs.] **(p: X)**

Tadeusz Józefiak and Bruce Sagan

1992a Free hyperplane arrangements interpolating between root system arrangements. In: *Séries formelles et combinatoire algébrique* (Actes du colloque, Montréal, 1992), pp. 265–270. Publ. Lab. Combin. Inform. Math., Vol. 11. Dép. de math. et d'informatique, Univ. de Québec à Montréal, 1992.

Summarizes the freeness results in (1993a). **(sg, gg: G, m, N)**

1993a Basic derivations for subarrangements of Coxeter arrangements. *J. Algebraic Combin.* 2 (1993), 291–320. MR 94j:52023. Zbl. 798.05069.

The hyperplane arrangements (over fields with characteristic $\neq 2$) corresponding to certain signed graphs are shown to be "free". Explicit bases and the exponents are given. The signed graphs are: $+K_{n-1} \subseteq \Sigma_1 \subseteq +K_n$ (known), $\pm K_n \subseteq \Sigma_2 \subseteq \pm K_n^\circ$, $\pm K_n \subseteq \Sigma_3 \subseteq \pm K_n^\circ$; also, those obtained from $+K_n$ or K_n° by adding all negative links in the order of their larger vertex (assuming ordered vertices) (Thms. 4.1, 4.2) or smaller vertex (Thms. 4.4, 4.5); and those obtained from $\pm K_{n-1}$ by adding positive edges ahead of negative ones (Thm. 4.3). [For further developments see Edelman and Reiner (1994a).] Similar theorems hold for complex arrangements when the sign group is replaced by the complex s-th roots of unity (§5). The Möbius functions of Σ_2, known from Hanlon (1988a), are deduced in §6. **(sg, gg: G, m, N)**

M. Jünger

See M. Grötschel.

Mark Jungerman and Gerhard Ringel

1978a The genus of the n-octahedron: Regular cases. *J. Graph Theory* 2 (1978), 69-75. MR 58 #5315. Zbl. 384.05037.

"Cascades": see Youngs (1968b). **(sg: O: Appl)**

Jerald A. Kabell

See also F. Harary.

1985a Co-balance in signed graphs. *J. Combin. Inform. System Sci.* 10 (1985), 5–8. MR 89i:05232. Zbl. 635.05028.

Cobalance means that every cutset has positive sign product. Thm.: Σ is cobalanced iff every vertex star has evenly many negative edges. For planar graphs, corollaries of this criterion and Harary's bipartition theorem result from duality. [The theorem follows easily by looking at the negative subgraph.] **(SG: B(D), B)**

1988a An algorithmic look at cycles in signed graphs. 250th Anniversary Conf. on Graph Theory (Fort Wayne, Ind., 1986). *Congressus Numerantium* 63 (1988), 229–230. MR 90d:05143. Zbl. 666.05046. **(SG, SD: B: Alg)**

Jeff Kahn and Joseph P.S. Kung

1980a Varieties and universal models in the theory of combinatorial geometries. *Bull. Amer. Math. Soc. (N.S.)* 3 (1980), 857–858. MR 81i:05051. Zbl. 473.05025.

Announcement of (1982a). **(gg: M)**

††1982a Varieties of combinatorial geometries. *Trans. Amer. Math. Soc.* 271 (1982), 485–499. MR 84j:05043. Zbl. 503.05010. Reprinted in: Joseph P.S. Kung, *A Source Book in Matroid Theory*, pp. 395–499, with commentary, pp. 335–338. Birkhäuser, Boston, 1986. MR 88e:05028. Zbl. 597.05019.

A "variety" is a class closed under deletion, contraction, and direct summation and having for each rank a "universal model", a single member containing all others. There are two nontrivial types of variety of finite matroids: matroids representable over GF(q), and gain-graphic matroids with gains in a finite group $\mathfrak{G}$. The universal models of the latter are the Dowling geometries $Q_n(\mathfrak{G})$.

It is incidentally proved that Dowling geometries of non-group quasigroups cannot exist in rank $n \geq 4$. **(gg: M)**

1986a A classification of modularly complemented geometric lattices. *European J. Combin.* 7 (1986), 243–248. MR 87i:06026. Zbl. 614.05018.

A geometric lattice of rank ≥ 4, if not a projective geometry with a few points deleted, is a Dowling lattice. **(gg: M)**

Jeff Kahn and Roy Meshulam

1998a On the number of group-weighted matchings. *J. Algebraic Combin.* 7 (1998), 285–290. MR 99b:05113. Zbl. 899.05042.

Continues Aharoni, Meshulam, and Wajnryb (1995a) (q.v., for definitions), generalizing its Thm. 1.3 (the case $|K| = 2$ of the following). Let $m =$ number of 0-weight matchings, $\delta =$ minimum degree. Thm. 1.1: If $m > 0$ then $m \geq (\delta - k + 1)!$ where $k = |K|$. *Conjecture* 1.2. k can be reduced. (See the paper for details.) [*Question.* Is there a generalization to weighted digraphs? One could have two kinds of arcs: some weighted from K, and some weighted 0. The perfect matching might be replaced by an alternating Hamilton cycle or a spanning union of disjoint alternating cycles.] **(WG)**

Thm. 2.1: Let D be a simple digraph with weights in an abelian group K. If all outdegrees are $> k$, where $k = |K|$, then there is a nonempty set of disjoint cycles whose total weight is 0. **(WD)**

Ajai Kapoor

See M. Conforti.

Roman Kapuscinski

See P. Doreian.

Richard M. Karp, Raymond E. Miller, and Shmuel Winograd

1967a The organization of computations for uniform recurrence equations. *J. Assoc. Computing Machinery* 14 (1967), 563–590. MR 38 #2920. Zbl. (e 171.38305). **(gd: cov)**

P.W. Kasteleyn

See also C.M. Fortuin.

P.W. Kasteleyn and C.M. Fortuin

1969a Phase transitions in lattice systems with random local properties. In: *International Conference on Statistical Mechanics* (Proc., Kyoto, 1968), pp. 11–14. Supplement to *J. Physical Soc. Japan*, Vol. 26, 1969. Physical Society of Japan, [Tokyo?], 1969.

A specialization of the parametrized dichromatic polynomial of a graph: $Q_\Gamma(q, p; x, 1)$ where $q_e = 1 - p_e$. [Essentially, announcing Fortuin and Kasteleyn (1972a).] **(sgc: Gen: N, Phys)**

Osamu Katai

1979a Studies on aggregation of group structures and group attributes through quantification methods. D.Eng. dissertation, Kyoto Univ., 1979.

Osamu Katai and Sousuke Iwai

1978a Studies on the balancing, the minimal balancing, and the minimum balancing processes for social groups with planar and nonplanar graph structures. *J. Math. Psychology* 18 (1978), 140–176. MR 83m:92072. Zbl. 394.92027. **(SG: Fr, Alg)**

1978b Graph-theoretic models of social group structures and indices of group structures. (In Japanese.) *Systems and Control* (*Shisutemu to Seigyo*) 22 (1978), 713–722. MR 80d:92038. **(Exp)**

1978c On the characterization of balancing processes of social systems and the derivation of the minimal balancing processes. *IEEE Trans. Systems Man Cybernetics* SMC-8 (1978), 337–348. MR 57 #18886. Zbl. 383.92025.

1978d Characterization of social balance by statistical and finite-state systems theoretical analysis. In: *Internat. Conf. Cybernetics and Society* (Proc. Conf., Tokyo, November, 1978).

Louis H. Kauffman

See also J.R. Goldman.

1986a Signed graphs. Abstract 828-57-12, *Abstracts Amer. Math. Soc.* 7, No. 5 (1986), p. 307.

Announcement of (1989a). **(SGc: Knot: N)**

1988a New invariants in the theory of knots. *Amer. Math. Monthly* 95 (1988), 195–242. MR 89d:57005. Zbl. 657.57001.

A leisurely development of Kauffman's combinatorial bracket polynomial of a link diagram and the Jones and other knot polynomials, including the basics of (1989a). **(Knot, SGc: N: Exp)**

†1989a A Tutte polynomial for signed graphs. *Discrete Appl. Math.* 25 (1989), 105–127. MR 91c:05082. Zbl. 698.05026.

The Tutte polynomial, also called "Kauffman's bracket of a signed graph" and equivalent to his bracket of a link diagram, is defined by a sum over spanning trees of terms that depend on the signs and activities of the edges and nonedges of the tree. The point is that the deletion-contraction recurrence over an edge has parameters dependent on the color of the edge; also, the parameters of the two colors are related. The purpose is to develop the bracket of a link diagram combinatorially. §3.2, "Link diagrams": how link diagrams correspond to signed plane graphs. §4, "A polynomial for signed graphs", defines the general sign-colored graph polynomial $Q[\Sigma](A, B, d)$ by deletion-contraction, modified multiplication on components, and evaluation on graphs of loops and isthmi. §5, "A spanning tree expansion for $Q[G]$" [G means Σ], proves $Q[\Sigma]$ exists by producing a spanning-tree expansion, shown independent of the edge ordering by a direct argument. [No dichromatic form of $Q[\Sigma]$ appears; but see successor articles.] §6, "Conclusion", remarks that $Q[\Sigma]$ is invariant under signed-graphic Reidemeister moves II and III. [This significant work, inspired by Thistlethwaite (1988a), led to independent but related generalizations by Przytycka and Przytycki (1988a), Schwärzler and Welsh (1993a), Traldi (1989a), and Zaslavsky (1992b) that were partially anticipated by Fortuin and Kasteleyn (1972a). Also see (1997a).] (**SGc: N, Knot**)

1997a Knots and electricity. In: S. Suzuki, ed., *Knots '96* (Proc. Fifth Internat. Research Institute Math. Soc. Japan, Tokyo, 1996), pp. 213–230. World Scientific, Singapore, 1997.

§2, "A state summation for classical electrical networks", uses a form of the parametrized dichromatic polynomial $Q_\Gamma(B, A; 1, 1)$ [as in Zaslavsky (1992b) et al.], where $A(e), B(e) \in \mathbb{C}^*$, to compute conductances as in Goldman and Kauffman (1993a). (**sgc: Gen: N: Exp**)

§3: "The bracket polynomial", discusses the connections with signed graphs and electricity. *Problem*: Is there a signed graph, not reducible by signed-graphic Reidemeister moves (see (1989a)) to a tree with loops, whose sign-colored dichromatic polynomial is trivial? If not, the Jones polynomial detects the unknot. (**SGc: N: Exp**)(**SGc: N**)

John G. Kemeny and J. Laurie Snell

1962a *Mathematical Models in the Social Sciences.* Blaisdell, Waltham, Mass., 1962. Reprinted by MIT Press, Cambridge, Mass., 1972. MR 25 #3797. Zbl. (256.92003).

Chapter VIII: "Organization theory: Applications of graph theory." See pp. 97–101 and 105–107. (**SG: B: Exp**)

John W. Kennedy

See M.L. Gargano.

Jeff L. Kennington and Richard V. Helgason

1980a *Algorithms for Network Programming.* Wiley, New York, 1980. MR 82a:9013. Zbl. 502.90056.

Ch. 5: "The simplex method for the generalized network problem." (**GN: M(Bases): Exp**)

F. Kharari and È. Palmer [Frank Harary and Edgar M. Palmer]

See F. Harary and E.M. Palmer (1977a).

A. Khelladi

1987a Nowhere-zero integral chains and flows in bidirected graphs. *J. Combin. Theory Ser. B* 43 (1987), 95–115. MR 88h:05045. Zbl. 617.90026.

Improves the result of Bouchet (1983a). (**SG: M, Flows**)

Shin'ichi Kinoshita

See also T. Yajima.

Shin'ichi Kinoshita and Hidetaka Terasaka

1957a On unions of knots. *Osaka Math J.* 9 (1957), 131–153. MR 20 #4846. Zbl. 080.17001.

Employs the sign-colored graph of a link diagram (Bankwitz 1930a) to form certain combinations of links. (**SGc: Knot**)

M. Kirby

See A. Charnes.

Scott Kirkpatrick

1977a Frustration and ground-state degeneracy in spin glasses. *Phys. Rev. B* 16, No. 10 (1977), 4630–4641. (**Phys: SG, B, Sw**)

Victor Klee

See also C. Jeffries.

1971a The greedy algorithm for finitary and cofinitary matroids. In: Theodore S. Motzkin, ed., *Combinatorics*, pp. 137–152. Proc. Symp. Pure Math., Vol. 19. Amer. Math. Soc., Providence, R.I., 1971. MR 48 #10865. Zbl. 229.05031.

Along with Simões-Pereira (1972a), invents the bicircular matroid (here, for infinite graphs). (**Bic**)

1989a Sign-patterns and stability. In: Fred Roberts, ed., *Applications of Combinatorics and Graph Theory to the Biological and Social Sciences*, pp. 203–219. IMA Vols. Math. Appl., Vol. 17. Springer-Verlag, New York, 1989. MR 90h:34081. Zbl. 747.05057.

When are various forms of stability of a linear differential equation $\dot{x} = Ax$ determined solely by the sign pattern of A? A survey of elegant combinatorial criteria. Signed digraphs [alas] play but a minor role. (**Sta, SD: Exp, Ref**)

Victor Klee, Richard Ladner, and Rachel Manber

1984a Signsolvability revisited. *Linear Algebra Appl.* 59 (1984), 131–157. MR 86a:15004. Zbl. 543.15016. (**SD, QM: Sol, Alg**)

Victor Klee and Pauline van den Driessche

1977a Linear algorithms for testing the sign stability of a matrix and for finding Z-maximum matchings in acyclic graphs. *Numer. Math* 28 (1977), 273–285. Zbl. 348.65032, (352.65020). (**SD: QM, Sta, Alg**)

Peter Kleinschmidt and Shmuel Onn

1995a Oriented matroid polytopes and polyhedral fans are signable. In: Egon Balas and Jens Clausen, eds., *Integer Programming and Combinatorial Optimization* (4th Internat. IPCO Conf., Copenhagen, 1995, Proc.), pp. 198–211. Lecture Notes in Computer Sci., Vol. 920. Springer, Berlin, 1995. MR 97b:05040.

In a graded partially ordered set with $\hat{0}$ and $\hat{1}$, assign a sign to each covering pair (x, y) where y is covered by $\hat{1}$. This is an "exact signing" if in every upper interval there is just one y whose coverings are all positive. Then the poset is "signable". (**S: G**)

1996a Signable posets and partitionable simplicial complexes. *Discrete Computat. Geom.* 15 (1996), 443–466. MR 97a:52014. Zbl. 853.52010.

See (1995a) for definition. Signability is a generalization to posets of partitionability of a simplicial complex (Prop. 3.1). Shellable posets, and face lattices of spherical polytopes and oriented matroid polytopes, are signable. A stronger property of a simplicial complex, "total signability", which applies for instance to simplicial oriented matroid polytopes (Thm. 5.12), implies the upper bound property (Thm. 4.4). Computational complexity of face counting and of deciding shellability and partitionability are discussed in §6. **(S: G, Alg)**

Joseph B. Klerlein

See also R.L. Hemminger.

1975a Characterizing line dipseudographs. In: F. Hoffman *et al.*, eds., *Proceedings of the Sixth Southeastern Conference on Combinatorics, Graph Theory and Computing* (Boca Raton, 1975), pp. 429–442. Congressus Numerantium, XIV. Utilitas Math. Publ. Inc., Winnipeg, Man., 1975. MR 53 #190. Zbl. 325.05106.

Continues the topic of Hemminger and Kerlein (1977a). **(sg: LG, o)**

Darwin Klingman

See J. Elam, F. Glover, and J. Hultz.

Elizabeth Klipsch

20xxa Some signed graphs that are forbidden link minors for orientation embedding. In preparation.

For each $n \geq 5$, either $-K_n$ or its 1-edge deletion, but not both, is a forbidden link minor. Which one it is, is controlled by Euler's polyhedral formula, provided $n \geq 7$. [A long version with excruciating detail is available.] **(SG: T, P)**

Muralidharan Kodialam and James B. Orlin

1991a Recognizing strong connectivity in (dynamic) periodic graphs and its relation to integer programming. In: *Proceedings of the Second Annual ACM-SIAM Symposium on Discrete Algorithms* (San Francisco, 1991), pp. 131–135. Assoc. for Computing Machinery, New York, 1991. Zbl. 800.68639.

Linear programming methods to find the strongly connected components of a periodic digraph from the static graph: i.e., of the covering digraph of a gain digraph Φ with gains in $\mathbb{Q}^d$ by looking at Φ. Cf. Cohen and Megiddo (1993a), whose goals are similar but algorithms differ. **(GD(Cov): B, Polygons: Alg)**

János Komlós

1997a Covering odd cycles. *Combinatorica* 17 (1997), 393–400. MR 99b:05114. Zbl. 902.05036.

Sharp asymptotic upper bounds on frustration index and vertex elimination number for all-negative signed graphs with fixed negative girth. Improves Bollobás, Erdős, Simonovits, and Szemerédi (1978a). [*Problem.* Generalize to arbitrary signed graphs or signed simple graphs.] **(P: Fr)**

Helene J. Kommel

See F. Harary.

Dénes König

1936a *Theorie der endlichen und unendlichen Graphen.* Mathematik und ihre Anwendungen, Band 16. Akademische Verlagsgesellschaft, Leipzig, 1936. Reprinted by Chelsea, New York, 1950. MR 12, 195. Zbl. 13, 228 (e: 013.22803).

§X.3, "Komposition von Büsheln", contains Thms. 9–16 of Ch. X. I restate them in terms of a signature on the edge set; König says subgraph or p-subgraph ("p-Teilgraph") to mean what we would call the negative edge set of a signature or a balanced signature. Instead of signed switching, König speaks of set summation ("composition") with a vertex star ("Büschel"). His theorems apply to finite and infinite graphs except where stated otherwise. Thm. 9: The edgewise product of balanced signatures is balanced. Thm.

10: Every balanced signing of a finite graph is a switching of the all-positive signature. Thm. 11: A signature is balanced iff it has a Harary bipartition [see Harary (1953a)]. Thm. 12 (cor. of 11): A graph is bicolorable iff every polygon has even length. [König makes this fundamental theorem a corollary of a signed-graph theorem!] Thm. 13: A signature is balanced if (not only if) every polygon of a fundamental system is positive. Thm. 14: A graph with n vertices (a finite number) and c components has 2^{n-c} balanced signings. Thm. 16: The set of all vertex switchings except for one in each finite component of Γ forms a basis for the space of all finitely generated switchings. **(sg: B, sw, E)**

1990a *Theory of Finite and Infinite Graphs.* Transl. Richard McCoart, commentary by W.T. Tutte, biographical sketch by T. Gallai. Birkhäuser, Boston, 1990. MR 91f:01026.

English translation of (1936a). §X.3: "Composition of stars". The term "Kreis" (circle, meaning polygon) is translated as "cycle"—one of the innumerable meanings of "cycle". **(sg: B, sw, E)**

Hideo Kosako, Suck Joong Moon, Katsumi Harashima, and Takeo Ikai

1993a Variable-signed graph. *Bull. Univ. Osaka Pref. Ser. A* 42 (1993), 37–49. MR 96e:05167. Zbl. 798.05070.

"Variable-signed graph" = signed simple (di)graph Σ with switching function p and switched graph Σ^p. Known basic properties of switching are established. More interesting: planar duality when $|\Sigma|$ is planar. The planar dual $|\Sigma|*$ inherits the same edge signs; a dual vertex has sign of the surrounding primal face boundary. Property 9 is in effect the statements: (1) If a signed plane graph has f negative face boundaries, then $l(\Sigma) \geq f/2$. (2) If the negative faces fall into two connected groups with oddly many faces in each, (1) can be improved to $\geq f/2+1$. Finally, incidence matrices are studied that are only superficially related to signs. [The paper is hard to interpret due to mathematical imprecision and language difficulty.] **(SG: Sw, fr, D, I)**

Alexandr V. Kostochka

See A.A. Ageev and E. Györi.

A. Kotzig

1968a Moves without forbidden transitions in a graph. *Mat. Časopis* 18 (1968), 76–80. MR 39 #4038. Zbl. (e: 155.31901). **(p: o)**

David Krackhardt

See P. Doreian.

M.A. Kramer and B.L. Palowitch, Jr.

1987a A rule-based approach to fault diagnosis using the signed directed graph. *AIChE J.* 33 (1987), 1067–1078. MR 88j:94060.

Vertex signs indicate directions of change in vertex variables; signed directed edges describe relations among these directions.

Truth tables for a signed edge as a function of endpoint signs. Algorithms for deducing logical rules about states (assignments of vertex signs) from the signed digraph. Has a useful discussion of previous literature, e.g., Iri, Aoki, O'Shima, and Matsuyama (1979a). **(SD, VS: Appl, Alg, Ref)**

I. Krasikov

1988a A note on the vertex-switching reconstruction. *Internat. J. Math. Math. Sci.* 11

(1988), 825–827. MR 89i:05204. Zbl. 663.05046.

Following up Stanley (1985a), a signed K_n is reconstructible from its single-vertex switching deck if its negative subgraph is disconnected [therefore also if its positive subgraph is disconnected] or if the minimum degree of its positive or negative subgraph is sufficiently small. All done in terms of Seidel switching of unsigned simple graphs. **(k: sw, TG)**

1994a Applications of balance equations to vertex switching reconstruction. *J. Graph Theory* 18 (1994), 217–225. MR 95d:05091. Zbl. 798.05039.

Following up Krasikov and Roditty (1987a), (K_n, σ) is reconstructible from its s-vertex switching deck if $s = \frac{1}{2}n - r$ where $r \in \{0, 2\}$ and $r \equiv n \pmod 4$, or $r = 1 \equiv n \pmod 2$; also, if $s = 2$ and the minimum degree of the positive or negative subgraph is sufficiently small. Also, bounds on $|E_-|$ if (K_n, σ) is not reconstructible. Negative-subgraph degree sequence: reconstructible when $s = 2$ and $n \geq 10$. Done in terms of Seidel switching of unsigned simple graphs. **(k: sw, TG)**

1996a Degree conditions for vertex switching reconstruction. *Discrete Math.* 160 (1996), 273–278. MR 97f:04137. Zbl. 863.05056.

If the minimum degrees of its positive and negative subgraphs obey certain bounds, a signed K_n is reconstructible from its s-switching deck. The main bound involves the least and greatest even zeros of the Krawtchouk polynomial $K_s^n(x)$. Done in terms of Seidel switching of unsigned simple graphs. [More details in Zbl.] **(k: sw, TG)**

Ilia Krasikov and Simon Litsyn

1996a On integral zeros of Krawtchouk polynomials. *J. Combin. Theory Ser. A* 74 (1996), 71–99. MR 97i:33005. Zbl. 853.33008.

Among the applications mentioned (pp. 72–73): 2. "Switching reconstruction problem", i.e., graph-switching reconstruction as in Stanley (1985a) etc. 4. "Sign reconstruction problem", i.e., reconstructing a signed graph from its s-edge negation deck, which is the multiset of signed graphs obtained by separately negating each subset of s edges (here called "switching signs", but it is not signed-graph switching); this is a new problem. **(k: sw, TG)(SG)**

I. Krasikov and Y. Roditty

1987a Balance equations for reconstruction problems. *Arch. Math. (Basel)* 48 (1987), 458–464. MR 88g:050996. Zbl. 594.05049.

§2: "Reconstruction of graphs from vertex switching". Corollary 2.3. If a signed K_n is not reconstructible from its s-vertex switching deck, a certain linear Diophantine system (the "balance equations") has a certain kind of solution. For $s = 1$ the balance equations are equivalent to Stanley's (1985a) theorem; for larger s they may or may not be. All is done in terms of Seidel switching of unsigned simple graphs. [Ellingham and Royle (1992a) note a gap in the proof of Lemma 2.5.] **(k: sw, TG)**

1992a Switching reconstruction and Diophantine equations. *J. Combin. Theory Ser. B* 54 (1992), 189–195. MR 93e:05072. Zbl. 702.05062 (749.05047).

Main Theorem. Fix $s \geq 4$. If n is large and (for odd s) not evenly even, every signed K_n is reconstructible from its s-vertex switching deck. Different results hold for $s = 2, 3$. (This is based on and strengthens Stanley (1985a).) Theorems 5 and 6 concern reconstructing subgraph numbers. All done in terms of Seidel switching of unsigned simple graphs. **(k: sw, TG)**

1994a More on vertex-switching reconstruction. *J. Combin. Theory Ser. B* 60 (1994), 40–55. MR 94j:05090. Zbl. 794.05092.

Based on (1987a) and strengthening Stanley (1985a): Theorem 7. A signed K_n is reconstructible if the Krawtchouk polynomial $K_s^n(x)$ "has one or two even roots [lying] far from $n/2$" (the precise statement is complicated). Numerous other partial results, e.g., a signed K_n is reconstructible if $s = \frac{1}{2}(n-r)$ where $r = 0, 1, 3$, or $2, 4, 5, 6$ with side conditions. All is done in terms of Seidel switching of unsigned simple graphs. **(k: sw, TG)**

Jan Kratochvíl, Jaroslav Nešetřil, and Ondřej Zýka

1992a On the computational complexity of Seidel's switching. In: Jaroslav Nešetřil and Miroslav Fiedler, eds., *Fourth Czechoslovak Symposium on Combinatorics, Graphs and Complexity* (Prachatice, 1990), pp. 161–166. Ann. Discrete Math., Vol. 51. North-Holland, Amsterdam, 1992. MR 93j:05156. Zbl. 768.68047.

Is a given graph switching equivalent to a graph with a specified property? (This is Seidel switching of simple graphs.) Depending on the property, this question may be in P or be NP-complete, whether the original property is in P or is NP-complete. Properties: containing a Hamilton path; containing a Hamilton polygon; no induced P_2; regularity; etc. Thm. 4.1: Switching isomorphism and graph isomorphism are polynomially equivalent. **(TG: Sw: Alg)**

Vijaya Kumar [G.R. Vijayakumar]

See G.R. Vijayakumar.

Joseph P.S. Kung

See also J.E. Bonin and J. Kahn.

1986a Numerically regular hereditary classes of combinatorial geometries. *Geom. Dedicata* 21 (1986), 85–105. MR 87m:05056. Zbl. 591.05019.

Examples include Dowling geometries, Ex. (6.2), and the bias matroids of full group expansions of graphs in certain classes; see pp. 98–99. **(GG: M)**

1990a Combinatorial geometries representable over $GF(3)$ and $GF(q)$. I. The number of points. *Discrete Computat. Geom.* 5 (1990), 83–95. MR 90i:05028. Zbl. 697.51007.

The Dowling geometry over the sign group is the largest simple ternary matroid not containing the "Reid matroid". **(sg: M: X)**

1990b The long-line graph of a combinatorial geometry. II. Geometries representable over two fields of different characteristic. *J. Combin. Theory Ser. B* 50 (1990), 41–53. MR 91m:51007. Zbl. 645.05026.

Dowling geometries used in the proof of Prop. (1.2). **(gg: M)**

1993a Extremal matroid theory. In: Neil Robertson and Paul Seymour, eds., *Graph Structure Theory* (Proc., Seattle, 1991), pp. 21–61. Contemp. Math., Vol. 147. Amer. Math. Soc., Providence, R.I., 1993. MR 94i:05022. Zbl. 791.05018.

Survey and new results. See: §2.7: "Gain-graphic matroids." P. 30, fn. 9. §4.3: "Varieties." §4.5. "Framed gain-graphic matroids." §6.4: "Matroids representable over two different characteristics." §8: "Concluding remarks," on a possible ternary analog of Seymour's decomposition theorem. **(GG: M: X, Str, Exp, Ref)**

1993b The Radon transforms of a combinatorial geometry. II. Partition lattices. *Adv. Math.* 101 (1993), 114–132. MR 95b:05051. Zbl. 786.05018.

Dowling lattices are lower-half Sperner. The proof is given only for partition lattices. **(gg: M)**

1996a Matroids. In: M. Hazewinkel, ed., *Handbook of Algebra*, Vol. 1, pp. 157–184. North-Holland (Elsevier), Amsterdam, 1996. MR 98c:05040. Zbl. 856.05001.

§6.2: "Gain-graphic matroids." **(GG: M: Exp)**

1996b Critical problems. In: Joseph E. Bonin, James G. Oxley, and Brigitte Servatius, eds., *Matroid Theory* (Proc., Seattle, 1995), pp. 1–127. Contemp. Math., Vol. 197. Amer. Math. Soc., Providence, R.I., 1996. MR 97k:05049. Zbl. 862.05019.

A remarkable more-than-survey with numerous new results and open problems. §4.5: "Abstract linear functionals in Dowling group geometries". §6: "Dowling geometries and linear codes", concentrates on higher-weight Dowling geometries, extending Bonin (1993b). §7.4: "Critical exponents of classes of gain-graphic geometries". §7.5: "Growth rates of classes of gain-graphic geometries". §8.5: "Jointless Dowling group geometries". Corollary 8.30. §8.11: "Tangential blocks in $\mathcal{Z}(A)$". Also see pp. 56, 61, 88, 92, 114. **(GG, Gen: M)**

1998a A geometric condition for a hyperplane arrangement to be free. *Adv. Math.* 135 (1998), 303–329. Zbl. 905.05017

Delete from a Dowling geometry a subset S that contains no whole plane. Found: necessary and sufficient conditions for the characteristic polynomial to factor completely over the integers. When the geometry corresponds to a hyperplane arrangement, many more of the arrangements are not free than are free; however, if S contains no whole line, all are free (so the characteristic polynomial factors completely over $\mathbb{Z}$) while many are not supersolvable. **(gg: M: N)**

Joseph P.S. Kung and James G. Oxley

1988a Combinatorial geometries representable over $GF(3)$ and $GF(q)$. II. Dowling geometries. *Graphs Combin.* 4 (1988), 323–332. MR 90i:05029. Zbl. 702.51004.

For $n \geq 4$, the Dowling geometry of rank n over the sign group is the unique largest simple matroid of rank n that is representable over GF(3) and GF(q). **(sg: M: X)**

David Kuo

See J.-H. Yan.

Richard Ladner

See V. Klee.

George M. Lady and John S. Maybee

1983a Qualitatively invertible matrices. *Math. Social Sci.* 6 (1983), 397–407. MR 85f:-15005. Zbl. 547.15002.

In terms of signed graphs, restates and completes the characterizations of sign-invertible matrices A due to Bassett, Maybee, and Quirk (1968a) and George M. Lady (The structure of qualitatively determinate relationships. *Econometrica* 51 (1983), 197–218. MR 85c:90019. Zbl. 517.15004) and reveals the sign pattern of A^{-1} in terms of path signs in the associated signed digraph. **(QM: Sol: SD)**

J.C. Lagarias

1985a The computational complexity of simultaneous diophantine approximation problems. *SIAM J. Computing* 14 (1985), 196–209. MR 86m:11048. Zbl. 563.10025.

Theorem F: Feasibility of integer linear programs with at most two variables per constraint is NP-complete. (**GN(I): D: Alg**)

Hong-Jian Lai and Xiankun Zhang

20xxa Group colorability of graphs. Submitted

Simple graphs only are considered. The [abelian] "group chromatic number" $\chi_1(\Gamma) = \min m$ such that Γ is $\mathfrak{A}$-colorable (as in Jaeger, Linial, Payan, and Tarsi (1992a)) for every abelian $\mathfrak{A}$ of order $\geq m$. Various results, e.g., Γ is $\mathbb{Z}_2$-colorable iff it is a forest; analog of Brooks' Theorem (stronger than the original because $\chi_1(\Gamma) \geq \chi(\Gamma)$; analog of Nordhaus-Gaddum Theorem involving the complementary graph. [Thus $\chi_1(\Gamma)$ seems to resemble ordinary chromatic number more than it does gain-graph coloring.] (**GG: Col**)

20xxb Coloring a graph with elements in an Abelian group. Submitted

Continues (20xxa). Thm.: If Γ is simple and has no K_5 minor, then $\chi_1(\Gamma) \leq 5$, improving on Jaeger, Linial, Payan, and Tarsi (1992a). (**GG: Col**)

Kelvin Lancaster

1981a Maybee's "Sign solvability". In: Harvey J. Greenberg and John S. Maybee, eds., *Computer-Assisted Analysis and Model Simplification* (Proc. Sympos., Boulder, Col., 1980), pp. 259–270. Academic Press, New York, 1981. MR 82g:00016 (book). Zbl. 495.93001 (book).

Comment on Maybee (1981a). (**QM: Sol: SD**)

Andrea S. LaPaugh and Christos H. Papadimitriou

1984a The even-path problem for graphs and digraphs. *Networks* 14 (1984), 507–513. MR 86g:05057. Zbl. 552.68059.

Fast algorithms for existence of even paths between two given vertices (or any two vertices) of a graph. The corresponding digraph problem is NP-complete. [Signed (di)graphs are similar, due to the standard reduction by negative subdivision.] [See also, e.g., works by Thomassen.]
(**P: Paths: Alg**)(**sd: P: Paths: Alg**)

Michel Las Vergnas

See A. Björner.

Monique Laurent

See M.M. Deza and A.M.H. Gerards.

Eugene L. Lawler

1976a *Combinatorial Optimization: Networks and Matroids.* Holt, Rinehart and Winston, New York, 1976. MR 55 #12005. Zbl. 413.90040.

Ch. 6: "Nonbipartite matching." §3: Bidirected flows. (**sg: O**)

Ch. 4: "Network flows." §8: "Networks with losses and gains." §12: "Integrality of flows and the unimodular property." (**GN**)(**sg: I, B**)

Jason Leasure

See L. Fern.

Bruno Leclerc

1981a Description combinatoire des ultramétriques. *Math. Sci. Humaines* No. 73 (1981), 5–37. MR 82m:05083. Zbl. 476.05079. (**SG: B**)

Jon Lee

1989a Subspaces with well-scaled frames. *Linear Algebra Appl.* 114/115 (1989), 21–56. MR 90k:90111. Zbl. 675.90061.

See Section 9. (**sg: O: I, Flows, Alg**)

Shyi-Long Lee

See also I. Gutman.

1989a Comment on 'Topological analysis of the eigenvalues of the adjacency matrices in graph theory: A difficulty with the concept of internal connectivity'. *J. Chinese Chem. Soc.* 36 (1989), 63–65.

Response to Gutman (1988a). Proposes weighted net sign: divide by number of nonzero vertex signs. The goal is to have the ordering of net signs correlate more closely with that of eigenvalues. (**VS, SGw, Chem**)

1989b Net sign analysis of eigenvectors and eigenvalues of the adjacency matrices in graph theory. *Bull. Inst. Chem., Academica Sinica* No. 36 (1989), 93–104.

Expounds principally Lee, Lucchese, and Chu (1987a) and Lee and Gutman (1989a). Examples include all connected, simple graphs of order ≤ 4 and some aromatics. (**VS, SGw, Exp, Chem**)

1992a Topological analysis of five-vertex clusters of group IVa elements. *Theoretica Chimica Acta* 81 (1992), 185–199.

See Lee, Lucchese, and Chu (1987a). More examples; again, eigenvalue and net-sign orderings are compared. (**VS, SGw, Chem**)

Shyi-Long Lee and Ivan Gutman

1989a Topological analysis of the eigenvectors of the adjacency matrices in graph theory: Degenerate case. *Chemical Physics Letters* 157 (1989), 229–232.

Supplements Lee, Lucchese, and Chu (1987a) to answer an objection by Gutman (1988a), by treating vertex signs corresponding to multidimensional eigenspaces. (**VS, SGw, Chem**)

Shyi-Long Lee and Chiuping Li

1994a Chemical signed graph theory. *Internat. J. Quantum Chem.* 49 (1994), 639–648.

Varies Lee, Lucchese, and Chu (1987a) by taking net signs of all balanced signings, instead of only those obtained from eigenvectors, for small paths, polygons, and polygons with short tails. The distribution of net sign, over all signings of each graph, is more or less binomial. (**VS, SGw, Chem**)

1994b On generating molecular orbital graphs: the first step in signed graph theory. *Bull. Inst. Chem., Academica Sinica* No. 41 (1994), 69–75.

Abbreviated presentation of (1994a). (**VS, SGw: Exp**)

Shyi-Long Lee and Feng-Yin Li

1990a Net sign approach in graph spectral theory. *J. Molecular Structure (Theochem)* 207 (1990), 301–317.

Similar topics to S.-L. Lee (1989a, 1989b). Several examples of order 6. (**VS, SGw, Exp, Chem**)

1990b Net sign analysis of five-vertex chemical graphs. *Bull. Inst. Chem., Academica Sinica* No. 37 (1990), 83–97.

See Lee, Lucchese, and Chu (1987a). Treats all connected, simple graphs of order 5. (**VS, SGw, Chem**)

Shyi-Long Lee, Feng-Yin Li, and Friday Lin

1991a Topological analysis of eigenvalues of particle [*sic*] in one- and two-dimensional simple quantal systems: Net sign approach. *Internat. J. Quantum Chem.* 39 (1991), 59–70.

See Lee, Lucchese, and Chu (1987a). §II: Net signs calculated for paths. §§III, IV: Planar graphs with two different types of potential, yielding complicated results. (**VS, SG, Chem**)

Shyi-Long Lee, Robert R. Lucchese, and San Yan Chu

1987a Topological analysis of eigenvectors of the adjacency matrices in graph theory: The concept of internal connectivity. *Chemical Physics Letters* 137 (1987), 279–284. MR 88i:05130. Zbl. none.

Introduces the net sign of a (balanced) signed graph. A graph has vertices signed according to the signs of an eigenvector X_i of the adjacency matrix, $\mu(v_r) = \mathrm{sgn}\,(X_{ir})$, and $\sigma(v_r v_s) = \mu(v_r)\mu(v_s)$ [hence Σ is balanced]. Note that a vertex can have 'sign' 0. Net sign of a [hydrocarbon] chemical graph is applied to prediction of properties of molecular orbitals. (**VS, SGw, Chem**)

Shyi-Long Lee, Yeung-Long Luo, and Yeong-Nan Yeh

1991a Topological analysis of some special graphs. III. Regular polyhedra. *J. Cluster Sci.* 2 (1991), 105–116.

See Lee, Lucchese, and Chu (1987a). Net signs for the Platonic polyhedra (Table I). (**VS, SGw, Chem**)

Shyi-Long Lee and Yeong-Nan Yeh

1990a Topological analysis of some special classes of graphs. Hypercubes. *Chemical Physics Letters* 171 (1990), 385–388.

Follows up Lee, Lucchese, and Chu (1987a) and Lee and Gutman (1989a), calculating net signs of eigenspatially signed hypercube graphs of dimensions up to 6 by means of a general graph-product formula. (**VS, SGw, Chem**)

1993a Topological analysis of some special classes of graphs. II. Steps, ladders, cylinders. *J. Math. Chem.* 14 (1993), 231–241. MR 95f:05079.

See Lee, Lucchese, and Chu (1987a). Net signs and eigenvalues are compared. (**VS, SGw, Chem**)

Samuel Leinhardt

See also J.A. Davis and P.W. Holland.

Samuel Leinhardt, ed.

1977a *Social Networks: A Developing Paradigm.* Academic Press, New York, 1977.

An anthology reprinting some basic papers in structural balance theory. (**PsS, SG: B, Cl**)

P.W.H. Lemmens and J.J. Seidel

1973a Equiangular lines. *J. Algebra* 24 (1973), 494–512. MR 46 #7084. Zbl. 255.50005. (**TG, G**)

Marianne Lepp [Marianne L. Gardner]

See R. Shull.

David W. Lewit

See E.G. Shrader.

Chiuping Li

See I. Gutman and S.-L. Lee.

Feng-Hin Li

See S.-L. Lee.

Hans Liebeck

See D. Harries.

Martin W. Liebeck

1980a Lie algebras, 2-graphs and permutation groups. *Bull. London Math. Soc.* 33 (1982), 76–85. MR 81f:05095. Zbl. 499.05031.

Examines the $F\,\mathrm{Aut}\,([\Sigma])$-module $F\,V(\Sigma)$, where Σ is a signed complete graph and F is a field of characteristic 2. (**TG: Aut**)

1982a Groups fixing graphs in switching classes. *J. Austral. Math. Soc. (A)* 33 (1982), 76–85. MR 83h:05048. Zbl. 499.05031.

Given an abstract group $\mathfrak{A}$, which of its permutation representations are exposable on every invariant switching class of signed complete graphs [see Harries and H. Liebeck (1978a) for definitions]? (**k: sw, TG: Aut**)

Thomas M. Liebling

See H. Groflin.

Magnhild Lien and William Watkins

20xxa Dual graphs and knot invariants. Submitted

The Kirchhoff ("Laplacian") matrices of a signed plane graph and its dual have the same invariant factors. The proof is via the signed graphs of knot diagrams. (**SGc: D, I, Knot**)

Ko-Wei Lih

See J.-H. Yan.

Friday Lin

See S.-L. Lee.

Bernt Lindström

See F. Harary.

Nathan Linial

See F. Jaeger.

Sóstenes Lins

1981a A minimax theorem on circuits in projective graphs. *J. Combin. Theory Ser. B* 30 (1981), 253–262. MR 82j:05074. Zbl. 457.05057.

For Eulerian Σ in projective plane, max. number of edge-disjoint negative polygons = min. number of edges cut by a noncontractible closed curve that avoids the vertices. [Generalized by Schrijver (1989a).] (**SG: T, fr, Alg**)

1982a Graph-encoded maps. *J. Combin. Theory Ser. B* 32 (1982), 171–181. MR 83e:05049. Zbl. 465.05031, (478.05040).

See §4. (**sg: T: b**)

1985a Combinatorics of orientation reversing polygons. *Aequationes Math.* 29 (1985), 123–131. MR 87c:05051. Zbl. 592.05019. (**sg, p: T, Fr**)

J.H. van Lint and J.J. Seidel

1966a Equilateral point sets in elliptic geometry. *Proc. Koninkl. Ned. Akad. Wetenschap. Ser. A* 69 (= *Indag. Math.* 28) (1966), 335–348. MR 34 #685. Zbl. 138, 417 (e: 138.41702). Reprinted in Seidel (1991a), pp. 3–16. (**TG, G**)

Marc J. Lipman and Richard D. Ringeisen

1978a Switching connectivity in graphs. In: F. Hoffman *et al.*, eds., *Proceedings of the Ninth Southeastern Conference. on Combinatorics, Graph Theory and Computing* (Boca Raton, 1978), pp. 471–478. Congressus Numerantium, XXI. Utilitas Math. Publ. Inc., Winnipeg, Man., 1978. MR 80k:05073. Zbl. 446.05033. (**TG**)

Simon Litsyn
See I. Krasikov.

Charles H.C. Little
See C.P. Bonnington.

M. Loebl
See Y. Crama.

D.O. Logofet and N.B. Ul'yanov
1982a Necessary and sufficient conditions for the sign stability of matrices. (In Russian.) *Dokl. Akad. Nauk SSSR* 264 (1982), 542–546. MR 84j:15018. Zbl. 509.15008.
Necessity of Jeffries' (1974a) sufficient conditions. (**Sta**)

D.O. Logofet and N.B. Ul'janov [N.B. Ul'yanov]
1982b Necessary and sufficient conditions for the sign stability of matrices. *Soviet Math. Dokl.* 25 (1982), 676–680. MR 84j:15018. Zbl. 509.15008.
English translation of (1982a). (**Sta**)

M. Loréa
1979a On matroidal families. *Discrete Math.* 28 (1979), 103–106. MR 81a:05029. Zbl. 409.05050.
Discovers the "count" matroids of graphs (see Whiteley (1996a)).
(**Bic: Gen**)

Janice R. Lourie
1964a Topology and computation of the generalized transportation problem. *Management Sci.* 11 (1965), No. 1 (Sept., 1964), 177–187. (**GN: M(bases)**)

L. Lovász
See also J.A. Bondy and Gerards *et al.* (1990a).

1965a On graphs not containing independent circuits. (In Hungarian.) *Mat. Lapok* 16 (1965), 289–299. MR 35 #2777. Zbl. 151, 334 (e 151.33403).
Characterization of the graphs having no two vertex-disjoint polygons. See Bollobás (1978a) for exposition in English. [**Major** *Problem.* Characterize the biased graphs having no two vertex-disjoint unbalanced polygons. This theorem is the contrabalanced case. The sign-biased case was also solved by Lovász; see Seymour (1995a). McQuaig (1993a) might be relevant.]
(**GG: Polygons**)

1983a Ear-decompositions of matching-covered graphs. *Combinatorica* 3 (1983), 105–117. MR 85b:05143. Zbl. 516.05047.
It is hard to escape the feeling that we are dealing with all-negative signed graphs and their $-K_4$ and $-K_2^\circ$ minors. [And indeed, see Gerards and Schrijver (1986a) and Gerards *et al.* (1990a) and the notes on Seymour (1995a).] (**P: Str**)

L. Lovász and M.D. Plummer
1986a *Matching Theory.* North-Holland Math. Stud., Vol. 121. Ann. Discrete Math., Vol. 29. Akadémiai Kiadó, Budapest, and North-Holland, Amsterdam, 1986. MR 88b:90087. Zbl. 618.05001.

L. Lovász, L. Pyber, D.J.A. Welsh, and G.M. Ziegler
1995a Combinatorics in pure mathematics. In: R.L. Graham, M. Grötschel, and L. Lovász, eds., *Handbook of Combinatorics*, Vol. II, Ch. 41, pp. 2039–2082. North-Holland (Elsevier), Amsterdam, and MIT Press, Cambridge, Mass., 1995. MR 97f:00003. Zbl. 851.52017.

§7: "Knots and the Tutte polynomial", considers the signed graph of a knot diagram (pp. 2076–77). (**SGc: Knot**)

Robert R. Lucchese
See S.-L. Lee.

Tomasz Łuczak
See E. Györi.

J. Richard Lundgren
See H.J. Greenberg and F. Harary.

Yeung-Long Luo
See I. Gutman and S.-L. Lee.

Enzo Maccioni
See F. Barahona.

Thomas L. Magnanti
See R.K. Ahuja.

N.V.R. Mahadev
See also P.L. Hammer.

N.V.R. Mahadev and U.N.Peled

1995a *Threshold Graphs and Related Topics.* Ann. Discrete Math., Vol. 56. North-Holland, Amsterdam, 1995. MR 97h:05001. Zbl. 950.36502.

§8.3: "Bithreshold graphs" (from Hammer and Mahadev (1985a)), and §8.4: "Strict 2-threshold graphs" (from Hammer, Mahadev, and Peled (1989a)), characterize two types of threshold-like graph. In each, a different signed graph H is defined on $E(\Gamma)$ so that Γ is of the specified type iff H is balanced. (The negative part of H is the "conflict graph", Γ^*.) The reason is that one wants Γ to decompose into two subgraphs, and the subgraphs, if they exist, must be the two parts of the Harary bipartition of H. [Thus one also gets a fast recognition algorithm (though not the fastest possible) for the desired type from the fast recognition of balance.] (**SG: B: Appl**)

§8.5: "Recognizing threshold dimension 2." Based on Raschle and Simon (1995a). Given: $\Gamma \subseteq K_n$ such that Γ^* is bipartite. Orient $-K_n$ so that Γ-edges are introverted and the other edges are extroverted. Their "alternating cycle" is a coherent closed walk in this orientation. Let us call it "black" (in a given black-white proper coloring of Γ^*) if its Γ-edges are all black. Thm. 8.5.2 (Hammer, Ibaraki, and Peled (1981a)): If there is a black coherent closed walk in E_0, then there is a coherent tour (closed trail) of length 6 (which is a pair of joined triangles or a hexagon—their AP_5 and AP_6). Thm. 8.5.4: Given that there is no black coherent hexagon, one can recolor quickly so there is no black coherent 6-tour. Thm. 8.5.9: Given that there is no 'double' coherent hexagon (the book's "double AP_6"), one can recolor quickly so there is no black coherent hexagon. Thm. 8.5.28: Any 2-coloring of Γ^* can be quickly transformed into one with no 'double' coherent hexagon. [*Question.* Can any of this, especially Thm. 8.5.2, be generalized to arbitrary oriented all-negative graphs B? Presumably, this would require first defining a conflict graph on the introverted edges of B. More remotely, consider generalizing to bidirected complete or arbitrary graphs.] (**p: o, Alg**)

§9.2.1: "Threshold signed graphs." In this version it's not clear where the signs are! (and their role is trivial). Real weights are assigned to the vertices and an edge receives the sign of the weight product of its endpoints.

(sg: b)

Ali Ridha Mahjoub
See F. Barahona.

J.M. Maillard
See J. Vannimenus.

M. Malek-Zavarei and J.K. Aggarwal
1971a Optimal flow in networks with gains and costs. *Networks* 1 (1971), 355–365. MR 45 #4896. Zbl. 236.90026. **(GN: b)**

R.B. Mallion
See A.C. Day.

C.L. Mallows and N.J.A. Sloane
1975a Two-graphs, switching classes and Euler graphs are equal in number. *SIAM J. Appl. Math.* 28 (1975), 876–880. MR 55 #164. Zbl. 275.05125, (297.05129).

Thm. 1: For all n, the number of unlabelled two-graphs of order n [i.e., switching isomorphism classes of signed K_n's] equals the number of unlabelled even-degree simple graphs on n vertices. The key to the proof is that a permutation fixing a switching class fixes a signing in the class. (Seidel (1974a) proved the odd case, where the fixing property is simple.) Thm. 2: The same for the labelled case. [More in Cameron (1977b), Cameron and Wells (1986a), Cheng and Wells (1984a, 1986a).]

To prove the fixing property they find the conditions under which a given permutation π of $V(K_n)$ and switching set C fix some signed K_n. [More in Harries and Liebeck (1978a), M. Liebeck (1982a), and Cameron (1977b).] **(TG: Aut, E)**

Rachel Manber
See also R. Aharoni and V. Klee.

1982a Graph-theoretical approach to qualitative solvability of linear systems. *Linear Algebra Appl.* 48 (1982), 457–470. MR 84g:68054. Zbl. 511.15008. **(SD, QM: Sol)**

Rachel Manber and Jia-Yu Shao
1986a On digraphs with the odd cycle property. *J. Graph Theory* 10 (1986), 155–165. MR 88i:05090. Zbl. 593.05032. **(SD, SG: P)**

Dănuţ Marcu
I cannot vouch for the authenticity of these articles. See MR 97a:05095 and Zbl. 701.51004. Also see MR 92a:51002, 92b:51026, 92h:11026, 97k:05050; and Marcu (1981b).

1980a On the gradable digraphs. *An. Ştiinţ. Univ. "Al. I. Cuza" Iaşi Secţ. I a Mat. (N.S.)* 26 (1980), 185–187. MR 82k:05056 (q.v.). Zbl. 438.05032.

See Harary, Norman, and Cartwright (1965a) for the definition. **(GD: b)**

1981a No tournament is gradable. *An. Univ. Bucureşti Mat.* 30 (1981), 27–28. MR 83c:05069. Zbl. 468.05028.

See Harary, Norman, and Cartwright (1965a) for the definition. The tournaments of order 3 are not gradable, whence the titular theorem. **(GD: b)**

1981b Some results concerning the even cycles of a connected digraph. *Studia Univ. Babeş-Bolyai Math.* 26 (1981), 24–28. MR 83e:05058. Zbl. 479.05032.

§1, "Preliminary considerations", appears to be an edited, unacknowledged transcription of portions of Harary, Norman, and Cartwright (1965a) (or

possibly (1968a)), pp. 341–345. Wording and notation have been modified, a trivial corollary has been added, and some errors have been introduced; but the mathematics is otherwise the same down to details of proofs. §2, "Results", is largely a list of the corollaries resulting from setting all signs negative. The exception is Thm. 2.5, for which I am not aware of a source; however, it is simple and well known. (**sg(SD): B**)

1987a Note on the matroidal families. *Riv. Math. Univ. Parma* (4) 13 (1987), 407–412. MR 89k:05025.

Matroidal families of (multi)graphs (see Simões-Pereira (1973a)) correspond to functions on all isomorphism types of graphs that are similar to matroid rank functions, e.g., submodular. This provides insight into matroidal families, e.g., it immediately shows there are infinitely many. (**Bic, EC: Gen**)

Harry Markowitz

1955a Concepts and computing procedures for certain X_{ij} programming problems. In: H.A. Antosiewicz, ed., *Proceedings of the Second Symposium in Linear Programming* (Washington, D.C., 1955), Vol. II, pp. 509–565. Nat. Bur. Standards of U.S. Dept. of Commerce, and Directorate of Management Analysis, DCS Comptroller, HQ, U.S. Air Force, 1955. Sponsored by Office of Scientific Res., Air Res. and Develop. Command. MR 17, 789.

Also see RAND Corporation Paper P-602, 1954. (**GN: m(bases)**)

Clifford W. Marshall

1971a *Applied Graph Theory*. Wiley-Interscience, New York, 1971. MR 48 #1951. Zbl. 226.05101.

"Consistency of choice" discusses signed graphs, pp. 262–266. (**SG: B, A: Exp**)

J.H. Mason

1977a Matroids as the study of geometrical configurations. In: *Higher Combinatorics* (Proc. NATO Adv. Study Inst., Berlin, 1976), pp. 133–176. NATO Adv. Study Inst. Ser., Ser. C: Math. Phys. Sci., Vol. 31. Reidel, Dordrecht, 1977. MR 80k:05037. Zbl. 358.05017.

§§2.5-2.6: "The lattice approach" and "Generalized coordinates", pp. 172–174, propose a purely matroidal and more general formulation of Dowling's construction of his lattices. (**gg(Gen): M**)

1981a Glueing matroids together: A study of Dilworth truncations and matroid analogues of exterior and symmetric powers. In: *Algebraic Methods in Graph Theory* (Proc., Szeged, 1978), Vol. II, pp. 519–561. Colloq. Math. Soc. János Bolyai, 25. North-Holland, Amsterdam, 1981. MR 84i:05041. Zbl. 477.05022.

Dowling matroids are an example in §1. (**gg: M**)

R.A. Mathon

See F.C. Bussemaker and Seidel (1991a).

Hisayoshi Matsuyama

See M. Iri.

Laurence R. Matthews

1977a Bicircular matroids. *Quart. J. Math. Oxford* (2) 28 (1977), 213–227. MR 58 #21732. Zbl. 386.05022.

Thorough study of bicircular matroids, introduced by Klee (1971a) and Simões-Pereira (1972a). (**Bic**)

1978a Properties of bicircular matroids. In: *Problèmes Combinatoires et Théorie des Graphes* (Colloq. Internat., Orsay, 1976), pp. 289–290. Colloques Internat. du CNRS, 260. Editions du C.N.R.S., Paris, 1978. MR 81a:05030. Zbl. 427.05021. (**Bic**)

1978b Matroids on the edge sets of directed graphs. In: *Optimization and Operations Research* (Proc. Workshop, Bonn, 1977), pp. 193–199. Lecture Notes in Economics and Math. Systems, 157. Springer, Berlin, 1978. MR 80a:05103. Zbl. 401.05031. (**gg: M**)

1978c Matroids from directed graphs. *Discrete Math.* 24 (1978), 47–61. MR 81e:05055. Zbl. 388.05005.

Invents poise, modular poise, and antidirection matroids of a digraph. (**gg: M**)

1979a Infinite subgraphs as matroid circuits. *J. Combin. Theory Ser. B* 27 (1979), 260–273. MR 81e:05056. Zbl. 433.05018. (**Bic: Gen**)

Laurence R. Matthews and James G. Oxley

1977a Infinite graphs and bicircular matroids. *Discrete Math.* 19 (1977), 61–65. MR 58 #16348. Zbl. 386.05021. (**Bic**)

Jean François Maurras

1972a Optimization of the flow through networks with gains. *Math. Programming* 3 (1972), 135–144. MR 47 #2993. Zbl. 243.90048. (**GN: M**)

John S. Maybee

See also L. Bassett, J. Genin, H.J. Greenberg, F. Harary, C.R. Johnson, and G.M. Lady.

1974a Combinatorially symmetric matrices. *Linear Algebra Appl.* 8 (1974), 529–537. MR 56 #11845. Zbl. (438.15021).

Survey and simple proofs. (**QM: sd, gg, Sta**)(**Exp**)

1980a Sign solvable graphs. *Discrete Appl. Math.* 2 (1980), 57–63. MR 81g:05063. Zbl. 439.05024. (**SD: QM: Sol**)

1981a Sign solvability. In: Harvey J. Greenberg and John S. Maybee eds., *Computer-Assisted Analysis and Model Simplification* (Proc. Sympos., Boulder, Col., 1980), pp. 201–257. Discussion, p. 321. Academic Press, New York, 1981. MR 82g:00016 (book). Zbl. 495.93001 (book).

For comments, see Lancaster (1981a). (**QM: Sol: SD**)

1989a Qualitatively stable matrices and convergent matrices. In: Fred Roberts, ed., *Applications of Combinatorics and Graph Theory to the Biological and Social Sciences*, pp. 245–258. IMA Vols. Math. Appl., Vol. 17. Springer-Verlag, New York, 1989. MR 90h:34082. Zbl.708.15007.

Signed (di)graphs play a role in characterizations. See e.g. §7. See also Roberts (1989a), §4. (**QM, SD**)

John S. Maybee and Stuart J. Maybee

1983a An algorithm for identifying Morishima and anti-Morishima matrices and balanced digraphs. *Math. Social Sci.* 6 (1983), 99–103. MR 85f:05084. Zbl. 567.05038.

A linear-time algorithm to determine balance or antibalance of the undirected signed graph of a signed digraph. The algorithm of Harary and Kabell (1980a) appears to be different. (**SG: B, P: Alg**)

John Maybee and James Quirk

1969a Qualitative problems in matrix theory. *SIAM Rev.* 11 (1969), 30–51. MR 40 #1127. Zbl. 186, 335 (e: 186.33503).

An important early survey with new results.

(**QM, SD: Sol, Sta, b; Exp(in part), Ref**)

John S. Maybee and Daniel J. Richman

1988a Some properties of GM-matrices and their inverses. *Linear Algebra Appl.* 107 (1988), 219–236. MR 89k:15039. Zbl. 659.15021.

Square matrix A is a GM-matrix if, for every positive and negative cycle P and N in its signed digraph, $V(P) \supseteq V(N)$. Classification of irreducible GM-matrices; connections with the property that each $p \times p$ principal minor has sign $(-1)^p$; some conclusions about the inverse. (**SD: QM**)

John S. Maybee and Gerry M. Weiner

1987a L-functions and their inverses. *SIAM J. Algebraic Discrete Methods* 8 (1987), 67–76. MR 88a:26021. Zbl. 613.15005.

An L-function is a nonlinear generalization of a qualitative linear function. Signed digraphs play a small role. (**QM, SD**)

Stuart J. Maybee

See J.S. Maybee.

W. Mayeda and M.E. Van Valkenburg

1965a Properties of lossy communication nets. *IEEE Trans. Circuit Theory* CT-12 (1965), 334–338. (**GN**)

R. Maynard

See F. Barahona and I. Bieche.

Richard D. McBride

See G.G. Brown.

H. Gilman McCann

See E.C. Johnsen.

William McCuaig

1993a Intercyclic digraphs. In: Neil Robertson and Paul Seymour, eds., *Graph Structure Theory* (Proc., Seattle, 1991), pp. 203–245. Contemp. Math., Vol. 147. Amer. Math. Soc., Providence, R.I., 1993. MR 94f:05062. Zbl. 789.05042.

Characterizes the digraphs with no two disjoint cycles as well as those with no two arc-disjoint cycles. [Since cycles do not form a linear subclass of polygons, this is not a biased-graphic theorem, but it might be of use in studying biased graphs that have no two disjoint balanced polygons. See Lovász (1965a).] (**Str**)

†20xxa Pólya's permanent problem. Submitted.

Question 1. Does a given digraph D have an even cycle? Question 2. Can a given digraph D be signed so that every cycle is negative? (These problems are easily seen to be equivalent.) The main theorem (the "Even Dicycle Thm.") is a structural characterization of digraphs that have a signing in which every cycle is negative. (These were previously characterized by forbidden minors in Seymour and Thomassen (1987a).)

The main theorem is proved also in Robertson, Seymour, and Thomas (20xx-a). (**SD: p: Str**)(**SG**)

20xxb When all dicycles have the same length. Submitted.

Uses the main theorem of (20xxa) and Robertson, Seymour, and Thomas (20xxa) to prove: a digraph has an edge weighting in which all cycles have equal nonzero total weight iff it does not contain a "double dicycle": a symmetric digraph whose underlying simple graph is a circle. There is also a structural description of such digraphs. (**SD: p: Str**)(**Sw**)

William McCuaig, Neil Robertson, P.D. Seymour, and Robin Thomas

20xxa Permanents, Pfaffian orientations, and even directed circuits. Extended abstract. In: *Proceedings of the 1997 Symposium on the Theory of Computing*

Extended abstract of McCuaig (20xxa) and Robertson, Seymour, and Thomas (20xxa). (**SD: p**)

W.D. McCuaig and M. Rosenfeld

1985a Parity of cycles containing specified edges. In: B.R. Alspach and C.D. Godsil, eds., *Cycles in Graphs*, pp. 419–431. Ann. Discrete Math., Vol. 27. North-Holland Math. Stud., Vol. 115. North-Holland, Amserdam, 1985. MR 87g:05139. Zbl. 583.05037.

In a 3-connected graph, almost any two edges are in an even and an odd polygon. [By the negative-subdivision trick this generalizes to signed graphs.] (**P, sg: B**)

T.A. McKee

1984a Balance and duality in signed graphs. Proc. Fifteenth Southeastern Conf. on Combinatorics, Graph Theory and Computing (Baton Rouge, 1984). *Congressus Numer.* 44 (1984), 11–18. MR 87b:05124. Zbl. 557.05046. (**SG: B: D**)

1987a A local analogy between directed and signed graphs. *Utilitas Math.* 32 (1987), 175–180. MR 89a:05075. Zbl. 642.05023. (**SG: D, Cl, B**)

Kathleen A. McKeon

See G. Chartrand.

Nimrod Megiddo

See E. Cohen and D. Hochbaum.

Roy Meshulam

See R. Aharoni and J. Kahn.

Robert Messer

See E.M. Brown.

Marc Mézard, Giorgio Parisi, and Miguel Angel Virasoro

1987a *Spin Glass Theory and Beyond.* World Scientific Lecture Notes in Physics, Vol. 9. World Scientific, Singapore, 1987. MR 91k:82066.

Focuses on the Sherrington-Kirkpatrick model, i.e., underlying complete graph, emphasising the Parisi-type model (see articles reprinted herein), which posits numerous metastable states, separated by energy barriers of greatly varying heights and subdividing as temperature decreases. Essentially heuristic (as noted in MR): that is, the ideas awaited [and still largely await] mathematical justification.

Many original articles on Ising and vector models (both of which are based on weighted signed graphs) are reprinted herein, though few are of general signed-graphic interest.

[See also, i.a., Toulouse (1977a, etc.), Chowdhury (1986a), Stern (1989a), Fischer and Hertz (1991a), Vincent, Hammann, and Ocio (1992a) for physics, Barahona (1982a, etc.), Grötschel, Jünger, and Reinelt (1987a) for mathematics.] (**Phys, SG: Fr: Exp, Ref**)

Ch. 0, "Introduction", briefly compares, in the obvious way, balance in social psychology with frustration in spin glasses. (**Phys, PsS: SG: B: Exp**)

Pt. I, "Spin glasses", Ch. 2, "The TAP approach": pp. 19–20 describe 1-vertex switching of a weighted signed graph to reduce frustration, not however necessarily producing the frustration index (minimum frustration). (**Phys: SG: Fr, Sw, Alg: Exp**)

Raymond E. Miller

See R.M. Karp.

William P. Miller

See J.E. Bonin.

Edward Minieka

1972a Optimal flow in a network with gains. *INFOR* 10 (1972), 171–178. Zbl. 234.90012. (**GN: M(indep), B**)

1978a *Optimization Algorithms for Networks and Graphs.* Marcel Dekker, New York and Basel, 1978. MR 80a:90066. Zbl. 427.90058.

§4.6: "Flows with gains," pp. 151–174. Also see pp. 80–81. (**GN: B, Sw, m(indep): Exp**)

1981a *Algoritmy Optimizatsii na Setyakh i Grafakh.* Transl. M.B. Katsnel'son and M.I. Rubinshteĭn; ed. E.K. Maslovskiĭ. Mir, Moskva, 1981. MR 83f:90118. Zbl. 523.90058.

Russian translation of (1978a). (**GN: B, Sw, m(indep): Exp**)

V. Mishra

1974a Graphs Associated With $(0,+1,-1)$ Arrays. Doctoral thesis, Indian Inst. of Technology, Bombay, 1974.

S. Mitra

1962a Letter to the editors. *Behavioral Sci.* 7 (1962), 107.

Treats only signed K_n. Viewed with hindsight, observes that balance holds iff $I + A(\Sigma) = vv^{\mathrm{T}}$ for some vector v of ± 1's; also, defines switching and observes [I call this the Switching Thm. of Frustration] that frustration index $l(\Sigma)$ = minimum number of negative edges over all switchings. States a simple "algorithm" for computing $l(\Sigma)$ [but without a stopping rule, and the obvious ones are invalid]. (**sg: k: A, sw, Fr**)

Bojan Mohar

1989a An obstruction to embedding graphs in surfaces. *Discrete Math.* 78 (1989), 135–142. MR 90h:05046. Zbl. 686.05019.

The "overlap matrix" of a signed graph with respect to a rotation system and a spanning tree provides a lower bound on the demigenus that sometimes improves on that from Euler's formula. (**SG: T**)

Elliott W. Montroll

1964a Lattice statistics. In: Edwin F. Beckenbach, ed., *Applied Combinatorial Mathematics*, Ch. 4, pp. 96–143. Wiley, New York, 1964. MR 30 #4687 (book). Zbl. 141, 155 (e: 141.15503).

§4.4: "The Pfaffian and the dimer problem". Exemplified by the square lattice, expounds Kasteleyn's method of signing edges to make the Pfaffian term signs all positive. Partial proofs. §4.7, "The Ising problem", pp. 127–129, explains application to the Ising model. Exceptionally readable. [Further development in, e.g., Vazirani and Yannakakis (1988a, 1989a).] (**SG, Phys: Exp**)

J.W. Moon and L. Moser

1966a An extremal problem in matrix theory. *Mat. Vesnik* 3 (18) (1966), 209–211. MR 34 #7385. Zbl. (e: 146.01401). **(sg: Fr)**

Suck Jung Moon

See H. Kosako.

M.A. Moore

See A.J. Bray.

Michio Morishima

1952a On the laws of change of the price-system in an economy which contains complementary commodities. *Osaka Economic Papers* 1 (1952), 101–113.

§4: "Alternative expression of the assumptions (1)," can be interpreted with hindsight as proving that, for a signed K_n, every triangle is positive iff the signature switches to all positive. (Everything is done with sign-symmetric matrices, not graphs, and switching is not mentioned in any form.) **(sg: b, sw)**

Julian O. Morrissette

1958a An experimental study of the theory of structural balance. *Human Relations* 11 (1958), 239–254.

Proposes that edges have strengths between -1 and $+1$ instead of pure signs. The Cartwright-Harary degree of balance (1956a), computed from polygons, is modified to take account of strength. In addition, signed graphs are allowed to have edges of two types, say U and A, and only short mixed-type polygons enter into the degree of balance. This is said to be more consistent with the experimental data reported herein. **(PsS, SG, Gen: Fr)**

Julian O. Morrissette and John C. Jahnke

1967a No relations and relations of strength zero in the theory of structural balance. *Human Relations* 20 (1967), 189–195.

Reports an experiment; then discusses problems with and alternatives to the Cartwright-Harary (1956a) polygon degree of balance. **(PsS: Fr)**

L. Moser

See J.W. Moon.

Sebastiano Mosterts

See E.L. Johnson.

Andrej Mrvar

See P. Doreian.

Luigi Muracchini and Anna Maria Ghirlanda

1965a Sui grafi segnati ed i grafi commutati. *Statistica* (Bologna) 25 (1965), 677–680. MR 33 #7272.

A partially successful attempt to use unoriented signed graphs to define a line graph of a digraph. [See Zaslavsky (20xxb) for the correct signed-graph approach.] The Harary-Norman line digraph is also discussed. **(SG: B, LG)**

Kunio Murasugi

1988a On the signature of a graph. *C.R. Math. Rep. Acad. Sci. Canada* 10 (1988), 107–111. MR 89h:05056.

The signature of a sign-colored graph (see 1989a) is an invariant of the sign-colored graphic matroid. **(SGc: I, m)**

1989a On invariants of graphs with applications to knot theory. *Trans. Amer. Math. Soc.* 314 (1989), 1–49. MR 89k:57016. Zbl. 726.05051.

Studies a dichromatic form, $P_\Sigma(x,y,z)$, of Kauffman's (1989a) Tutte polynomial of a sign-colored graph. The deletion-contraction parameters are $a_\epsilon = 1$, $b_\epsilon = x^\epsilon$ for $\epsilon = \pm 1$; the initial values are such that $P_\Sigma(x,y,z) = y^{-1}Q_\Sigma(a,b;y,z)$ of Zaslavsky (1992b). The polynomial is shown to be, in effect, an invariant of the sign-colored graphic matroid.

Much unusual graph theory is in here. A special focus is the degrees of the polynomial. First Main Thm. 3.1: Formulas for the maximum and minimum combined degrees of $P_\Sigma(x,y,z)$. §7, "Signature of a graph", studies the signature (σ in the paper, s here) of the Kirchhoff matrix B_Σ obtained by changing the diagonal of $A(\Sigma)$ so the row sums are 0. Prop. 7.2 is a matrix-tree theorem [entirely different from that of Zaslavsky (1982a)]. The Second Main Thm. 8.1 bounds the signature: $|V| - 2\beta_0(\Sigma_-) + 1 \leq s \leq |V| - 2\beta_0(\Sigma_+) + 1$ ($\beta_0 =$ number of components), with equality characterized. The Kirchhoff matrix is further examined later on. §9, "Dual graphs": Differing from most studies, here the dual of a sign-colored plane graph is the planar dual with same edge signs [however, negating all colors is a triviality]. §10, "Periodic graphs": These graphs might be called branched covering graphs of signed gain graphs with finite cyclic gain group. [Thus they generalize the periodic graphs of Collatz (1978a) and others.] §§12–15 concern applications to knot theory. (**SGc: N, I, GG(Cov), D, Knot**)

1991a Invariants of graphs and their applications to knot theory. In: S. Jackowski, B. Oliver, and K. Pawałowski, eds., *Algebraic topology Poznań 1989* (Proc., Poznań, 1989), pp. 83–97. Lecture Notes in Math., Vol. 1474. Springer-Verlag, Berlin, 1991. MR 92m:57015. Zbl. 751.57007.

§§1–3 expound results from (1989a) on the dichromatic polynomial and the signature of a sign-colored graph and knot applications. §5 discusses the signed Seifert graph of a link diagram. (**SGc: N, I, Knot: Exp**)

1993a *Musubime riron to sono ōnō.* [Knot Theory and Its Applications.] (In Japanese.) 1993.

See (1996a). (**SGc: Knot**)

1996a *Knot Theory and Its Applications.* Birkhäuser, Boston, 1996. MR 97g:57011. Zbl. 864.57001.

Updated translation of (1993a) by Bohdan Kurpita. Pp. 36–37: Construction of signed plane graph from link diagram, and conversely. (**SGc: Knot**)

Kunio Murasugi and Jozef H. Przytycki

1993a *An Index of a Graph with Applications to Knot Theory.* Mem. Amer. Math. Soc., Vol. 106, No. 158. Amer. Math. Soc., Providence, R.I., 1993. MR 94d:57025. Zbl. 792.05047.

Ch. I, "Index of a graph". The "index" is the largest number of "independent" edges, where "independent" has a complicated recursive definition (unrelated to matchings), one of whose requirements is that the edges be "singular" (simple, i.e., nonmultiple links). The positive or negative index of a sign-colored graph is similar except that the independent edges must all be positive or negative. [The general notion is that of the index of a graph-subgraph pair. The signs pick out complementary subgraphs.] Thm. 2.4: Each of these indices is additive on blocks of a bipartite graph. The main interest, because of applications to knot theory, is in bipartite plane graphs. Ch. II,

"Link theory": Pp. 26–27 define the sign-colored Seifert graph of an oriented link diagram and apply the graphical index theory. (**SGc: N, D, Knot**)

Tadao Murata

1965a Analysis of lossy communication nets by modified incidence matrices. In: M.E. Van Valkenburg, ed., *Proceedings, Third Annual Allerton Conference on Circuit and System Theory* (Monticello, Ill., 1965), pp. 751–761. Dept. of Electrical Eng. and Coordinated Sci. Lab., Univ. of Illinois, Urbana, Ill.; and Circuit Theory Group, Inst. of Electrical and Electronics Engineers, [1965]. (**GN: I**)

Takeshi Naitoh

See K. Ando.

Kazuo Nakajima

See H. Choi.

Daishin Nakamura and Akihisa Tamura

1998a The generalized stable set problem for claw-free bidirected graphs. In: Robert E. Bixby, E. Andrew Boyd, and Roger Z. Ríos-Mercado, eds., *Integer Programming and Combinatorial Optimization* (6th Internat. IPCO Conf., Houston, 1998, Proc.), pp. 69–83. Lecture Notes in Computer Sci., Vol. 1412. Springer, Berlin, 1998. Zbl. 907.90272

The problem of the title is solvable in polynomial time. See Johnson and Padberg (1982a), Tamura (1997a) for definitions. They reduce to simple graphs, transitively bidirected with no sink or introverted edge (called "canonical" bidirected graphs). (**sg: O: G, Sw, Alg**)

20xxa A linear time algorithm for the generalized stable set problem on triangulated bidirected graphs. Submitted (**sg: O: G. Alg**)

L. Nanjundaswamy

See E. Sampathkumar.

Joseph (Seffi) Naor

See D. Hochbaum.

C.St.J.A. Nash-Williams

1960a On orientations, connectivity, and odd-vertex-pairings in finite graphs. *Canad. J. Math.* 12 (1960), 555–567. MR 22 #9455. Zbl. 96, 380 (e: 096.38002).

1969a Well-balanced orientations of finite graphs and unobtrusive odd-vertex-pairings. In: W.T. Tutte, ed., *Recent Progress in Combinatorics* (Proc. Third Waterloo Conf., 1968), pp. 133–149. Academic Press, New York, 1969. MR 40 #7146. Zbl. 209, 557 (e: 209.55701).

Roman Nedela and Martin Škoviera

1996a Regular embeddings of canonical double coverings of graphs. *J. Combin. Theory Ser. B* 67 (1996), 249–277. MR 97e:05078. Zbl. 856.05029.

By "canonical double covering" of Γ they mean the signed covering graph $\tilde{\Sigma}$ of $\Sigma = -\Gamma$, but without reversing orientation at the negative covering vertex [as one would do in a signed covering graph (cf. e.g. Zaslavsky 1992a)], because orientable embeddings of Γ are being lifted to orientable embeddings of $\tilde{\Sigma}$. [Thus these can be thought of as not signed graphs but rather voltage (i.e., gain) graphs with 2-element voltage group.] Instead of reversal they twist the negative-vertex rotations by taking a suitable power. In some cases this allows classifying the orientable, regular embeddings of $\tilde{\Sigma}$. (**P: Cov, T, Aut**)

1997a Exponents of orientable maps. *Proc. London Math. Soc.* (3) 75 (1997), 1–31. MR 98i:05059. Zbl. 877.05012.

Main topic: the theory of twisting of rotations as in (1996a).

(**GG: Cov, T, Aut**)

Portions concern double covering graphs of signed graphs. §7: "Antipodal and algebraically antipodal maps". A map is "antipodal" if it is the orientable double covering of a nonorientable map; that is, as a graph it is the canonical double covering of an unbalanced signed graph. A partial algebraic criterion for a map to be antipodal. §9: "Regular embeddings of canonical double coverings of graphs". See (1996a). (**sg, P: Cov, T, Aut**)

1997b Regular maps from voltage assignments and exponent groups. *European J. Combin.* 18 (1997), 807–823. MR 98j:05061. Zbl. 908.05036.

Cases in which the classification of (1996a) is necessarily incomplete are studied by taking larger voltage (i.e., gain) groups and twisting the rotations at covering vertices by taking a power that depends on the position of the vertex in its fiber. Main result: the (very special) conditions on twisting under which a regular map lifts to a regular map. (**GG: Cov, T, Aut**)

Toshio Nemoto

See K. Ando.

H. Nencka

See Ph. Combe.

Jaroslav Nešetřil

See J. Kratochvíl.

A. Neumaier

1982a Completely regular twographs. *Arch. Math. (Basel)* 38 (1982), 378–384. MR 83g:05066. Zbl. 475.05045.

In the signed graph (K_n, σ) of a two-graph (see D.E. Taylor 1977a), a "clique" is a vertex set that induces an antibalanced subgraph. A two-graph is "completely regular" if every clique of size i lies in the same number of cliques of size $i+1$, for all i. Thm. 1.4 implies there is only a small finite number of completely regular two-graphs. (**TG**)

Sang Nguyen

See P.L. Hammer.

Juhani Nieminen

1976a Weak balance: a combination of Heider's theory and cycle and path-balance. *Control Cybernet.* 5 (1976), 69–73. MR 55 #2639. (**SD: B**)

Peter Nijkamp

See F. Brouwer.

Robert Z. Norman

See also F. Harary.

Robert Z. Norman and Fred S. Roberts

1972a A derivation of a measure of relative balance for social structures and a characterization of extensive ratio systems. *J. Math. Psychology* 9 (1972), 66–91. MR 45 # 2121. Zbl. 233.92006.

Polygon ("cycle") indices of imbalance: the proportion of polygons that are unbalanced, with polygons weighted nonincreasingly according to length.

1972b A measure of relative balance for social structures. In: Joseph Berger, Morris Zelditch, Jr., and Bo Anderson, eds., *Sociological Theories in Progress*, Ch. 14, pp. 358–391. Houghton Mifflin, Boston, 1972.

Exposition and application of (1972a).

Beth Novick and András Sebö

1995a On combinatorial properties of binary spaces. In: Egon Balas and Jens Clausen, eds., *Integer Programming and Combinatorial Optimization* (4th Internat. IPCO Conf., Copenhagen, 1995, Proc.), pp. 212–227. Lecture Notes in Computer Sci., Vol. 920. Springer-Verlag, Berlin, 1995. MR 96h:0503.

The clutter of negative circuits of a signed binary matroid (M, σ). Important are the lift and extended lift matroids, $L(M, \sigma)$ and $L_0(M, \sigma)$, defined as in signed graph theory. An elementary result: the clutter is signed-graphic iff $L_0(M, \sigma)/e_0$ is graphic (which is obvious). There are also more substantial but complicated results. [See Cornuéjols (20xxa), §8.4.] **(S(M): M)**

1996a On ideal clutters, metrics and multiflows. In: William H. Cunningham, S. Thomas McCormick, and Maurice Queyrann, eds., *Integer Programming and Combinatorial Optimization* (5th Internat. IPCO Conf., Vancouver, 1996, Proc.), pp. 275–287. Lecture Notes in Computer Sci., Vol. 1084. Springer-Verlag, Berlin, 1996. MR 98i:90075. **(S(M): M)**

Cyriel van Nuffelen

1973a On the rank of the incidence matrix of a graph. Colloque sur la Theorie des Graphes (Bruxelles, 1973). *Cahiers Centre Etudes Rech. Oper.* 15 (1973), 363–365. MR 50 #162. Zbl. 269.05116.

The unoriented incidence matrix has rank = rank$(G(-\Gamma))$. [Because the matrix represents $G(-\Gamma)$.] **(p: I, ec)**

1976a On the incidence matrix of a graph. *IEEE Trans. Circuits Systems* CAS-23 (1976), 572. MR 56 #186.

Summarizes (1973a). **(p: I, ec)**

M. Ocio

See E. Vincent.

E. Olaru

See St. Antohe.

D.D. Olesky

See C.R. Johnson.

Kenji Onaga

1966a Dynamic programming of optimum flows in lossy communication nets. *IEEE Trans. Circuit Theory* CT-13 (1966), 282–287. **(GN)**

1967a Optimal flows in general communication networks. *J. Franklin Inst.* 283 (1967), 308–327. MR 36 #1189. Zbl. (e: 203.22402). **(GN)**

Shmuel Onn

See also P. Kleinschmidt.

1997a Strongly signable and partitionable posets. *European J. Combin.* 18 (1997), 921–938. MR 99d:06007. Zbl. 887.06003.

For "signability" see Kleinschmidt and Onn (1995a). A strong signing is an exact signing that satisfies a recursive condition on lower intervals. **(S, G)**

Rikio Onodera

1968a On signed tree-graphs and cotree-graphs. *RAAG Res. Notes* (3) No. 133 (1968), ii + 29 pp. MR 38 #5671. Zbl. (e: 182.58201). **(SG: B)**

The Open University

1981a Graphs and Digraphs. Unit 2 in Course TM361: Graphs, Networks and Design. The Open University Press, Walton Hall, Milton Keynes, England, 1981. MR none. Zbl. none.

Social sciences (pp. 21–23). Signed digraphs (pp. 50–52). [Published version: see Wilson and Watkins (1990a).] **(SG, PsS, SD: Exp)**

Peter Orlik and Louis Solomon

1980a Unitary reflection groups and cohomology. *Invent. Math.* 59 (1980), 77–94. MR 81f:32017. Zbl. 452.20050. **(gg: M, G)**

1982a Arrangements defined by unitary reflection groups. *Math. Ann.* 261 (1982), 339–357. MR 84h:14006. Zbl. 491.51018. **(gg: M, G)**

1983a Coxeter arrangements. In: Peter Orlik, ed., *Singularities* (Arcata, Calif., 1981), Part 2, pp. 269–291. Proc. Symp. Pure Math., Vol. 40. Amer. Math. Soc., Providence, R.I., 1983. MR 85b:32016. **(gg: M, G)**

James B. Orlin

See also R.K. Ahuja, M. Kodialam, and R. Shull.

1984a Some problems on dynamic/periodic graphs. In: *Progress in combinatorial optimization* (Proc. Conf., Waterloo, Ont., 1982), pp. 273–293. Academic Press, Toronto, 1984. MR 86m:90058. Zbl. 547.05060.

Problems on 1-dimensional periodic graphs (i.e., covering (di)graphs of $\mathbb{Z}$-gain graphs Φ) that can be solved in Φ: connected components, strongly connected components, directed path from one vertex to another, Eulerian trail (directed or not), bicolorability, and spanning tree with minimum average cost. **(GG, GD: Cov: Paths, Polygons, Col: Alg)**

1985a On the simplex algorithm for networks and generalized networks. *Math. Programming Study* 24 (1985), 166–178. MR 87k:90102. Zbl. 592.90031. **(GN: M(Bases): Alg)**

Charles E. Osgood and Percy H. Tannenbaum

1955a The principle of congruity in the prediction of attitude change. *Psychological Rev.* 62 (1955), 42–55. **(VS: PsS)**

Eiji O'Shima

See M. Iri.

James G. Oxley

See also J.P.S. Kung and L.R. Matthews.

1992a Infinite matroids. In: Neil White, ed., *Matroid Applications*, Ch. 3, pp. 73–90. Encycl. Math. Appl., Vol. 40. Cambridge Univ. Press, Cambridge, Eng., 1992. MR 93f:05027. Zbl. 766.05016.

See Exercise 3.20. **(Bic: Exp)**

1992a *Matroid Theory.* Oxford Univ. Press, Oxford, 1992. MR 94d:05033. Zbl. 784.-05002.

§10.3, Exercise 3 concerns the Dowling lattices of $\mathrm{GF}(q)^*$. **(gg: M: Exp)**

Manfred W. Padberg

See E.L. Johnson

Steven R. Pagano

†1998a Separability and Representability of Bias Matroids of Signed Graphs. Ph.D. thesis, Dept. of Mathematical Sciences, Binghamton University, 1998.

Ch. 1: "Separability". Graphical characterization of bias-matroid k-separations of a biased graph. Also, some results on the possibility of k-separations in which one or both sides are connected subgraphs. (**GG: M: Str**)

Ch. 2: "Representability". The bias matroid of every signed graph is representable over all fields with characteristic $\neq 2$. For which signed graphs is it representable in characteristic 2 (and therefore representable over GF(4), by the theorem of Geoff Whittle, A characterization of the matroids representable over GF(3) and the rationals. *J. Combin. Theory Ser. B* 65 (1995), 222–261. MR 96m:05046. Zbl. 835.05015.)? Solved (for 3-connected signed graphs having vertex-disjoint negative polygons and hence nonregular matroid). There are two essentially different types: (i) two balanced graphs joined by three independent unbalanced digons; (ii) a cylindrical signed graph, possibly with balanced graphs adjoined by 3-sums. [See notes on Seymour (1995a) for definition of (ii) and for Lovász's structure theorem in the case without vertex-disjoint negative polygons.] (**SG: M: I, Str, T**)

Ch. 3: "Miscellaneous results". (**SG: M: I, Str**)

20xxa Binary signed graphs. Submitted (**SG: M: I, Str**)

20xxb Signed graphic GF(4) forbidden minors. Submitted (**SG: M**)

20xxc GF(4)-representations of bias matroids of signed graphs: The 3-connected case. Submitted (**SG: M: I, Str, T**)

Edgar M. Palmer

See F. Harary and F. Kharari.

B.L. Palowitch, Jr.

See M.A. Kramer.

Christos H. Papadimitriou

See also E.M. Arkin and A.S. LaPaugh.

Christos H. Papadimitriou and Kenneth Steiglitz

1982a *Combinatorial Optimization: Algorithms and Complexity.* Prentice-Hall, Englewood Cliffs, N.J., 1982. MR 84k:90036. Zbl. 503.90060.

See Ch. 10, Problems 6–7, p. 244, for bidirected graphs and flows in relation to the matching problem. (**sg: O: Flows**)

1985a *Kombinatornaya optimiztsiya. Algoritmy i Slozhnost'.* Transl. V.B. Alekseev. Mir, Moskva, 1985. MR 86i:90067. Zbl. 598.90067.

Russian translation of (1982a). (**sg: O: Flows**)

Giorgio Parisi

See M. Mézard.

Philippa Pattison

1993a *Algebraic Models for Social Networks.* Structural Analysis in the Social Sciences, 7. Cambridge Univ. Press, Cambridge, 1993.

Ch. 8, pp. 258–9: "The balance model. The complete clustering model." Embedded in a more general framework. (**SG, S: A, B, Cl: Exp**)

G.A. Patwardhan

See B.D. Acharya and M.K. Gill.

Charles Payan

See F. Jaeger.

Edmund R. Peay

1977a Matrix operations and the properties of networks and directed graphs. *J. Math. Psychology* 15 (1977), 89–101. MR 56 #2690 (q.v.). Zbl. 352.05039.

(**SD, WD: A: Gen**)

1977b Indices for consistency in qualitative and quantitative structures. *Human Relations* 30 (1977), 343–361.

Proposes an index of nonclusterability for signed graphs and generalizes to edges weighted by a linearly ordered set. (**SG, Gen: Cl: Fr**)

1982a Structural models with qualitative values. *J. Math. Sociology* 8 (1982), 161–192. MR 83d:92107. Zbl. 486.05060.

See mainly §3: "Structural consistency." (**sd: Gen: B, Cl**)

Uri N. Peled

See S.R. Arikati, P.L. Hammer, T. Ibaraki, and N.V.R. Mahadev.

Francisco Pereira

See A.J. Hoffman.

M. Petersdorf

1966a Einige Bemerkungen über vollständige Bigraphen. *Wiss. Z. Techn. Hochsch. Ilmenau* 12 (1966), 257–260. MR 37 #1275. Zbl. (e: 156.44302).

Treats signed K_n's. Satz 1: $\max l(\Sigma) = \lfloor (n-1)^2/4 \rfloor$ with equality iff Σ is antibalanced. [From which follows easily the full Thm. 14 of Abelson and Rosenberg (1958a).] Also, some further discussion of antibalanced and unbalanced cases. [For extensions of this problem see notes on Erdős, Győri, and Simonovits (1992a).] (**SG: Fr**)

J.L. Phillips

1967a A model for cognitive balance. *Psychological Rev.* 74 (1967), 481–495.

Proposes to measure imbalance of a signed (di)graph by largest eigenvalue of a matrix close to $I + A(\Sigma)$. (Cf. Abelson (1967a).) Possibly, means to treat only graphs that are complete aside from isolated vertices. [Somewhat imprecise.] Summary of Ph.D. thesis. (**SG: B, Fr, A**)

Nancy V. Phillips

See F. Glover.

Jean-Claude Picard and H. Donald Ratliff

1973a A graph-theoretic equivalence for integer programs. *Oper. Res.* 21 (1973), 261–269. MR 50 #12240. Zbl. 263.90021.

A minor application of signed switching to a weighted graph arising from an integer linear program. (**sg: sw**)

P. Pincus

See S. Alexander.

Tomaž Pisanski and Jože Vrabec

1982a Graph bundles. Preprint Ser., Dept. Math., Univ. Ljubljana, 1982.

Definition (see Pisanski, Shawe-Taylor, and Vrabec (1983a)), examples, superimposed structure, classification. (**GG: Cov(Gen)**)

Tomaž Pisanski, John Shawe-Taylor, and Jože Vrabec

1983a Edge-colorability of graph bundles. *J. Combin. Theory Ser. B* 35 (1983), 12–19. MR 85b:05086. Zbl. 505.05034, (515.05031).

A graph bundle is, roughly, a covering graph with an arbitrary graph F_v (the "fibre") over each vertex v, so that the edges covering $e: vw$ induce an isomorphism $F_v \to F_w$. (**GG: Cov(Gen): ECol**)

Michael Plantholt

See F. Harary.

M.D. Plummer

See L. Lovász.

Svatopluk Poljak

See also Y. Crama.

Svatopluk Poljak and Daniel Turzík

1982a A polynomial algorithm for constructing a large bipartite subgraph, with an application to a satisfiability problem. *Canad. J. Math.* 34 (1982), 519–524. MR 83j:05048. Zbl. 471.68041, (487.68058).

Main Theorem: For a simple, connected signed graph of order n and size $|E| = m$, the frustration index $l(\Sigma) \le g(m,n) := \frac{1}{2}m - \frac{1}{2}\lceil\frac{1}{2}(n-1)\rceil$. The proof is algorithmic, by constructing a (relatively) small deletion set. Dictionary: Σ is an "edge-2-colored graph" (G,c), E_+ and E_- are called E_1 and E_2, a balanced subgraph is "generalized bipartite", and $m - l(\Sigma)$ is what is calculated. [Thus for a connected, simple graph, $D(\Gamma) \le g(m,n)$: see Akiyama, Avis, Chvátal, and Era (1981a).] (**SG: Fr, Alg**)

1986a A polynomial time heuristic for certain subgraph optimization problems with guaranteed worst case bound. *Discrete Math.* 58 (1986), 99–104. MR 87h:68131. Zbl. 585.05032.

Generalizes (1982a), with application to signed graphs in Cor. 3. (**SG: Fr, Alg**)

1987a On a facet of the balanced subgraph polytope. *Časopis Pěst. Mat.* 112 (1987), 373–380. MR 89g:57009. Zbl. 643.05059.

The polytope $P_B(\Sigma)$ (the authors write P_{BL}) is the convex hull in $\mathbb{R}^E$ of incidence vectors of balanced edge sets. It generalizes the bipartite subgraph polytope $P_B(\Gamma) = P_B(-\Gamma)$ (see Barahona, Grötschel, and Mahjoub (1985a)), but is essentially equivalent to it according to Prop. 2: The negative-subdivision trick preserves facets of the polytope. Thm. 1 gives new facets, corresponding to certain circulant subgraphs. (They are certain unions of two Hamilton polygons, each having constant sign.) (**SG: Fr, G**)

1992a Max-cut in circulant graphs. *Discrete Math.* 108 (1992), 379–392. MR 93k:05101.

Further development of (1987a) for all-negative Σ. The import for general signed graphs is not discussed. (**P: Fr, G**)

Svatopluk Poljak and Zsolt Tuza

1995a Maximum cuts and large bipartite subgraphs. In: W. Cook, L. Lovaśz, and P. Seymour, eds., *Combinatorial Optimization* (Papers from the DIMACS Special Year), pp. 181–244. DIMACS Ser. Discrete Math. Theoret. Computer Sci., Vol. 20. Amer. Math. Soc., Providence, R.I., 1995. MR 95m:90008. Zbl. 819.00048.

Surveys max-cut and weighted max-cut [that is, max. size balanced subgraph and max. weight balanced subgraph in all-negative signed graphs]. See esp. §2.9: "Bipartite subgraph polytope and weakly bipartite graphs". [The weakly bipartite classes announced by Gerards suggested that a signed-graph characterization of weakly bipartite graphs is called for. This is provided by Guenin (20xxa).]

§1.2, "Lower bounds, expected size, and heuristics", surveys results for all-negative signed graphs that are analogous to results in Akiyama, Avis, Chvátal, and Era (1981a) (q.v.), etc. [*Problem.* Generalize any of these results, that are not already generalized, to signed simple graphs and to simply signed graphs.] **(p: Fr, tg(Sw): Exp, Ref)**

Y. Pomeau

See B. Derrida.

Dragos Popescu [Dragoş-Radu Popescu]

See Dragoş-Radu Popescu.

Dragoş-Radu Popescu [Dragos Popescu]

1979a Proprietati ale grafurilor semnate. [Properties of signed graphs.] (In Romanian. French summary.) *Stud. Cerc. Mat.* 31 (1979), 433–452. MR 82b:05111. Zbl. 426.05048.

A signed K_n is balanced or antibalanced or has a positive and a negative polygon of every length $k = 3, \ldots, n$. For odd n, the signed K_n if not balanced has at least $\frac{n-1}{2}$ Hamiltonian polygons. For even n, $-K_n$ does not maximize the number of negative polygons. A "polygon basis" is a set of the smallest number of polygons whose signs determine all polygon signs. This is proved to have $\binom{n-1}{2}$ members. Furthermore, there is a basis consisting of k-gons for each $k = 3, \ldots, n$. [A polygon basis in this sense is the same as a basis of polygons for the binary cycle space. See Zaslavsky (1981b), Topp and Ulatowski (1987a).] **(SG: Fr)**

1991a Cicluri în grafuri semnate. [Cycles in signed graphs.] (In Romanian; French summary.) *Stud. Cercet. Mat.* 43, No. 3/4 (1991), 85–219. MR 92j:05114. Zbl. 751.05060.

Ch. 1: "A-balance" (p. 91). Let F be a spanning subgraph of K_n and A a signed K_n. The "product" of signed graphs is $\Sigma_1 * \Sigma_2$ whose underlying graph is $|\Sigma_1| \cup |\Sigma_2|$, signed as in Σ_i for an edge in only one Σ_i but with sign $\sigma_1(e)\sigma_2(e)$ if in both. Let $\mathcal{G}_F$ denote the group of all signings of F; let $\mathcal{G}_F(A)$ be the group generated by the set of restrictions to F of isomorphs of A. A member of $\mathcal{G}_F(A)$ is "A-balanced"; other members of $\mathcal{G}_F$ are A-unbalanced. We let $\hat{\Sigma}$ denote the coset of Σ and $\approx$ the "isomorphism" of cosets induced by graph isomorphism, i.e., cosets are isomorphic if they have isomorphic members. Let $\dot{\Sigma}$ be the isomorphism class of Σ, $\hat{\hat{\Sigma}}$ the isomorphism class of $\hat{\Sigma}$, and $\mathring{\Sigma} := \bigcup \hat{\hat{\Sigma}}$. Now choose a system of representatives of the coset isomorphism classes, $R = \{\Sigma_1, \ldots, \Sigma_l\}$. Prop. 1.4.1. Each $\dot{\Sigma}$ intersects exactly one $\hat{\Sigma}_i$. Let $R_i = \{\Sigma_{i1}, \ldots, \Sigma_{ia_i}\}$ be a system of representatives of $\hat{\Sigma}_i/\cong$, arranged so that $|E_-(\Sigma_{ij})|$ is a minimum when $j = 1$. This minimum value is the "[line] index of A-imbalance" of each $\Sigma \in \mathring{\Sigma}_i$ and is denoted by $\delta_A(\Sigma)$. (§2.1: Taking A to be K_n with one vertex star all negative makes this equal the frustration index $l(\Sigma)$.) Prop. 1.5.1. $\delta_A(\Sigma)$ is the least number of edges whose sign needs to be changed to make Σ A-balanced. Prop. 1.5.2. $\delta_A(\Sigma) = |E_-(\Sigma)|$ iff $|E_-(\Sigma) \cap E_-(F, \beta)| \leq \frac{1}{2}|E_-(F, \beta)|$ for every signing β)

of F. Finally, for each $\Sigma \in \mathcal{G}_F$ define the "Σ-relation" on coset isomorphism classes $\hat{\hat{\Sigma}}_i$ to be the relation generated by negating in Σ_1 all the edges of $E_-(\Sigma)$, extended by isomorphism and transitivity. This is well defined (Prop. 1.6.1) and symmetric (Prop. 1.6.2) and is preserved under negation of coset isomorphism classes (Prop. 1.6.4, 1.6.5). Self-negative classes, such that $\hat{\hat{\Sigma}} \approx -\hat{\hat{\Sigma}}$, are the subject of Prop. 1.6.3.

Ch. 2: "Signed complete graphs" (p. 106). §2.5: "H-graphs". If H is a signed K_h, a "standard H-graph" Σ is a signed K_n such that $\Sigma_- \cong H_- \cup K^c_{n-h}$. Prop. 2.5.3. Assume certain hypotheses on n, $|X_0|$ for $X_0 \subseteq V(\Sigma)$, and a quantity $D^-(H)$ derived from negative degrees. Then $|E_-| = l(\Sigma) \Rightarrow$ the induced subgraph $G{:}X_0$ is a standard H-graph with $|E_-(\Sigma{:}X_0)| = l(\Sigma{:}X_0)$. The cases $H_- = K_1$, K_2, and a 2-edge path are worked out. For the former, Prop. 2.5.3 reduces to Sozański's (1976a) Thm. 3.

Ch. 3: "Frustration index" (p. 158). Some upper bounds.

Ch. 4: "Evaluations, divisibility properties" (p. 174). Similar to parts of (1996a) and Popescu and Tomescu (1996b).

Ch. 5: "Maximal properties" (p. 198). §5.1: "Minimum number and maximum number of negative stars, resp. 2-stars". §5.2 is a special case of Popescu and Tomescu (1996a), Thm. 2. §5.3: "On the maximum number of negative cycles in some signed complete graphs". Shows that Conjecture 1 is false for even $n \geq 6$. Some results on the odd case.

Conjecture 1 (Tomescu). A signed complete graph of odd order has the most negative polygons iff it is antibalanced. (Partial results are in §5.3.) [This example maximizes $l(\Sigma)$. A somewhat related conjecture is in Zaslavsky (1997b).] *Conjecture* 2. See (1993a). *Conjecture* 3. Given k and m, there is $n(k,m)$ so that for any $n \geq n(k,m)$, a signed K_n with m negative edges has (a) the most negative k-gons iff the negative edges are pairwise nonadjacent; (b) the fewest iff the negative edges form a star.

(**SG: B(Gen), K, Fr, E: Polygons, Paths**)

1993a Problem 17. *Research Problems* at the Internat. Conf. on Combinatorics (Keszthely, 1993). Unpublished manuscript. János Bolyai Math. Soc., Budapest, 1993.

Conjecture. An unbalanced signed complete graph has the minimum number of negative polygons iff its frustration index equals 1. (**SG: Fr**)

1996a Une méthode d'énumération des cycles négatifs d'un graphe signé. *Discrete Math.* 150 (1996), 337–345. MR 97c:05077. Zbl. 960.39919.

The numbers of negative subgraphs, especially polygons and paths of length k, in an arbitrarily signed K_n. Formulas and divisibility and congruence properties. Extends part of Popescu and Tomescu (1996a).

(**SG: K, E: Polygons, Paths**)

Dragoş-Radu Popescu and Ioan Tomescu

1996a Negative cycles in complete signed graphs. *Discrete Appl. Math.* 68 (1996), 145–152. MR 98f:05098. Zbl. 960.35935.

The number c_p of negative polygons of length p in a signed K_n with s negative edges. Thm. 1. For n sufficiently large compared to p and s, c_p is minimized if E_- is a star (iff, when $s > 3$) and is maximized iff E_- is a matching. Thm. 2. c_p is divisible by $2^{p-2-\lfloor \log_2(p-1) \rfloor}$. Thm. 3. If $s \sim \lambda n$ and $p \sim \mu n$ and the negative-subgraph degrees are bounded (this is essential), then asymptotically the fraction of negative p-gons is $\frac{1}{2}(1 - e^{-4\lambda\mu})$. **(SG: K: Fr, E: Polygons)**

1996b Bonferroni inequalities and negative cycles in large complete signed graphs. *European J. Combin.* 17 (1996), 479–483. MR 97d:05177. Zbl. 861.05036.

A much earlier version of (1996a) with delayed publication. Contains part of (1996a): a version of Thm. 1 and a restricted form of Thm. 3. **(SG: K: Fr, E: Polygons)**

Alexander Postnikov

1997a Intransitive trees. *J. Combin. Theory Ser. A* 79 (1997), 360–366. MR 98b:05036. Zbl. 876.05042.

§4.2 mentions the lift matroid of the integral poise gains of a transitively oriented complete graph. [See also Stanley (1996a).] **(GG: M, G)**

Alexander Postnikov and Richard P. Stanley

20xxa Deformations of Coxeter hyperplane arrangements. Submitted.

Geert Prins

See F. Harary.

Sharon Pronchik

See L. Fern.

Andrzey Proskurowski

See A.M. Farley.

J. Scott Provan

1983a Determinacy in linear systems and networks. *SIAM J. Algebraic Discrete Methods* 4 (1983), 262–278. MR 84g:90061. Zbl. 558.93018. **(Sol, GN)**

1987a Substitutes and complements in constrained linear models. *SIAM J. Algebraic Discrete Methods* 8 (1987), 585–603. MR 89c:90072. Zbl. 645.90049.

§4: "Determinacy in a class of network models." [Fig. 1 and Thm. 4.7 hint at possible digraph version of signed-graph or gain-graph bias matroid.] **(?sg, gg: m(?bases): gen)**

Teresa M. Przytycka and Józef H. Przytycki

1988a Invariants of chromatic graphs. Tech. Rep. No. 88-22, Univ. of British Columbia, Vancouver, B.C., 1988.

Generalizing concepts from Kauffman (1989a). [See also Traldi (1989a) and Zaslavsky (1992b).] **(SGc: Gen: N, Knot)**

1993a Subexponentially computable truncations of Jones-type polynomials. In: Neil Robertson and Paul Seymour, eds., *Graph Structure Theory* (Proc., Seattle, 1991), pp. 63–108. Contemp. Math., Vol. 147. Amer. Math. Soc., Providence, R.I., 1993. MR 95c:57016. Zbl. 812.57010.

A "chromatic graph" is a graph with edges weighted from the set $Z \times \{d, l\}$, Z being [apparently] an arbitrary set of "colors". A "dichromatic graph" has $Z = \{+, -\}$. Such graphs have general dichromatic polynomials [see Przytycka and Przytycki (1988a), Traldi (1989a), and Zaslavsky (1992b)], as [partially] anticipated by Fortuin and Kasteleyn (1972a). I will not attempt to summarize this paper. **(SGc: N, Knot, Ref)**

Jozef H. Przytycki

See K. Murasugi and T.M. Przytycka.

Vlastimil Ptak
See M. Fiedler.

William R. Pulleyblank
See J.-M. Bourjolly and M. Grötschel.

L. Pyber
See L. Lovász.

Hongxun Qin
See J.E. Bonin.

Louis V. Quintas
See M. Gargano.

James P. Quirk
See also L. Bassett and J.S. Maybee.

1974a A class of generalized Metzlerian matrices. In: George Horwich and Paul A. Samuelson, eds., *Trade, Stability, and Macroeconomics: Essays in Honor of Lloyd A. Metzler*, pp. 203–220. Academic Press, New York, 1974. (**QM: Sta: sd**)

1981a Qualitative stability of matrices and economic theory: a survey article. In: Harvey J. Greenberg and John S. Maybee, eds., *Computer-Assisted Analysis and Model Simplification* (Proc. Sympos., Boulder, Col., 1980), pp. 113–164. Discussion, pp. 193–199. Academic Press, New York, 1981. MR 82g:00016 (book). Zbl. 495.93001 (book).

Comments by W.M. Gorman (pp. 175–189) and Eli Hellerman (pp. 191–192). Discussion: see pp. 193–196. (**QM: Sta: sd, b: Exp**)

James Quirk and Richard Ruppert

1965a Qualitative economics and the stability of equilibrium. *Rev. Economic Stud.* 32 (1965), 311–326. (**QM: Sta: sd**)

W.M. Raike
See A. Charnes.

R. Rammal
See F. Barahona and I. Bieche.

M.R. Rao
See Y.M.I. Dirickx.

S.B. Rao
See also P. Das and [G.R.] Vijaya Kumar.

1984a Characterizations of harmonious marked graphs and consistent nets. *J. Combin. Inform. System Sci.* 9 (1984), 97–112. MR 89h:05048. Zbl. 625.05049.

A complicated solution, with a polynomial-time algorithm, to the problem of characterizing consistency in vertex-signed graphs. Thm. 4.1 points out that graphs with signed vertices and edges can be easily converted to graphs with signed vertices only; thus harmony in graphs with signed vertices and edges is characterized as well. [See Hoede (1992a) for the last word.]
(**SG, VS: B, Alg**)

S.B. Rao, N.M. Singhi, and K.S. Vijayan

1981a The minimal forbidden subgraphs for generalized line graphs. In: S.B. Rao, ed., *Combinatorics and Graph Theory* (Proc. Sympos., Calcutta, 1980), pp. 459–472. Lecture Notes in Math., 885. Springer-Verlag, Berlin, 1981. MR 83i:05062. Zbl. 494.05053. (**sg: LG, p**)

Thomas Raschle and Klaus Simon

1995a Recognition of graphs with threshold dimension two. In: *Proceedings of the Twenty-Seventh Annual ACM Symposium on the Theory of Computing* (Las Vegas, 1995), pp. 650–661.

Expounded by Mahadev and Peled (1995a), Sect.. 8.5 (q.v.). (**p: o, Alg**)

H. Donald Ratliff

See J.-Cl. Picard.

Bertram H. Raven

See B.E. Collins.

D.K. Ray-Chaudhuri, N.M. Singhi, and G.R. Vijayakumar

1992a Signed graphs having least eigenvalue around -2. *J. Combin. Inform. System Sci.* 17 (1992), 148–165. MR 94g:05056. (**SG: A, G: Exp**)

Margaret A. Readdy

See R. Ehrenborg.

P. Reed

See A.J. Bray.

F. Regonati

See E. Damiani.

G. Reinelt

See M. Grötschel.

Victor Reiner

See also P. Edelman.

1993a Signed posets. *J. Combin. Theory Ser. A* 62 (1993), 324–360. MR 94d:06011. Zbl. 773.06008.

They are equivalent to acyclic bidirected graphs. (**S, sg: O: Str, g**)

Daniel J. Richman

See J.S. Maybee.

Robert G. Rieper

See J. Chen.

M.J. Rigby

See A.C. Day.

Chong S. Rim

See H. Choi.

R.D. Ringeisen

See also M.J. Lipman.

1974a Isolation, a game on a graph. *Math. Mag.* 47, No. 3 (May, 1974), 132–138. MR 51 #9842. (**TG**)

Gerhard Ringel

See also N. Hartsfield and M. Jungerman.

1974a *Map Color Theorem.* Grundl. math. Wiss., B. 209. Springer-Verlag, Berlin, 1974. MR 50 #2860. Zbl. 287.05102.

"Cascades": see Youngs (1968b). (**sg: O: Appl**)

1974a *Teorema o Raskraske Kart.* Transl. V.B. Alekseev. Ed. G.P. Gavrilov. "Mir", Moskva, 1977. MR 57 #5809. Zbl. 439.05019.

Russian translation of (1974a). (**sg: O: Appl**)

1977a The combinatorial map color theorem. *J. Graph Theory* 1 (1977), 141–155. MR 56 #2860. Zbl. 386.05030.

Signed rotation systems for graphs. Thm. 12: Signed rotation systems describe all cellular embeddings of a graph; an embedding is orientable iff its signature is balanced. Compare Stahl (1978a). Dictionary: "Triple" means graph with signed rotation system. "Orientable" triple means balanced signature. "Oriented" means all positive. (**SG: T, Sw**)

Fred S. Roberts

See also T. A. Brown and R.Z. Norman.

1974a Structural characterizations of stability of signed digraphs under pulse processes. In: Ruth A. Bari and Frank Harary, eds., *Graphs and Combinatorics* (Proc. Capital Conf., Geo. Washington Univ., 1973), pp. 330–338. Lecture Notes in Math., 406. Springer-Verlag, Berlin, 1974. MR 50 #12792. Zbl. 302.05107. (**SDw**)

1976b *Discrete Mathematical Models, With Applications to Social, Biological, and Environmental Problems.* Prentice-Hall, Englewood Cliffs, N.J., 1976. Zbl. 363.90002.

§3.1: "Signed graphs and the theory of structural balance." Many topics are developed in the exercises. Exercise 4.2.7 (from Phillips (1967a)).

(**SG, SD: B, Alg, A, Cl, Fr, PsS: Exp, Exr**)

Ch. 4: "Weighted digraphs and pulse processes." Signed digraphs here are treated as unit-weighted digraphs. Note esp.: §4.3: "The signed or weighted digraph as a tool for modelling complex systems." Conclusions about models are drawn from very simple properties of their signed digraphs. §4.4: "Pulse processes." §4.5: "Stability in pulse processes." Stability is connected to eigenvalues of $A(\Sigma)$. (**SDw, SD, WD: B, A, PsS: Exp, Exr, Ref**)

1978a *Graph Theory and Its Applications to Problems of Society.* CBMS-NSF Regional Conf. Ser. in Appl. Math., 29. Soc. Indust. Appl. Math., Philadelphia, 1978. MR 80g:90036. Zbl. 452.05001.

Ch. 9: "Balance theory and social inequalities." Ch. 10: "Pulse processes and their applications." Ch. 11: "Qualitative matrices."

(**SG, SD, SDw: B, PsS, QM: Exp, Ref**)

1979a Graph theory and the social sciences. In: Robin J. Wilson and Lowell W. Beineke, eds., *Applications of Graph Theory*, Ch. 9, pp. 255–291. Academic Press, London, 1979. MR 81h:05050 (book). Zbl. 444.92018.

§2: "Balance and clusterability." Basics in brief. §7: "Signed and weighted digraphs as decision-making models." Cursory.

(**SG, PsS, SD, SDw: B, Cl, K: Exp, Ref**)

1986a *Diskretnye matematicheskie modeli s prilozheniyami k sotsialnym, biologicheskim i ekologicheskim zadacham.* Transl. A. M. Rappoport and S. I. Travkin. Ed. and preface by A. I. Teĭman. Teoriya i Metody Sistemnogo Analiza. [Theory and Methods of Systems Analysis.] "Nauka", Moscow, 1986. MR 88e:00020. Zbl.662.90002.

Russian edition of (1976b). (**SG, SD: B, Alg, A, Cl, Fr, PsS: Exp, Exr**)
(**SDw, SD, WD: B, A, PsS: Exp, Exr, Ref**)

1989a Applications of combinatorics and graph theory to the biological and social sciences: Seven fundamental ideas. In: Fred Roberts, ed., *Applications of Combinatorics and Graph Theory to the Biological and Social Sciences*, pp. 1–37. IMA Vols. Math. Appl., Vol. 17. Springer-Verlag, New York, 1989. MR 91c:92001.

§4: "Qualitative stability." A fine, concise basic survey.

(**QM: SD: Exp, Ref**)

§5: "Balanced signed graphs." Another concise basic survey, and two open problems (p. 20). **(SG: B: Exp, Ref)**

1995a On the problem of consistent marking of a graph. *Linear Algebra Appl.* 217 (1995), 255–263. MR 95k:05157. Zbl. 830.05059.
[See Hoede (1992a).] **(VS: B)**

Fred S. Roberts and Thomas A. Brown

1975a Signed digraphs and the energy crisis. *Amer. Math. Monthly* 82, No. 6 (June–July, 1975), 577–594. MR 51 #5195. Zbl. 357.90070. **(SD, SDw)**

1977a [Reply to Waterhouse (1977a)]. *Amer. Math. Monthly* 84 (1977), 27.

Neil Robertson, P.D. Seymour, and Robin Thomas

See also W. McCuaig.

†20xxa Permanents, Pfaffian orientations, and even directed circuits. Submitted.
The main theorem is proved also in McCuaig (20xxa).

Robert W. Robinson

See also Harary, Palmer, Robinson, and Schwenk (1977a) and Harary and Robinson (1977a).

1981a Counting graphs with a duality property. In: H.N.V. Temperley, ed., *Combinatorics* (Proc. Eighth British Combinatorial Conf., Swansea, 1981), pp. 156–186. London Math. Soc. Lecture Note Ser., 52. Cambridge Univ. Press, Cambridge, England, 1981. MR 83c:05071. Zbl. 462.05035.
The "bilayered digraphs" of §7 are identical to simply signed, loop-free digraphs (where multiple arcs are allowed if they differ in sign or direction). Thm. 1: Their number b_n = number of self-complementary digraphs of order $2n$. Cor. 1: Equality holds if the vertices are signed and k-colored. In §8, Cor. 2 concerns vertex-signed and 2-colored digraphs; Cor. 3 concerns vertex-signed tournaments. Assorted remarks on previous signed enumerations, mainly from Harary, Palmer, Robinson, and Schwenk (1977a), are scattered about the article. **(SD, VS, SG: E)**

Y. Roditty

See I. Krasikov.

Vojtěch Rödl

See R.A. Duke.

Milton J. Rosenberg

See also R.P. Abelson.

Milton J. Rosenberg and Robert P. Abelson

1960a An analyisis of cognitive balancing. In: Milton J. Rosenberg *et al.*, eds., *Attitude Organization and Change: An Analysis of Consistency Among Attitude Components*, Ch. 4, pp. 112–163. Yale Univ. Press, New Haven, 1960.
An attempt to test structural balance theory experimentally. The test involves, in effect, a signed K_4 [an unusually large graph for such an experiment]. Conclusion: there is a tendency to balance but it competes with other forces. **(PsS)**

Seymour Rosenberg

1968a Mathematical models of social behavior. In: Gardner Lindzey and Elliot Aronson, eds., *The Handbook of Social Psychology*, Second Edition, Vol. 1, Ch. 3, pp. 179–244. Addison-Wesley, Reading, Mass., 1968.
"Balance model," pp. 196–199. "Congruity model," pp. 199–203.

(PsS, SG: B: Exp, Ref)

M. Rosenfeld
See W.D. McCuaig.

Gian-Carlo Rota
See P. Doubilet.

Uriel G. Rothblum and Hans Schneider

1980a Characterizations of optimal scalings of matrices. *Math. Programming* 19 (1980), 121–136. MR 81j:65064. Zbl. 437.65038. **(gg: m, Sw)**

1982a Characterizations of extreme normalized circulations satisfying linear constraints. *Linear Algebra Appl.* 46 (1982), 61–72. MR 84d:90047. Zbl. 503.05032. **(gg: m)**

Bernard Roy

1959a Contribution de la théorie des graphes à l'étude de certains problèmes linéaires. *C.R. Acad. Sci. Paris* 248 (1959), 2437–2439. MR 22 #2493. **(WD: OG)**

1970a *Algèbre moderne et théorie des graphes, orientées vers les sciences économiques et sociales. Tome II: Applications et problèmes spécifiques.* Dunod, Paris, 1970. MR 41 #5039. Zbl. 238.90073.
§IX.B.3.b: "Flots multiplicatifs et non conditionnels, ou k-flots." §IX.E.1.b: "Extension du problème central aux k-flots." §IX.E.2.c: "Quelques utilisations concretes des k-flots." **(GN: m(circuit): Exp)**

Gordon F. Royle
See M.N. Ellingham.

G. Rozenberg
See A. Ehrenfeucht.

Arthur L. Rubin
See P. Erdős.

Richard Ruppert
See J. Quirk.

Herbert J. Ryser
See Richard A. Brualdi.

Rachid Saad

1996a Finding a longest alternating cycle in a 2-edge-coloured complete graph is in RP. *Combin. Probab. Computing* 5 (1996), 297–306. MR 97g:05156. Zbl. 865.05054.
Thm.: In a bidirected all-negative complete graph with a suitable extra hypothesis, the maximum length of a coherent polygon equals the maximum order of a coherent degree-2 subgraph. More or less generalizes part of Bánkfalvi and Bánkfalvi (1968a) (q.v.). [Generalized in Bang-Jensen and Gutin (1998a).] [*Problem.* Generalize to signed complete graphs or further.] **(p: o: Paths, Alg)**

Horst Sachs
See D.M. Cvetković.

Bruce Sagan
See also C. Bennett, A. Björner, A. Blass, F. Harary, and T. Józefiak.

1995a Why the characteristic polynomial factors. *Sém. Lotharingien Combin.* 35 (1995) [1998], Article B35a, iii + 20 pp. (electronic). MR 98a:06006. Zbl. 855.05012.
A shorter predecessor of (1999a). **(SG, Gen: N: Col, G: Exp)**

1999a Why the characteristic polynomial factors. *Bull. Amer. Math. Soc. (N.S.)* 36 (1999), 113–133.

In Section 4, coloring of a signed graph Σ, especially of $\pm K_n^\bullet$ and $\pm K_n$, is used to calculate and factor the characteristic polynomial of $G(\Sigma)$. Presents the geometrical reinterpretation and generalization by Blass and Sagan (1998a). In Sections 5 and 6, other methods of calculation and factorization are applied to some signed graphs (in their geometrical representation).

(**SG, Gen: N: Col, G: Exp**)

Michael Saks

See P.H. Edelman.

Nicolau C. Saldanha

20xxa Generalized Kasteleyn matrices and their singular values. Manuscript, 1997. WorldWideWeb URL (7/99) http://www.mat.puc-rio.br/~nicolau/publ.html

A generalized Kasteleyn matrix is the left-right adjacency matrix of a bipartite gain graph with the complex units as gain group. (A Kasteleyn matrix has for gain group the sign group.) The object is to interpret combinatorially the singular values. The approach is cohomological (cf. Cameron 1977b).

(**GG, SG: A, Sw**)

E. Sampathkumar

1972a Point-signed and line-signed graphs. Karnatak Univ. Graph Theory Res. Rep. 1, 1972.

See *Graph Theory Newsletter* 2, No. 2 (Nov., 1972), Abstract No. 7.

(**SG, VS: B**)

1984a Point signed and line signed graphs. *Nat. Acad. Sci. Letters (India)* 7 (1984), 91–93. Zbl. 552.05051.

Consider a simple graph with an edge signature σ and a vertex signature μ. [I write $\partial\sigma(v) := \prod\{\sigma(e) : e \text{ incident with } v\}$.] Thm. 1: (Γ, μ) is consistent iff $\mu = \partial\sigma$ for some σ. [Necessity is incorrect: consider $K_{2,3}$ with every vertex negative. Sufficiency is incorrect: attach a pendant edge to every vertex of Γ, so $\partial\sigma|_{V(\Gamma)}$ can be chosen arbitrarily. The corollaries are incorrect.] Thm. 2: If Σ is balanced and $(|\Sigma|, \partial\sigma)$ is consistent, then there exist all-negative, pairwise edge-disjoint paths connecting the negative vertices in pairs. [This is Listing's Theorem applied to Σ_-. The assumptions are unnecessary.] (**SG, VS: B**)

E. Sampathkumar and V.N. Bhave

1973a Group valued graphs. *J. Karnatak Univ. Sci.* 18 (1973), 325–328. MR 50 #177. Zbl. 284.05113.

Group-weighted graphs, both in general and where the group has exponent 2 (so all $x^{-1} = x$). Analogs of elementary theorems of Harary and Flament. Here balance of a polygon means that the weight product around the polygon, taking for each edge either $w(e)$ or $w(e)^{-1}$ arbitrarily, equals 1 for some choice of where to invert. (**WG, GG: B**)

E. Sampathkumar and L. Nanjundaswamy

1973a Complete signed graphs and a measure of rank correlation. *J. Karnatak Univ. Sci.* 18 (1973), 308–311. MR 54 #11649 (q.v.). Zbl. 291.62066.

Given a permutation of $\{1, 2, \ldots, n\}$, sign K_n so edge ij is negative if the permutation reverses the order of i and j and is positive otherwise. Kendall's measure τ of correlation of rankings (i.e., permutations) A and B equals $(|E_+| - |E_-|)/|E|$ in the signature due to AB^{-1}. **(SG: K)**

B. David Saunders

See also A. Berman.

B. David Saunders and Hans Schneider

1978a Flows on graphs applied to diagonal similarity and diagonal equivalence for matrices. *Discrete Math.* 24 (1978), 205–220. MR 80e:15008. Zbl. 393.94046. **(gg: Sw)**

1979a Cones, graphs and optimal scalings of matrices. *Linear Multilinear Algebra* 8 (1979), 121–135. MR 80k:15036. Zbl. 433.15005. **(gg: Sw)(Ref)**

R.H. Schelp

See P. Erdős.

Baruch Schieber

See L. Cai.

Rüdiger Schmidt

1979a On the existence of uncountably many matroidal families. *Discrete Math.* 27 (1979), 93–97. MR 80i:05029. Zbl. 427.05024.

The "count" matroids of graphs (see Whiteley (1996a)) and an extensive further generalization of bicircular matroids that includes bias matroids of biased graphs. His "partly closed set" is a linear class of circuits in an arbitrary "count" matroid. **(GG: M, Bic, EC: Gen)**

Hans Schneider

See G.M. Engel, U.G. Rothblum, and B.D. Saunders.

Alexander Schrijver

See also A.M.H. Gerards.

1986a *Theory of Linear and Integer Programming.* Wiley, Chichester, 1986. MR 88m:-90090. Zbl. 665.90063.

Remark 21.2 (p. 308) cites Truemper's (1982a) definition of balance of a $0, \pm 1$-matrix. **(sg: p: I: Exp)**

1989a The Klein bottle and multicommodity flows. *Combinatorica* 9 (1989), 375–384. MR 92b:90083. Zbl. 708.05019.

Assume Σ embedded in the Klein bottle. If Σ is bipartite, negative girth = max. number of disjoint balancing edge sets. If Σ is Eulerian, frustration index = max. number of edge-disjoint negative polygons. Proved via polyhedral combinatorics. **(SG: T, G, Fr)**

1990a Applications of polyhedral combinatorics to multicommodity flows and compact surfaces. In: William Cook and P.D. Seymour, eds., *Polyhedral Combinatorics*, pp. 119–137. DIMACS Ser. in Discrete Math. and Theoret. Comp. Sci., Vol. 1. Amer. Math. Soc. and Soc. Indust. Appl. Math., Providence, R.I., 1990. MR 92d:05057. Zbl. 727.90025.

§2: "The Klein bottle," surveys Schrijver (1989a). **(SG: T, G, Fr: Exp)**

1990b Homotopic routing methods. In: B. Korte, L. Lovász, H.J. Prömel, and A. Schrijver, eds., *Paths, Flows, and VLSI-Layout*, pp. 329–371. Algorithms and Combinatorics, Vol. 9. Springer-Verlag, Berlin, 1990. MR 92f:68139. Zbl. 732.90087.

§4: "Edge-disjoint paths in planar graphs," pp. 342–345, "The projective plane and the Klein bottle," surveys Schrijver (1989a). **(SG: T, G, Fr: Exp)**

§3: "Edge-disjoint paths and multicommodity flows," pp. 334 ff. [This work suggests there may be a signed-graph generalization with the theorems discussed corresponding to all-negative signatures.] (**p: Paths: Exp**)

1991a Disjoint circuits of prescribed homotopies in a graph on a compact surface. *J. Combin. Theory Ser. B* 51 (1991), 127–159. MR 92a:05048. Zbl. 723.05050.

§2: "An auxiliary theorem on linear inequalities," concerns feasibility of inequalities with coefficient matrix containing incidence matrix of $-\Gamma$. [See Hurkens (1988a).] (**ec: I**)

1991b (As "A. Skhreĭver") *Teoriya lineĭnogo i tselochislennogo programmirovaniya*, Vols. 1 and 2. Mir, Moscow, 1991. MR 94c:90003, 94g:90005.

Russian transl. of Schrijver (1986a). (**sg: p: I: Exp**)

Michelle Schultz

See G. Chartrand.

W. Schwärzler and D.J.A. Welsh

1993a Knots, matroids and the Ising model. *Math. Proc. Cambridge Philos. Soc.* 13 (1993), 107–139. MR 94c:57019. Zbl. 797.57002.

Tutte and dichromatic polynomials of signed matroids, generalized from Kauffman (1989a); this is the 2-colored case of Zaslavsky's (1992b) strong Tutte functions of colored matroids. [For terminology see Zaslavsky 1992b.] Applications to knot theory.

§2, "A matroid polynomial", is foundational. Prop. 2.1 characterizes strong Tutte functions of signed matroids by two equations connecting their parameters and their values on signed coloops and loops. [If the function is 0 on positive coloops, the proof is incomplete and the functions = 0 except on $M = \emptyset$ are missed.] Prop. 2.2: The Tutte (basis-expansion) polynomial of a function W of signed matroids is well defined iff W is a strong Tutte function. Eq. (2.8) says $W =$ the rank generating polynomial Q_Σ (here also called W) if certain variables are nonzero; (2.9) shows there are only 3 essential variables since, generically, only the ratio of parameters is essential [an observation that applies to general strong Tutte functions]. Prop. 2.5 computes Q_Σ of a 2-sum.

§3 adapts Q_Σ to Kauffman's and Murasugi's (1989a) signed-graph polynomials and simplifies some of the latter's results (esp. his chromatic degree). §4, "The anisotropic Ising model", concerns the Hamiltonian of a state of a signed graph. The partition function is essentially an evaluation of Q_Σ. §5, "The bracket polynomial", and §6, "The span of the bracket polynomial": Certain substitutions reduce Q_Σ to 1 variable; its properties are examined, esp. in light of knot-theoretic questions. Thm. 6.4 characterizes signed matroids with "full span" (a degree property). §7, "Adequate and semi-adequate link diagrams", generalizes those notions to signed matroids. §8, "Zero span matroids": when does span(bracket) = 0? Yes if $M = M(\Sigma)$ where Σ reduces by Reidemeister moves to K_1, but the converse is open (and significant if true). (**Sc(M), SGc: N, Knot, Phys**)

Allen J. Schwenk

See Harary, Palmer, Robinson, and Schwenk (1977a).

András Sebö

See also B. Novick.

1990a Undirected distances and the postman-structure of graphs. *J. Combin. Theory*

Ser. B 49 (1990), 10–39. MR 91h:05049. Zbl. 638.05032.
See A. Frank (1996a). (**SGw: Str**)

J.J. Seidel
See also F.C. Bussemaker, P.J. Cameron, P.W.H. Lemmens, and J.H. van Lint.

1968a Strongly regular graphs with $(-1, 1, 0)$ adjacency matrix having eigenvalue 3. *Linear Algebra and Appl.* 1 (1968), 281–298. MR 38 #3175. Zbl. 159, 254 (e: 159.25403). Reprinted in Seidel (1991a), pp. 26–43. (**tg**)

1969a Strongly regular graphs. In: W.T. Tutte, ed., *Recent Progress in Combinatorics* (Proc. Third Waterloo Conf. on Combinatorics, 1968), pp. 185–198. Academic Press, New York, 1969. MR 54 #10047. Zbl. 191, 552 (e: 191.55202). (**TG**)

1974a Graphs and two-graphs. In: F. Hoffman *et al.*, eds., *Proceedings of the Fifth Southeastern Conference on Combinatorics, Graph Theory, and Computing* (Boca Raton, 1974), pp. 125–143. Congressus Numerantium X. Utilitas Math. Publ. Inc., Winnipeg, Man., 1974. MR 51 #283. Zbl. 308.05120. (**TG**)

†1976a A survey of two-graphs. In: *Colloquio Internazionale sulle Teorie Combinatorie* (Rome, 1973), Tomo I, pp. 481–511. Atti dei Convegni Lincei, No. 17. Accad. Naz. Lincei, Rome, 1976. MR 58 #27659. Zbl. 352.05016. Reprinted in Seidel (1991a), pp. 146–176. (**TG: A, Cov, Aut**)

1978a Eutactic stars. In: A. Hajnal and Vera T. Sós, eds., *Combinatorics* (Proc. Fifth Hungar. Colloq., Keszthely, 1976), Vol. 2, pp. 983–999. Colloq. Math. Soc. János Bolyai, 18. North-Holland, Amsterdam, 1978. MR 80d:05016. Zbl. 391.05050.

1979a The pentagon. In: Allan Gewirtz and Louis V. Quintas, eds., Second International Conference on Combinatorial Mathematics (New York, 1978). *Ann. New York Acad. Sci.* 319 (1979), 497–507. MR 81e:05004. Zbl. 417.51005. (**TG: A**)

1979b The pentagon. In: P.C. Baayen *et al.*, eds., *Proceedings, Bicentennial Congress, Wiskundig Genootschaap* (Amsterdam, 1978), Part I, pp. 80–96. Mathematical Center Tracts, 100. Mathematisch Centrum, Amsterdam, 1979. MR 80f:51008. Zbl. 417.51005.
Same as (1979a), with photograph. (**TG: A**)

1991a *Geometry and Combinatorics: Selected Works of J.J. Seidel.* D.G. Corneil and R. Mathon, eds. Academic Press, Boston, 1991. MR 92m:01098. Zbl. 770.05001.
Reprints many articles on two-graphs and related systems. (**TG: Sw, G**)

1992a More about two-graphs. In: Jaroslav Nešetřil and Miroslav Fiedler, eds., *Fourth Czechoslovakian Symposium on Combinatorics, Graphs and Complexity* (Prachatice, 1990), pp. 297–308. Ann. Discrete Math., Vol. 51. North-Holland, Amsterdam, 1992. MR 94h:05040. Zbl. 764.05036. (**TG: Exp, Ref**)

1995a Geometric representations of graphs. *Linear Multilinear Algebra* 39 (1995), 45–57. MR 97e:05149a. Zbl. 832.05079. Errata. *Linear Multilinear Algebra* 39 (1995), 405. MR 97e:05149b. Zbl. 843.05078.
See §4. (**SG: A, G: Exp**)

1995b Discrete non-Euclidean geometry. In: F. Buekenhout, ed., *Handbook of Incidence Geometry: Buildings and Foundations*, Ch. 15, pp. 843–920. North-Holland (Elsevier), Amsterdam, 1995. MR 96m:52001. Zbl. 826.51012.
§3.2: "Equidistant sets in elliptic $(d-1)$-space." §3.3: "Regular two-graphs." (**TG: A, G: Exp**)

J.J. Seidel and D.E. Taylor

1981a Two-graphs, a second survey. In: L. Lovász and Vera T. Sós, eds., *Algebraic Methods in Graph Theory* (Proc. Internat. Colloq., Szeged, 1978), Vol. II, pp. 689–711. Colloq. Math. Soc. János Bolyai, 25. János Bolyai Math. Soc., Budapest, and North-Holland, Amsterdam, 1981. MR 83f:05070. Zbl. 475.05073. Reprinted in Seidel (1991a), pp. 231–254. **(TG)**

J.J. Seidel and S.V. Tsaranov

1990a Two-graphs, related groups, and root systems. Algebra, Groups and Geometry. *Bull. Soc. Math. Belg. Ser. A* 42 (1990), 695–711. MR 95m:20046. Zbl. 736.05048.

A group $\mathrm{Ts}(\Sigma)$ is defined from a signed complete graph Σ: its generators are the vertices and its relations are $(uv^{-\sigma(uv)})^2 = 1$ for each edge uv. It is invariant under switching, hence determined by the two-graph of Σ. A certain subgraph of a Coxeter group of a tree T is isomorphic to $\mathrm{Ts}(\Sigma)$ for suitable Σ_T constructed from T. [Generalized in Cameron, Seidel, and Tsaranov (1994a). More on Σ_T under Tsaranov (1992a). The construction of Σ_T is simplified in Cameron (1994a).] **(TG: A, G)**

Charles Semple and Geoff Whittle

1996a Partial fields and matroid representation. *Adv. Appl. Math.* 17 (1996), 184–208. MR 97g:05046. Zbl. 859.05035.

§7: "Dowling group geometries". A Dowling geometry of a group $\mathfrak{G}$ has a partial-field representation iff G is abelian and has at most one involution. **(gg: M: I)**

E.C. Sewell

1996a Binary integer programs with two variables per inequality. *Math. Programming* 75 (1996), Ser. A, 467–476. MR 97m:90059 . Zbl. 874.90138.

See Johnson and Padberg (1982a) for definitions. §2, "Equivalence to stable set problem": Optimization on the bidirected stable set polytope is reduced to optimization on a stable set polytope with no more variables. Results of Bourjolly (1988a) and Hochbaum, Megiddo, Naor, and Tamir (1993a) can thereby be explained. §3, "Perfect bigraphs", proves the conjectures of Johnson and Padberg (1982a): a transitively closed bidirection of a simple graph is perfect iff its underlying graph is perfect. [Also proved by Ikebe and Tamura (20xxa).] Dictionary: "Bigraph" = bidirected graph B. "Stable" set in B = vertex set inducing no introverted edge. **(SG: O: I, G, sw)**

P.D. Seymour

See also Gerards, Lovász, *et al.* (1990a), W. McCuaig, and N. Robertson.

1974a On the two-colouring of hypergraphs. *Quart. J. Math. Oxford* (2) 25 (1974), 303–312. MR 51 #7927. Zbl. 299.05122. **(sd: P: b)**

1977a The matroids with the max-flow min-cut property. *J. Combin. Theory Ser. B* 23 (1977), 189–222. MR 57 #2960. Zbl. 375.05022.

The central example is $Q_6 = \mathcal{C}_-(-K_4)$, the clutter of (edge sets of) negative polygons in $-K_4$. P. 199: the extended lift matroid $L_0(-K_4) = F_7^*$, the dual Fano matroid. Result (3.4) readily generalizes (by the negative-subdivision trick) to: every $\mathcal{C}_-(\Sigma)$ is a binary clutter, that is, a port of a binary matroid. [This is also immediate from the construction of $L_0(\Sigma)$.]

P. 200, (i)–(iii): amongst minor-minimal binary clutters without the "weak MFMC property" are the circuit clutter of F_7^* and $\mathcal{C}(-K_5)$ and its blocker.

Main Thm. (§5): A binary clutter is "Mengerian" (I omit the definition) iff it does not have $\mathcal{C}_{-}(-K_4)$ as a minor. (See p. 200 for the antecedent theorem of Gallai.)

[See Cornuéjols (20xxa), Guenin (1998b) for more.] **(sg, P: M, G)**

1981a Matroids and multicommodity flows. *European J. Combin.* 2 (1981), 257–290. MR 82m:05030. Zbl. 479.05023.

Conjecture (based on (1977a)). A binary clutter has the weak MFMC property iff no minor is either the circuit clutter of F_7 or $\mathcal{C}_{-}(-K_5)$ or its blocker. **(s(m), sg: M)**

†1995a Matroid minors. In: R.L. Graham, M. Grötschel, and L. Lovász, eds., *Handbook of Combinatorics*, Vol. I, Ch. 10, pp. 527–550. North-Holland (Elsevier), Amsterdam, and MIT Press, Cambridge, Mass., 1995. MR 97a:05055. Zbl. 960.24825.

In Thm. 6.6, p. 546, interpreting G as a signed graph and an "odd-K_4" as a subdivision of $-K_4$ gives the signed graph generalization, due to Gerards and Schrijver (1986a) [also Gerards (1990a), Thm. 3.2.3]. Let Σ be a signed simple, 3-connected graph in which no 3-separation has > 4 edges on both sides. Then Σ has no $-K_4$ minor iff either (i) deleting some vertex makes it balanced (the complete lift matroid of this type is graphic); or (ii) it is cylindrical: it can be drawn on a cylindrical surface that has a lengthwise red line so that an edge is negative iff it crosses the red line an odd number of times [Note: the extended lift matroid of this type is cographic, as observed by, I think, Gerards and Schrijver or by Lovász]. [See Pagano (1998a) for another use of cylindrical signed graphs.] [*Problem.* Find the forbidden topological subgraphs, link minors, and $Y\Delta$ graphs for cylindrical signed graphs.] [*Question.* Embed a signed graph in the plane with k distinguished faces so that a polygon's sign is the parity of the number of distinguished faces it surrounds. Cylindrical embedding is $k = 1$. For each k, which signed graphs are so embeddable?] **(SG: Str, T)**

Thm. 6.7, pp. 546–547, generalizes to signed graphs, interpreting G as a signed graph and an "odd cycle" as a negative polygon. Take a signed simple, 3-connected, internally 4-connected graph. It has no two vertex-disjoint negative polygons iff it is one of four types: (i) deleting some vertex makes it balanced; (ii) deleting the edges of an unbalanced triangle makes it balanced; (iii) it has order ≤ 5; (iv) it can be orientation-embedded in the projective plane. This is due to Lovász; see, if you can, Gerards *et al.* (1990a). [A 2-connected Σ has no vertex-disjoint negative polygons iff $G(\Sigma)$ is binary iff $G(\Sigma)$ is regular iff the lift matroid $L(\Sigma)$ is regular. See Pagano (1998a) for classification of Σ with vertex-disjoint negative polygons according to representability of the bias matroid.] **(SG: Str, m, T)**

Paul Seymour and Carsten Thomassen

1987a Characterization of even directed graphs. *J. Combin. Theory Ser. B* 42 (1987), 36–45. MR 88c:05089. Zbl. 607.05037.

"Even" means every signing contains a positive cycle. A digraph is even iff it contains a subdigraph that is obtained from a symmetric odd-polygon digraph by subdivision and a vertex-splitting operation. [Cf. Thomassen (1985a).] **(sd: p: Str)**

L. de Sèze

See J. Vannimenus.

Bryan L. Shader
See Richard A. Brualdi.

Jia-Yu Shao
See R. Manber.

John Shawe-Taylor
See T. Pisanski.

F.B. Shepherd
See A.M.H. Gerards.

Ronald G. Sherwin

1975a Structural balance and the sociomatrix: Finding triadic valence structures in signed adjacency matrices. *Human Relations* 28 (1975), 175–189.

A very simple [but not efficient] matrix algorithm for counting different types of polygons in a signed (di)graph. ["Valence" means sign, unfortunately.] (**sg, SD: B: Alg**)

Jeng-Horng Sheu
See I. Gutman.

Elizabeth G. Shrader and David W. Lewit

1962a Structural factors in cognitive balancing behavior. *Human Relations* 15 (a962), 265–276.

For $\Gamma \subset K_n$ and signing σ of Γ, "plausibility" = mean and "differentiability" = standard deviation of $f(K_n, \sigma')$ over all extensions of σ to K_n, where f is any function that measures degree of balance. Proposed: tendency toward balance is high when plausibility and differentiability are high. A specific f, based on triangles and quite complicated, is studied for $n = 4$, with experiments. (**sg, fr, PsS**)

Alan Shuchat
See R. Shull.

Randy Shull, James B. Orlin, Alan Shuchat, and Marianne L. Gardner

1989a The structure of bases in bicircular matroids. *Discrete Appl. Math.* 23 (1989), 267–283. MR 90h:05040. Zbl. 698.05022.

[See Coullard, del Greco, and Wagner (1991a).] (**Bic(Bases)**)

Randy Shull, Alan Shuchat, James B. Orlin, and Marianne Lepp

1993a Recognizing hidden bicircular networks. *Discrete Appl. Math.* 41 (1993), 13–53. MR 94e:90122. Zbl. 781.90089. (**GN: Bic: I, Alg**)

1997a Arc weighting in hidden bicircular networks. Proc. Twenty-eighth Southeastern Internat. Conf. on Combinatorics, Graph Theory and Computing (Boca Raton, Fla., 1997). *Congressus Numer.* 125 (1997), 161–171. MR 98m:05181. Zbl. 902.90157. (**GN: Bic: I, Alg**)

E.E. Shult
See P.J. Cameron.

B. Simeone
See C. Benzaken, J.-M. Bourjolly, P.L. Hammer, and P. Hansen.

Slobodan K. Simić
See also D.M. Cvetković.

1980a Graphs which are switching equivalent to their complementary line graphs I. *Publ. Inst. Math. (Beograd) (N.S.)* 27 (41) (1980), 229–235. MR 82m:05077. Zbl. 531.05050. (**TG: LG**)

1982a Graphs which are switching equivalent to their complementary line graphs II. *Publ. Inst. Math. (Beograd) (N.S.)* 31 (45) (1982), 183–194. MR 85d:05207. Zbl. 531.05051. (**TG: LG**)

R. Simion and D.-S. Cao

1989a Solution to a problem of C. D. Godsil regarding bipartite graphs with unique perfect matching. *Combinatorica* 9 (1989), 85–89. MR 90f:05113. Zbl. 688.05056.

Answering Godsil (1985a): $|\Sigma| = \Gamma$ iff Γ consists of a bipartite graph with a pendant edge attached to every vertex. [Surely there is a signed-graphic generalization of Godsil's and this theorem in which bipartiteness becomes balance or something like it.] (**sg: A, b**)

J.M.S. Simões-Pereira

1972a On subgraphs as matroid cells. *Math. Z.* 127 (1972), 315–322. MR 47 #6522. Zbl. 226.05016, (243.05022).

"Cell" = circuit. Along with Klee (1971a), invents the bicircular matroid (here, for finite graphs) (Thm. 1). Suppose we have matroids on the edge sets of all [simple] graphs, such that the class of circuits is a [nonempty] union of homeomorphism classes of connected graphs. Thm. 2: The polygon and bicircular matroids [and free matroids] are the only such matroids. (**Bic**)

1973a On matroids on edge sets of graphs with connected subgraphs as circuits. *Proc. Amer. Math. Soc.* 38 (1973), 503–506. MR 47 #3214. Zbl. 241.05114, 264.05126.

A family of (isomorphism types of) [simple] connected graphs is "matroidal" if for any Γ the class of subgraphs of Γ that are in the family constitute the circuits of a matroid on $E(\Gamma)$. Bicircular and even-cycle matroids are the two nicest examples. A referee contributes the even-cycle matroid [cf. Doob (1973a)]. Thm.: The family cannot be finite [unless it is void or consists of K_2]. [See Marcu (1987a) for a valuable new viewpoint.] (**Bic, EC, Gen**)

1975a On matroids on edge sets of graphs with connected subgraphs as circuits II. *Discrete Math.* 12 (1975), 55–78. MR 54 #7298. Zbl. 307.05129.

Partial results on describing matroidal families of simple, connected graphs. Five basic types: free [omitted in the paper], cofree, polygon, bicircular, and even-cycle. If the family does not correspond to one of these, then every member has ≥ 3 independent polygons and minimum degree ≥ 3. (**Bic, EC: Gen**)

1978a A comment on matroidal families. In: *Problèmes Combinatoires et Théorie des Graphes* (Colloq. Internat., Orsay, 1976), pp. 385–387. Colloques Internat. du CNRS, 260. Editions du C.N.R.S., Paris, 1978. MR 81b:05031. Zbl. 412.05023.

Two small additions to (1973a, 1975a); one is that a matroidal family not one of the five basic types must contain $K_{p,q(p)}$ for each $m \geq 3$, with $q(p) \geq p$. (**Bic, EC: Gen**)

1992a Matroidal families of graphs. In: Neil White, ed., *Matroid Applications*, Ch. 4, pp. 91–105. Encycl. Math. Appl., Vol. 40. Cambridge Univ. Press, Cambridge, Eng., 1992. MR 93c:05036. Zbl. 768.05024.

"Count" matroids (see N. White (1996a)) in §4.3; Schmidt's (1979a) remarkable generalization in §4.4. (**GG: M, Bic, EC: Gen: Exp, Exr, Ref**)

Klaus Simon
See T. Raschle.

M. Simonovits
See B. Bollobás, J.A. Bondy, and P. Erdős.

N.M. Singhi
See also S.B. Rao, D.K. Ray-Chaudhuri, and G.R. Vijayakumar.

N.M. Singhi and G.R. Vijayakumar

1992a Signed graphs with least eigenvalue < -2. *European J. Combin.* 13 (1992), 219–220. MR 93e:05069. Zbl. 769.05065. (**SG: A**)

Jozef Širáň
See also D. Archdeacon.

1991a Characterization of signed graphs which are cellularly embeddable in no more than one surface. *Discrete Math.* 94 (1991), 39–44. MR 92i:05086. Zbl. 742.05035.

A signed graph orientation-embeds in only one surface iff any two polygons are vertex disjoint. (**SG: T**)

1991b Duke's theorem does not extend to signed graph embeddings. *Discrete Math.* 94 (1991), 233–238. MR 92j:05065. Zbl. 742.05036.

Richard A. Duke (The genus, regional number, and Betti number of a graph. *Canad. J. Math.* 18 (1966), 817–822. MR 33 #4917.) proved that the (orientable) genus range of a graph forms a contiguous set of integers. Stahl (1978a) proved the analog for nonorientable embeddings. Širáň shows this need not be the case for the demigenus range of an unbalanced signed graph. However, any gaps consist of a single integer each. The main examples with gaps are vertex amalgamations of balanced and uniquely embeddable unbalanced signed graphs, but a 3-connected example is $+W_6$ together with the negative diameters of the rim. *Question* 1 (Širáň). Do all gaps occur at the bottom of the demigenus range? [*Question* 2. Can one in some way derive almost all signed graphs with gaps from balanced ones?] (**SG: T**)

Jozef Širáň and Martin Škoviera

††1991a Characterization of the maximum genus of a signed graph. *J. Combin. Theory Ser. B* 52 (1991), 124–146. MR 92b:05033. Zbl. 742.05037.

The maximum demigenus $d_M(\Sigma)$ = the largest demigenus of a closed surface in which Σ orientation embeds. Two formulas are proved for $d_M(\Sigma)$: one a minimum and the other a maximum of readily computable numbers. Thus $d_M(\Sigma)$ has a "good" (polynomial) characterization. Along the way, several results are proved about single-face embeddings. *Problem* (§11). Characterize those edge-2-connected Σ such that Σ and all $\Sigma \backslash e$ have single-face embeddings. [A complex and lovely paper.] (**SG: T**)

A. Skhreĭver [A. Schrijver]
See A. Schrijver.

Martin Škoviera
See also R. Nedela and J. Širáň.

1983a Equivalence and regularity of coverings generated by voltage graphs. In: Miroslav Fiedler, ed., *Graphs and Other Combinatorial Topics* (Proc. Third Czechoslovak Sympos. on Graph Theory, Prague, 1982), pp. 269–272. Teubner-Texte Math., 59. Teubner, Leipzig, 1983. MR 85e:05064. Zbl. 536.05019. (**GG: T, Cov, Sw**)

1986a A contribution to the theory of voltage graphs. *Discrete Math.* 61 (1986), 281–292. MR 88a:05060. Zbl. 594.05029.

Automorphisms of covering projections of canonical covering graphs of gain graphs. **(GG: T, Cov, Aut, Sw)**

1992a Random signed graphs with an application to topological graph theory. In: Alan Frieze and Tomasz Luczak, eds., *Random Graphs, Vol. 2* (Proc., Poznań, 1989), Ch. 17, pp. 237–246. Wiley, New York, 1992. MR 93g:05126. Zbl. 817.05059.

The model: each edge is selected with probability p, positive with probability s. Under mild hypotheses on p and s, Σ is almost surely unbalanced and almost surely has a 1-face orientation embedding. **(SG: Rand, E, T)**

N.J.A. Sloane

See P.C. Fishburn, R.L. Graham, and C.L. Mallows.

J. Laurie Snell

See J. Berger and J.G. Kemeny.

Patrick Solé and Thomas Zaslavsky

1994a A coding approach to signed graphs. *SIAM J. Discrete Math.* 7 (1994), 544–553. MR 95k:94041. Zbl. 811.05034.

Among other things, improves some results in Akiyama, Avis, Chvátal, and Era (1981a). Thm. 1: For a loopless graph with c components, $D(\Gamma) \geq \frac{1}{2}m - \sqrt{\frac{1}{2}\ln 2}\sqrt{m(n-c)}$. Thm. 2: For a simple, bipartite graph, $D(\Gamma) \leq \frac{1}{2}(m - \sqrt{m})$. *Conjecture.* The best general asymptotic lower bound is $D(\Gamma) \geq \frac{1}{2}m - c_1\sqrt{mn} + o(\sqrt{mn})$ where c_1 is some constant between $\sqrt{\frac{1}{2}\ln 2}$ and $\frac{1}{2}\pi$. *Question.* What is c_1 for, e.g., k-connected graphs? Thm. 4 gives girth-based upper bounds on $D(\Gamma)$. §5, "Embedded graphs", has bounds for several examples obtained by surface duality. All proofs are via covering radius of the cutset code of Γ. **(SG: Fr, T)**

Louis Solomon

See P. Orlik.

Tadeusz Sozański

1976a Processus d'équilibration et sous-graphes équilibrés d'un graphe signé complet. *Math. Sci. Humaines*, No. 55 (1976), 25–36, 83. MR 58 #27613.

Σ denotes a signed K_n. The "level of balance" ("indice du niveau d'équilibre") $\rho(\Sigma)$:= maximum order of a balanced subgraph. [Complement of the vertex deletion number.] Define distance $d(\Sigma_1, \Sigma_2) := |E_{1+} \triangle E_{2+}|$. Say Σ is p-clusterable if Σ_+ consists of p disjoint cliques [its "clusters"]. Thm. 1 evaluates the frustration index of a p-clusterable Σ. Thm. 2 bounds $l(\Sigma)$ in terms of n and $\rho(\Sigma)$. A negation set U for Σ "conserves" a balanced induced subgraph if they are edge-disjoint; it is "(strongly) conservative" if it conserves some (resp., every) maximum-order balanced induced subgraph. Thm. 3: Every minimum negation set conserves every balanced induced subgraph of order $> \frac{2}{3}n$. Thm. 4: A minimum negation set can be ordered so that, successively negating its edges one by one, ρ never decreases. **(SG: K: Fr, Cl)**

1980a Enumeration of weak isomorphism classes of signed graphs. *J. Graph Theory* 4 (1980), 127–144. MR 81g:05070. Zbl. 434.05059.

"Weak isomorphism" = switching isomorphism. Principal results: The number of switching nonisomorphic signed K_n's. (Cf. Mallows and Sloane (1975a).) The number that are switching isomorphic to their negations. The number of nonisomorphic (not switching nonisomorphic!) balanced signings of a given graph. **(SG, K: Sw: E)**

1982a Model rownowagi strukturalnej. Teoria grafow oznakowanych i jej zastosowania w naukach spotecznych. [The structural balance model. The theory of signed graphs and its applications in the social sciences.] (In Polish.) Ph.D. thesis, Jagellonian Univ., Krakow, 1982. **(SG, PsS: B, Fr, Cl, Aut, A, Ref)**

Joel Spencer

See T.A. Brown.

Murali K. Srinivasan

1998a Boolean packings in Dowling geometries. *European J. Combin.* 19 (1998), 727–731.

Decomposes the Dowling lattice $Q_n(\mathfrak{G})$ into Boolean algebras, indexed in part by integer compositions, that are cover-preserving and centered above the middle rank. **(GG: M)**

Saul Stahl

1978a Generalized embedding schemes. *J. Graph Theory* 2 (1978), 41–52. MR 58 #5318. Zbl. 396.05013.

A generalized embedding scheme for a graph is identical to a rotation system for a signing of the graph. Thm. 2: Signed rotation systems describe all cellular embeddings of a graph. Thm. 4: Embeddings are homeomorphic iff their signed rotation systems are switching equivalent. Thm. 5: An embedding is orientable iff its signature is balanced. Compare Ringel (1977a). Dictionary: λ is the signature. "λ-trivial" means balanced. **(sg: T, Sw)**

1978b The embeddings of a graph—a survey. *J. Graph Theory* 2 (1978), 275–298. MR 80a:05085. Zbl. 406.05027. **(sg: T)**

Richard P. Stanley

See also P. Doubilet and A. Postnikov.

1985a Reconstruction from vertex-switching. *J. Combin. Theory Ser. B* 38 (1985), 132–138. MR 86f:05096. Zbl. 572.05046.

From the 1-vertex switching deck (the multiset of isomorphism types of signed graphs resulting by separately switching each vertex) of $\Sigma = (K_n, \sigma)$, Σ can be reconstructed, provided that $4 \nmid n$. The same for i-vertex switchings, provided that the Krawtchouk polynomial $K_i^n(x)$ has no even zeros from 0 to n. When $i = 1$, the negative-subgraph degree sequence is always reconstructible. All done in terms of Seidel (graph) switching of unsigned simple graphs. [See Ellingham; Ellingham and Royle; Krasikov; Krasikov and Roditty for further developments. *Problem* 1. Generalize to signings of other highly symmetric graphs. *Problem* 2. Prove a similar theorem for switching of a bidirected K_n.] **(k: sw, TG)**

1986a *Enumerative Combinatorics, Vol. I.* Wadsworth and Brooks/Cole, Monterey, Cal., 1986. MR 87j:05003. Zbl. 608.05001.

Ch. 3, "Partially ordered sets": Exercise 51, pp. 165 and 191, concerns the Dowling lattices of a group and mentions Zaslavsky's generalizations [signed and biased graphs]. **(GG: M, N: Exr, Exp)**

1990a (As "R. Stenli") *Perechislitel'naya kombinatorika.* "Mir", Moscow, 1990. MR

91m:05002.

Russian translation of Stanley (1986). (**GG: M, N: Exr, Exp**)

1991a A zonotope associated with graphical degree sequences. In: Peter Gritzmann and Bernd Sturmfels, eds., *Applied Geometry and Discrete Mathematics: The Victor Klee Festschrift*, pp. 555–570. DIMACS Ser. Discrete Math. and Theoret. Computer Sci., Vol. 4. American Mathematical Soc. and Assoc. for Computing Machinery, Providence and Baltimore, 1991. MR 92k:52020. Zbl. 737.05057.

All-negative complete graphs (implicit in §3) and signed colorings (§4) are used to find the number of ordered degree sequences of n-vertex graphs and to study their convex hull. (**SG: G, Col**)

1996a Hyperplane arrangements, interval orders, and trees. *Proc. Nat. Acad. Sci. USA* 93 (1996), 2620–2625. MR 97i:52013. Zbl. 848.05005.

Deformed braid hyperplane arrangements, i.e., hyperplane representations of $\mathrm{Lat^b}\,(K_n,\varphi)$ with gains $\varphi(ij)=l_i\in\mathbb{Z}$ where $i<j$. ($\mathrm{Lat^b}$ denotes the geometric semilattice of balanced flats of the bias or lift matroid. Write φ_l if all $l_i=l$.) In particular (§4), $\varphi=\varphi_1$. Also (§5), the "Shi" arrangement, which represents $\mathrm{Lat^b}\,\Phi$ where $\Phi=(K_n,\varphi_0)\cup(K_n,\varphi_1)$.

(**gg: G, M, N: Exp**)

1997a *Enumerative Combinatorics, Vol. I.* Corrected [and enlarged] reprint. Cambridge Stud. Adv. Math., Vol. 49. Cambridge University Press, Cambridge, Eng., 1997. MR 98a:05001. Zbl. 970.29805.

Additional exercises and some updating. (**GG: M, N: Exr, Exp**)

1998a Hyperplane arrangements, parking functions and tree inversions. In: B.E. Sagan and R. Stanley, eds., *Mathematical Essays in Honor of Gian-Carlo Rota*, Progress in Math., Vol. 161, pp. 359–375. Birkhäuser, Boston, 1998. MR 99f:05006. Zbl. 980.39546. (**gg: G, M, N: Exp**)

Kenneth Steiglitz

See C.H. Papadimitriou.

R. Stenli [Richard P. Stanley]

See R.P. Stanley.

Daniel L. Stern

1989a Spin glasses. *Scientific American*, July 1989, 52–59.

Informally describes frustration in spin glasses in terms of randomly ferromagnetic and antiferromagnetic interactions (see Toulouse (1977a)) and gives some history and applications. (**Phys:: sg: b, Rand: Exp**)

B.M. Stewart

1966a Magic graphs. *Canad. J. Math.* 18 (1966), 1031–1059. MR 33 #5523. Zbl. 149, 214 (e: 149.21401). (**ec: I**)

Allen H. Stix

1974a An improved measure of structural balance. *Human Relations* 27 (1974), 439–455. (**SG: Fr**)

J. Randolph Stonesifer

1975a Logarithmic concavity for a class of geometric lattices. *J. Combin. Theory Ser. A* 18 (1975), 216–218. MR 50 #9637. Zbl. 312.05019.

The second kind of Whitney numbers of a Dowling lattice are binomially concave, hence strongly logarithmically concave, hence unimodal. [*Famous Problem* (Rota). Generalize this.] (**gg: M: N**)

Bernd Sturmfels
See A. Björner.

J. Stutz
See F. Glover.

Benjamin Sudakov
See G. Gutin.

Janusz Szczypula
See P. Doreian.

E. Szemerédi
See B. Bollobás.

Z. Szigeti
See A.A. Ageev.

Irving Tallman

1967a The balance principle and normative discrepancy. *Human Relations* 20 (1967), 341–355. **(PsS: ECol)**

Arie Tamir
See also D. Hochbaum.

1976a On totally unimodular matrices. *Networks* 6 (1976), 373–382. MR 57 #12553. Zbl. 356.15020. **(SD: B)**

Akihisa Tamura
See also Y.T. Ikebe and D. Nakamura.

1997a The generalized stable set problem for perfect bidirected graphs. *J. Oper. Res. Soc. Japan* 40 (1997), 401–414. MR 99e:05063. Zbl. 894.90156.

The problem: maximize an integral weight function over the bidirected stable set polytope (cf. Johnson and Padberg 1982a). §3 concerns the effect on perfection of deleting all incoming edges at a vertex. §4 reduces the "generalized stable set problem" for bidirected graphs to the maximum weighted stable set problem for ordinary graphs, whence the problem for perfect bidirected graphs is solvable in polynomial time. **(sg: O:I, G, Sw, Alg)**

20xxa Perfect $(0, \pm 1)$-matrices and perfect bidirected graphs. Submitted. **(sg: O: G, Alg)**

Percy H. Tannenbaum
See C.E. Osgood.

Èva Tardos and Kevin D. Wayne

1998a Simple generalized maximum flow algorithms. In: Robert E. Bixby, E. Andrew Boyd, and Roger Z. Ríos-Mercado, eds., *Integer Programming and Combinatorial Optimization* (6th Internat. IPCO Conf., Houston, 1998, Proc.), pp. 310–324. Lecture Notes in Computer Sci., Vol. 1412. Springer, Berlin, 1998. Zbl. 911.90156.

Max. flow in a network with positive rational gains. Multiple sources and sinks are allowed. "Relabeling" is switching the gains. Useful references to previous work. **(GN: Sw, Alg, Ref)**

Michael Tarsi
See F. Jaeger.

D.E. Taylor
See also J.J. Seidel.

1977a Regular 2-graphs. *Proc. Lond. Math. Soc.* (3) 35 (1977), 257–274. MR 57 #16147. Zbl. 362.05065.

Introducing two-graphs and regular two-graphs (defined by G. Higman, unpublished). [See Seidel (1976a) etc. for more.] A "two-graph" is the class $\mathcal{C}_{3-}$ of negative triangles of a signed complete graph (K_n, σ). (See §2. p. 258, where the group is $\mathbb{Z}_2 \cong \{+,-\}$ and the definition is in terms of the 2-coboundary operator.) Two-graphs and switching classes of signed complete graphs are equivalent concepts. (Stated in terms of Seidel switching in §2, p. 260.) A two-graph is "regular" if every edge lies in the same number of negative triangles. Thm.: $\mathcal{C}_{3-}$ is regular iff $A(K_n, \sigma)$ has at most two eigenvalues. Various parameters of regular two-graphs are calculated.

(**TG: A. G**)

Herbert Taylor

See P. Erdős.

Howard F. Taylor

1970a *Balance in Small Groups.* Van Nostrand Reinhold, New York, 1970.

A thorough and pleasantly written survey of psychological theories of balance, including formalizations by signed graphs (Chs. 3 and 6), experimental tests and critical evaluation of the formalisms, and so forth. Ch. 2: "Substantive models of balance", takes the perspective of social psychology. §2.2: "Varieties of balance theory", reviews the theories of Heider (1946a) (the source of Harary's (1953a) invention of signed graphs), Osgood and Tannenbaum (1955a), and others. §2.2e: "The Rosenberg-Abelson modifications", discusses their introduction of the "cost" of change of relations, which led them (Abelson and Rosenberg 1958a) to propose the frustration index as a measure of imbalance. (**PsS, SG, WG: Exp, Ref**)

Ch. 3: "Formal models of balance", reviews various graph-theoretic models: signed and weighted signed, different ways to weigh imbalance, etc., the relationship to theories in social psychology being constantly kept in mind. §3.1: "Graph theory and balance theory", presents the basics of balance, measures of degree of balance by polygons (Cartwright and Harary (1956a)), polygons with strengths of edges (Morrissette (1958a)), local balance and N-balance (Harary (1955a)), edge deletion and negation (Abelson and Rosenberg (1958a), Harary (1959b)), vertex elimination number (Harary (1959b)). §3.2: "Evaluation of formalizations: strong points", and §3.3: "Evaluation of formalizations: weak points", judged from the applied standpoint. §3.3a: "Discrepancies between cycles or subsets of cycles", suggests that differing degrees of imbalance among certain different subsets of the vertices may be significant [Is this reasonable?] and proposes measures, e.g., a variance measure (p. 71), of this "discrepancy". (**SG, WG: B, Fr: Exp**)

Ch. 6: "Issues involving formalization", goes into more detail. §6.1: "Indices of balance", compares five indices, in particular Phillips' (1967a) eigenvalue index (also in Abelson (1967a)) with examples to show that the index differentiates among different balanced signings of the same graph. §6.2: "Extra-balance properties", discusses Davis's (1967a) clustering (§6.2b) and indices of clustering (§6.2c). §6.3: "The problem of cycle length and non-local cycles". Are long polygons less important? Do polygons at a distance from an actor (that is, a vertex) have less effect on the actor in balancing processes?

(**SG: Fr, A: Exp**)

Hidetaka Terasaka
See S. Kinoshita.

Morwen B. Thistlethwaite

1988a On the Kauffman polynomial of an adequate link. *Invent. Math.* 93 (1988), 285–296. MR 89g:57009. Zbl. 645.57007.

A 1-variable Tutte-style polynomial Γ_Σ of a sign-colored graph. Fix an edge ordering. For each spanning tree T and edge e, let $\mu_T(e) = -A^{3\tau_T(e)\sigma(e)}$ if e is active with respect to T, $A^{\tau_T(e)\sigma(e)}$ if it is inactive, where $\tau_T(e) = +1$ if $e \in T$, -1 if $e \notin T$. Then $\Gamma_\Sigma(A) = \sum_T \prod_{e \in T} \mu_T(e)$. [In the notation of Zaslavsky (1992a), $\Gamma_\Sigma(A) = Q_\Sigma$ with $a_\epsilon = A^{-\epsilon}, b_\epsilon = A^\epsilon$ for $\epsilon = \pm 1$ and $u = v = -(A^2 + A^{-2})$.] §§3 and 4 show Γ_Σ is independent of the ordering. Other sections derive consequences for knot theory. [This marks the invention of a Tutte-style polynomial of a colored, or parametrized or weighted, graph or matroid, developed in Kauffman (1989a) and successors.] **(SGc: Knot: N)**

A.D. Thomas
See F.W. Clarke.

Robin Thomas
See W. McCuaig and N. Robertson.

Carsten Thomassen
See also Paul Seymour.

1985a Even cycles in directed graphs. *European J. Combin.* 6 (1985), 85–89. MR 86i:05098. Zbl. 606.05039.

It is an NP-complete problem to decide whether a given signed digraph has a positive but not all-positive cycle, even if there are only 2 negative arcs. This follows from Lemma 3 of Steven Fortune, John Hopcroft, and James Wyllie, The directed subgraph homeomorphism problem (*Theoret. Computer Sci.* 10 (1980), 111–121. MR 81e:68079. Zbl. 419.05028.) by the simple argument in the proof of Prop. 2.1 here.

To decide whether a specified arc of a digraph lies in an even cycle, or in an odd cycle, are NP-complete problems (Prop. 2.1). To decide existence of an even cycle [hence, by the negative subdivision trick, of a positive cycle in a signed digraph] is difficult [but is solvable in polynomial time; see Robertson, Seymour, and Thomas (20xxa)], although existence of an odd cycle [resp., of a negative cycle] is easy, by a trick here attributed to Edmonds (unpublished). Prop. 2.2: Deciding existence of a positive cycle in a signed digraph is polynomial-time solvable if $|E_-|$ is bounded. Thm. 3.2: If the outdegrees of a digraph are all $> \log_2 n$, then *every signing* has a positive cycle, and this bound is best possible; restricting to the all-negative signature, the lower bound might (it's not known) go down by a factor of up to 2, but certainly (Thm. 3.1) a constant minimum on outdegree does not imply existence of an even cycle. [See (1992a) for the effect of connectivity.] **(SD, P: B, Alg)**

1986a Sign-nonsingular matrices and even cycles in directed graphs. *Linear Algebra Appl.* 75 (1986), 27–41. MR 87k:05120. Zbl. 589.05050. Erratum, *Linear Algebra Appl.* 240 (1996), 238. **(QM, sd: p: Sol, b, Alg)**

1988a Paths, circuits and subdivisions. In: Lowell W. Beineke and Robin J. Wilson, eds., *Selected Topics in Graph Theory 3*, Ch. 5, pp. 97–131. Academic Press, London, 1988. MR 93h:05003 (book). Zbl. 659.05062.

§8: "Even directed circuits and sign-nonsingular matrices." **(SD, QM: B, Sol: Exp)**

§§8–10 treat even cycles in digraphs. **(SD: B: Exp)**

[*General Problem.* Generalize even-cycle and odd-cycle results to positive and negative cycles in signed digraphs, the unsigned results corresponding to all-negative signatures.]

1989a When the sign pattern of a square matrix determines uniquely the sign pattern of its inverse. *Linear Algebra Appl.* 119 (1989), 27–34. MR 90f:05099. Zbl. 673.05067. **(QM, SD: Sol, A)**

1990a Embeddings of graphs with no short noncontractible cycles. *J. Combin. Theory Ser. B* 48 (1990), 155–177. MR 91b:05069. Zbl. 704.05011.

§5 describes the "fundamental cycle method", a simple algorithm for a shortest unbalanced polygon in a biased graph (Thm. 5.1). Thus the method finds a shortest noncontractible polygon (Thm. 5.2). A noteworthy linear class: the surface-separating ("Π-separating") polygons (p. 166). Dictionary: "3-path-condition" on a class F of polygons = property that F^c is a linear class. "Möbius cycle" = negative polygon in the signature induced by a nonorientable embedding. **(gg, sg: Alg, T)**

1992a The even cycle problem for directed graphs. *J. Amer. Math. Soc.* 5 (1992), 217–229. MR 93b:05064. Zbl. 760.05051.

A digraph that is strongly connected and has all in- and out-degrees ≥ 3 contains an even cycle. **(sd: p: b)**

1993a The even cycle problem for planar digraphs. *J. Algorithms* 15 (1993), 61–75. MR 94d:05077. Zbl. 784.68045.

A polynomial-time algorithm for deciding the existence of an even cycle in a planar digraph. **(sd: p: b: Alg)**

G.L. Thompson

See V. Balachandran.

R.L. Tobin

1975a Minimal complete matchings and negative cycles. *Networks* 5 (1975), 371–387. MR 52 #16578. Zbl. 348.90151.

Bjarne Toft

See T.R. Jensen.

Ioan Tomescu

See also D.R. Popescu.

1973a Note sur une caracterisation des graphes dont le degré de deséquilibre est maximal. *Math. Sci. Humaines*, No. 42 (1973), 37–40. MR 51 #3003. Zbl. 266.05115.

Independent proof of Petersdorf's (1966a) Satz 1. Also, treats similarly a variation on the frustration index. **(SG: Fr)**

1974a La réduction minimale d'un graphe à une réunion de cliques. *Discrete Math.* 10 (1974), 173–179. MR 51 #247. Zbl. 288.05127. **(SG: B, Cl)**

1976a Sur le nombre des cycles négatifs d'un graphe complet signé. *Math. Sci. Humaines*, No. 53 (1976), 63–67. MR 56 #15493. Zbl. 327.05119.

The parity of the number of negative triangles = that of $n|E_-|$. The number of negative t-gons is even when $n,\ t \geq 4$ [strengthened in Popescu (1991a), (1996a)]. **(SG: B)**

1978a Problem 2. In: A. Hajnal and Vera T. Sós, eds., *Combinatorics* (Proc. Fifth Hungarian Colloq., Keszthely, 1976), Vol. 2, p. 1217. Colloq. Math. Soc. János Bolyai, 18. János Bolyai Math. Soc., Budapest, and North-Holland, Amsterdam, 1978. MR 80a:05002 (book). Zbl. 378.00007. (**SG: B**)

C.B. Tompkins

See I. Heller.

J. Topp and W. Ulatowski

1987a On functions which sum to zero on semicycles. *Zastosowanie Mat.* (*Applicationes Math.*) 19 (1987), 611–617. MR 89i:05138. Zbl. 719.05044.

An additive real gain graph is balanced iff every polygon in a polygon basis is balanced, iff the gains are induced by a vertex labelling [in effect, switch to 0], iff every two paths with the same endpoints have the same gains. A digraph is gradable (Harary, Norman, and Cartwright (1965a); also see Marcu (1980a)) iff φ_1 is balanced, where for each arc e, $\varphi_1(e) = 1 \in \mathbb{Z}$ (Thm. 3). The Windy Postman Problem (Thms. 4, 5). (**GG, GD: B**)

Aleksandar Torgašev

See also D.M. Cvetković.

1982a The spectrum of line graphs of some infinite graphs. *Publ. Inst. Math. (Beograd) (N.S.)* 31 (45) (1982), 209–222. MR 85d:05175. Zbl. 526.05039.

An infinite analog of Doob's (1973a) characterization via the even-cycle matroid of when a line graph has -2 as an eigenvalue. [*Problem.* Generalize to line graphs of infinite signed graphs.] (**p: A(LG)**)

1983a A note on infinite generalized line graphs. In: D. Cvetković *et al.*, eds., *Graph Theory* (Proc. Fourth Yugoslav Seminar, Novi Sad, 1983), pp. 291–297. Univ. Novom Sadu, Inst. Mat., Novi Sad, 1984. MR 85i:05168. Zbl. 541.05042.

An infinite graph is a generalized line graph iff its least "limit" eigenvalue ≥ -2. [*Problem.* Generalize to line graphs of infinite signed graphs.] (**p: A(LG)**)

Gérard Toulouse

See also B. Derrida and J. Vannimenus.

1977a Theory of the frustration effect in spin glasses: I. *Commun. Phys.* 2 (1977), 115–119. Reprinted in M. Mézard, G. Parisi, and M.A. Virasoro, *Spin Glass Theory and Beyond*, pp. 99–103. World Scientific Lecture Notes in Physics, Vol. 9. World Scientific, Singapore, 1987.

Introduces the notion of imbalance ("frustration") of a signed graph to account for inherent disorder in an Ising model (here synonymous with a signed graph, usually a lattice graph). (Positive and negative edges are called "ferromagnetic and antiferromagnetic bonds".) Observes that switching the edge signs from all positive (the model of D.D. Mattis, *Phys. Lett.* 56A (1976), 421–?) makes no essential difference. In a planar lattice [or any plane graph] frustration of face boundaries ("plaquettes") can be thought of as curvature, i.e., failure of flatness. Proposes two kinds of asymptotic behavior of frustration as a polygon encloses more plaquettes. The planar-duality approach for finding the states with minimum frustration (i.e., switchings with fewest negative edges); the number of such states is the "ground-state degeneracy" and is important. Ideas are sketched; no proofs. (**SG: Phys, Sw, B**)

1979a Symmetry and topology concepts for spin glasses and other glasses. In: Nonperturbative Aspects in Quantum Field Theory (Proc. Les Houches Winter Adv.

Study Inst., 1978). *Physics Rep.* 49, No. 2 (1979), 267–272. MR 82j:82063.
(**Phys: SG: Exp**)

Gérard Toulouse and Jean Vannimenus

1977a La frustration: un monde semé de contradictions. *La Recherche* No. 83, Vol. 8 (Nov., 1977), 980–981.

Popular exposition of the elements of frustration in relation to the Ising model [evidently based on Toulouse (1977a)]. Briefly mentions the social psychology application. [See also Stern (1989a).] (**Phys: SG, B: Exp**)(**SG: PsS: Exp**)

Lorenzo Traldi

1989a A dichromatic polynomial for weighted graphs and link polynomials. *Proc. Amer. Math. Soc.* 106 (1989), 279–286. MR 90a:57013. Zbl. 713.57003.

Generalizing Kauffman's (1989a) Tutte polynomial of a sign-colored graph, Traldi's "weighted dichromatic polynomial" $Q(\Gamma; t, z)$ is the $Q_\Gamma(1, w; t, z)$ of Zaslavsky (1992b), in which the deletion-contraction parameters $a_e = 1$ and $b_e = w(e)$, the weight of e. Thm. 2 gives the Tutte-style spanning-tree expansion. Thm. 4: Kauffman's Tutte polynomial $Q[\Sigma](A, B, d) = d^{-1}A^{|E_+|}B^{|E_-|}Q_{|\Sigma|}(1, w; d, d)$ for connected Σ, with $w(e) = (AB^{-1})^{\sigma(e)}$. [See Kauffman (1989a) for other generalizations. Traldi gives perhaps too much credit to Fortuin and Kasteleyn (1972a).]

P. 284: Invariance under Reidemeister moves of type II constrains the weighted dichromatic polynomial to, in essence, equal Kauffman's. Thus no generalization is evident in connection with general link diagrams. There is an interesting application to special link diagrams. (**SGc: Gen: N, Knot**)

Marián Trenkler

See S. Jezný.

Nenad Trinajstić

See also A. Graovac.

1983a *Chemical Graph Theory.* 2 vols. CRC Press, Boca Raton, Florida, 1983. MR 86g:92044.

Vol. I: Ch. 3, §VI: " Möbius graphs." Ch. 5, §VI: "Extension of Sachs formula to Möbius systems." §VII: "The characteristic polynomial of a Möbius cycle." Ch. 6, §VIII: "Eigenvalues of Möbius annulenes."
(**SG: Chem, A: Exp**)

1992a *Chemical Graph Theory, Second Ed.* CRC Press, Boca Raton, Florida, 1992. MR 93g:92034.

Ch. 3, §V.B: "Möbius graphs." Ch. 4, §I: "The adjacency matrix": see pp. 42–43. Ch. 5: "The characteristic polynomial of a graph", §II.B: "The extension of the Sachs formula to Möbius systems"; §III.D: "Möbius cycles". Ch. 6, §VIII: "Eigenvalues of Möbius annulenes" (i.e., unbalanced polygons); §IX: "A classification scheme for moncyclic systems" (i.e., characteristic polynomials of polygons). (**SG: A, Chem**)

Ch. 7: "Topological resonance energy," §V.C: "Möbius annulenes"; §V.G: "Aromaticity in the lowest excited state of annulenes". (**Chem**)

K. Truemper

See also Gerards *et al.* (1990a).

1976a An efficient scaling procedure for gain networks. *Networks* 6 (1976), 151–159. MR 56 #10882. Zbl. 331.90027. (**gg: GN, sg: B, Sw**)

1977a On max flows with gains and pure min-cost flows. *SIAM J. Appl. Math.* 32 (1977), 450–456. MR 55 #5197. Zbl. 352.90069. **(GG, OG, GN, B)**

1977b Unimodular matrices of flow problems with additional constraints. *Networks* 7 (1977), 343–358. MR 58 #20352. Zbl. 373.90023. **(sg: I: B)**

1978a Optimal flows in nonlinear gain networks. *Networks* 8 (1978), 17–36. MR 57 #5041. Zbl. 381.90039. **(GN)**

††1982a Alpha-balanced graphs and matrices and $GF(3)$-representability of matroids. *J. Combin. Theory Ser. B* 32 (1982), 112–139. MR 83i:05025. Zbl. 478.05026.

A $0, \pm 1$-matrix is called "balanced" if it contains no submatrix that is the incidence matrix of a negative polygon. More generally, α-balance of a $0, \pm 1$-matrix corresponds to prescribing the signs of holes in a signed graph. Main theorem characterizes the sets of holes (chordless polygons) in a graph that can be the balanced holes in some signing. [A major result. See Conforti and Kapoor (1998a) for a new proof and discussion of applications.] **(sg: B, I)**

1992a *Matroid Decomposition.* Academic Press, San Diego, 1992. MR 93h:05046. Zbl. 760.05001.

§12.1: "Overview." §12.2: "Characterization of alpha-balanced graphs," exposition of (1982a). **(sg: B, Sw)**

Marcello Truzzi

See F. Harary.

S.V. Tsaranov

See also F.C. Bussemaker, P.J. Cameron, and J.J. Seidel.

1992a On spectra of trees and related two-graphs. In: Jaroslav Nešetřil and Miroslav Fiedler, eds., *Fourth Czechoslovak Symposium on Combinatorics, Graphs and Complexity* (Prachatice, 1990), pp. 337–340. Ann. Discrete Math., Vol. 51. North-Holland, Amsterdam, 1992. MR 93i:05004 (book). Zbl. 776.05077.

A two-graph whose points are the edges of a tree T and whose triples are the nonseparating triples of edges of T (from Seidel and Tsaranov (1990a) via Cameron (1994a)). An associated signed complete graph Σ_T on vertex set $E(T)$ is obtained by orienting T arbitrarily, then taking $\sigma_T(ef) = +$ or $-$ depending on whether e and f are similarly or oppositely oriented in the path of T that contains both. Reorienting edges corresponds to switching Σ_T. Thm.: Letting $n = |V(T)|$, the matrices $3I_n + A(\Sigma_T)$ and $2I_{n+1} - A(T)$ have the same numbers of zero and negative eigenvalues. **(TG: A, G)**

1993a Trees, two-graphs, and related groups. In: D. Jungnickel and S.A. Vanstone, eds., *Coding Theory, Design Theory, Group Theory* (Proc. Marshall Hall Conf., Burlington, Vt., 1990), pp. 275–281. Wiley, New York, 1993. MR 94j:05062.

New proof of theorem on the group (Seidel and Tsaranov 1990a) of the two-graph (Tsaranov 1992a) of a tree. **(TG: A, G)**

Michael Tsatsomeros

See C.R. Johnson.

Thomas W. Tucker

See J.L. Gross.

Vanda Tulli

See A. Bellacicco.

Edward C. Turner
See R.Z. Goldstein.

Daniel Turzík
See S. Poljak.

W.T. Tutte
1981a On chain-groups and the factors of graphs. In: L. Lovász and Vera T. Sós, eds., *Algebraic Methods in Graph Theory* (Proc. Colloq., Szeged, 1978), Vol. 2, pp. 793–818. Colloq. Math. Soc. János Bolyai, 25. János Bolyai Math. Soc., Budapest, and North-Holland, Amsterdam, 1981. MR 83b:05104. Zbl. 473.05023.
(**sg: EC, D**)

Zsolt Tuza
See S. Poljak.

J.P. Uhry
See F. Barahona and I. Bieche.

Włodzimierz Ulatowski
See also J. Topp.

1991a On Kirchhoff's voltage law in Z_n. *Discussiones Math.* 11 (1991), 35–50. MR 93g:05121. Zbl. 757.05058.
Examines injective, nowhere zero, balanced gains (called "graceful labellings") from Z_{m+1}, $m = |E|$, on arbitrarily oriented polygons and variously oriented paths. [*Question.* Does this work generalize to bidirected polygons and paths?] (**GD: b: Polygons, Paths**)

N.B. Ul'janov [N.B. Ul'yanov]
See N.B. Ul'yanov.

N.B. Ul'yanov
See D.O. Logofet.

M.E. Van Valkenburg
See W. Mayeda.

Pauline van den Driessche
See J. Bélair, C. Jeffries, C.R. Johnson, and V. Klee.

Jean Vannimenus
See also B. Derrida and G. Toulouse.

J. Vannimenus, J.M. Maillard, and L. de Sèze
1979a Ground-state correlations in the two-dimensional Ising frustration model. *J. Phys. C: Solid State Phys.* 12 (1979), 4523–4532. (**Phys: SG: Fr**)

J. Vannimenus and G. Toulouse
1977a Theory of the frustration effect: II. Ising spins on a square lattice. *J. Phys. C: Solid State Phys.* 10 (1977), L537–L541. (**SG: Phys: Fr**)

Vijay V. Vazirani and Mihalis Yannakakis
1988a Pfaffian orientations, 0/1 permanents, and even cycles in directed graphs. In: Timo Lepistö and Arto Salomaa, eds., *Automata, Languages and Programming* (Proc. 15th Internat. Colloq., Tampere, Finland, 1988), pp. 667–681. Lecture Notes in Computer Sci., Vol. 317. Springer-Verlag, Berlin, 1988. MR 90k:68078. Zbl. 648.68060.
Slightly abridged version of (1989a). (**SD: A, B: Alg**)

Vijay V. Vazirani and Milhalis [Mihalis] Yannakakis

1989a Pfaffian orientations, 0–1 permanents, and even cycles in directed graphs. *Discrete Appl. Math.* 25 (1989), 179–190. MR 91e:05080. Zbl. 696.68076.

"Evenness" of a digraph (i.e., every signing contains a positive cycle) is polynomial-time equivalent to evaluability of a certain 0–1 permanent by a determinant and to parts of the existence and recognition problems for Pfaffian orientations of a graph. Briefly expounded in Brundage (1996a).]

(**SD: A, B: Alg**)

G.K. Vijayakumar

See G.R. Vijayakumar.

G.R. Vijayakumar

See also P.D. Chawathe, D.K. Ray-Chaudhuri, and N.M. Singhi.

1984a (As "G.K. Vijayakumar") A characterization of generalized line graphs and classification of graphs with eigenvalues at least 2 [misprint for -2]. *J. Combin. Inform. Syst. Sci.* 9 (1984), 182–192. MR 89g:05055. Zbl. 629.05046. (**sg: A, lg**)

1987a Signed graphs represented by D_∞. *European J. Combin.* 8 (1987), 103–112. MR 88b:05111. Zbl. 678.05058. (**SG: A, G, lg**)

1992a Signed graphs represented by root system E_8. Combinatorial Math. and Appl. (Proc., Calcutta, 1988). *Sankhya Ser. A* 54 (1992), 511–517. MR 94d:05072. Zbl. 882.05118. (**SG: A, G**)

1993a Algebraic equivalence of signed graphs with all eigenvalues ≥ -2. *Ars Combin.* 35 (1993), 173–191. MR 93m:05134. Zbl. 786.05059. (**SG: A, G**)

1994a Representation of signed graphs by root system E_8. *Graphs Combin.* 10 (1994), 383–388. MR 96a:05128. Zbl. 821.05040. (**SG: G**)

G.R. Vijayakumar and N.M. Singhi

1990a Some recent results on signed graphs with least eigenvalues ≥ -2. In: Dijen Ray-Chaudhuri, ed., *Coding Theory and Design Theory Part I: Coding Theory* (Proc. Workshop IMA Program Appl. Combin., Minneapolis, 1987–88), pp. 213–218. IMA Vol. Math. Appl., Vol. 20. Springer-Verlag, New York, 1990. MR 91e:05069. Zbl. 711.05033. (**SG: G, lg, A: Exp**)

G.R. Vijayakumar (as "Vijaya Kumar"), S.B. Rao, and N.M. Singhi

1982a Graphs with eigenvalues at least -2. *Linear Algebra Appl.* 46 (1982), 27–42. MR 83m:05099. Zbl. 494.05044. (**sg: A, G, lg**)

K.S. Vijayan

See S.B. Rao.

Jacques Villain

1977a Spin glass with non-random interactions. *J. Phys. C: Solid State Phys.* 10, No. 10 (1977), 1717–1734. (**SG: Phys, Fr, Sw**)

1977b Two-level systems in a spin-glass model: I. General formalism and two-dimensional model. *J. Phys. C: Solid State Phys.* 10 (1977), 4793–4803. (**Phys: SG: Fr**)

1978a Two-level systems in a spin-glass model: II. Three-dimensional model and effect of a magnetic field. *J. Phys. C: Solid State Phys.* 11, No. 4 (1978), 745–752.

(**Phys: SG: Fr**)(**GG: Phys, Sw, B**)

Andrew Vince

1983a Combinatorial maps. *J. Combin. Theory Ser. B* 34 (1983), 1–21. MR 84i:05048. Zbl. 505.05054.

See Theorem 6.1. (**sg: b: T**)

E. Vincent, J. Hammann, and M. Ocio

1992a Slow dynamics in spin glasses and other complex systems. In: D.H. Ryan, ed., *Recent Progress in Random Magnets*, pp. 207–236. World Scientific, Singapore, 1992.

Surveys experiments with spin glass materials, especially their aging behavior. Interprets results as tending to support the Parisi-type model (see notes on Mézard, Parisi, and Virasoro (1987a)). (**Phys**)

Miguel Angel Virasoro

See M. Mézard.

Jože Vrabek

See T. Pisanski.

Kristina Vušković

See M. Conforti.

Donald K. Wagner

See also V. Chandru and C.R. Coullard.

††1985a Connectivity in bicircular matroids. *J. Combin. Theory Ser. B* 39 (1985), 308–324. MR 87c:05041. Zbl. 584.05019.

Prop. 1 and Thm. 2 show that n-connectivity of the bicircular matroid $B(\Gamma)$ is equivalent to "n-biconnectivity" of Γ.

When do two 3-biconnected graphs have isomorphic bicircular matroids? §5 proves that 3-biconnected graphs with > 4 vertices have isomorphic bicircular matroids iff one is obtained from the other by a sequence of operations called "edge rolling" and "3-star rotation". This is the bicircular analog of Whitney's polygon-matroid isomorphism theorem, but it is complicated. [An important theorem, generalized to all bicircular matroids in Coullard, del Greco, and Wagner (1991a). *Major Research Problems.* Generalize to bias matroids of biased graphs. Find the analog for lift matroids.]

(**Bic: Str**)

1988a Equivalent factor matroids of graphs. *Combinatorica* 8 (1988), 373–377. MR 90d:05071. Zbl. 717.05022.

"Factor matroid" = even-cycle matroid $G(-\Gamma)$. Decides when $G(-\Gamma) \cong G(B)$ where B is a given bipartite, 4-connected graph. (**EC: Str**)

Bronislaw Wajnryb

See R. Aharoni.

Derek A. Waller

See also F.W. Clarke.

1976a Double covers of graphs. *Bull. Austral. Math. Soc.* 14 (1976), 233–248. MR 53 #10662. Zbl. 318.05113. (**SG: Cov**)

Egon Wanke

See also F. Höfting.

1993a Paths and cycles in finite periodic graphs. In: Andrzej M. Borzyszkowski and Stefan Sokołowski, eds., *Mathematical Foundations of Computer Science 1993* (Proc., 18th Internat. Sympos., MFCS '93, Gdańsk, 1993), pp. 751–760. MR 95c:05077.

Broadly resembles Höfting and Wanke (1994a) but omits those edges of $\tilde{\Phi}$ that are affected by the modulus α. (**GD(Cov): Alg**)

20xxa Paths and cycles in finite periodic graphs. Submitted.
Full version of (1993a). (**GD(Cov): Alg**)

G.H. Wannier
1950a Antiferromagnetism. The triangular Ising net. *Phys. Rev.* (2) 79 (1950), 357–364. MR 12, 576. Zbl. 38, 419 (e: 038.41904). (**Phys: P: Fr**)

Stanley Wasserman and Katherine Faust
1994a *Social Network Analysis: Methods and Applications.* Structural Anal. Soc. Sci., 8. Cambridge Univ. Press, Cambridge, 1994. Zbl. 980.24676.
§1.2: "Historical and theoretical foundations." A brief summary of various network methods in sociometry, signed graphs and digraphs among them. §4.4: "Signed graphs and signed directed graphs." Mathematical basics. Ch. 6: "Structural balance and transitivity." Application of balance of signed (di)graphs and of ensuing notions like clusterability, historically evolving into transitivity of unsigned digraphs. History and evaluation.
(**PsS, SG, SD: B, Fr, Cl, Gen: Exp, Ref**)

William C. Waterhouse
1977a Some errors in applied mathematics. *Amer. Math. Monthly* 84, No. 1 (January, 1977), 25–27. Zbl. 376.9001 (q.v.).
Criticizes Roberts and Brown (1975a, 1977a). See rebuttal in the Zbl. review.

John J. Watkins
See R.J. Wilson.

Kevin D. Wayne
See È. Tardos.

Jeffrey R. Weeks and Kenneth P. Bogart
1979a Consensus signed digraphs. *SIAM J. Appl. Math.* 36 (1979), 1–14. MR 81i:92026. Zbl. 411.05042. (**SD**)

Gerry M. Weiner
See J.S. Maybee.

Volkmar Welker
1997a Colored partitions and a generalization of the braid arrangement. *Electronic J. Combin.* 4 (1) (1997), Article R4, 12 pp. (electronic). MR 98b:57026. Zbl. 883.52010.
The arrangement is the affine part (that is, where $x_0 = 1$) of the projective representation of $G(\Phi)$, where Φ is the complex multiplicative gain graph $\Phi = \{1\}K_{n+1} \cup \{re_{0i} : 1 \leq i \leq n \text{ and } 2 \leq r \leq s\}$. Here the vertex set is $\{0, 1, \ldots, n\}$, s is any positive integer, and re_{0i} (in the paper, $e_{0i}(r)$) denotes an edge v_0v_i with gain r. The topics of interest are those related to the complex complement. The study is based on the combinatorics of the intersection semilattice [that is, the geometric semilattice $\mathrm{Lat}^{\mathrm{b}}\,\Phi$ of balanced flats], including the Poincaré polynomial of the arrangement [equivalent to the balanced chromatic polynomial of Φ]. (**gg: M, G, N**)

Albert L. Wells, Jnr.
See also P.J. Cameron and Y. Cheng.

1982a Regular generalized switching classes and related topics. D. Phil. thesis, Oxford Univ., 1982. (**SG: Sw, A, E, TG, G, Cov, Aut**)

1984a Even signings, signed switching classes, and $(-1, 1)$-matrices. *J. Combin. Theory Ser. B* 36 (1984), 194–212. MR 85i:05206. Zbl. 527.05007. (**SG: Sw, E, Aut**)

D.J.A. Welsh [Dominic Welsh]
See also L. Lovász and W. Schwaärzler.

1976a *Matroid Theory.* L.M.S. Monographs, Vol. 8. Academic Press, London, 1976. MR 55 #148. Zbl. 343.05002.
§11.4: "Partition matroids determined by finite groups", sketches the most basic parts of Dowling (1973b). **(gg: M: Exp)**

1992a On the number of knots and links. In: G. Halász, L. Lovász, D. Miklós, and T. Szönyi, eds., *Sets, Graphs and Numbers* (Proc., Budapest, 1991), pp. 713–718. Colloq. Math. Soc. János Bolyai, Vol. 60. János Bolyai Math. Soc., Budapest, and North-Holland, Amsterdam, 1992. MR 94f:57010. Zbl. 799.57001.
The signed graph of a link diagram is employed to get an upper bound. **(SGc: E)**

1993a *Complexity: Knots, Colourings and Counting.* London Math. Soc. Lecture Note Ser., 186. Cambridge Univ. Press, Cambridge, Eng., 1993. MR 94m:57027. Zbl. 799.68008.
Includes very brief treatments of some appearances of signed graphs.
§2.2, "Tait colourings", defines the signed graph of a link diagram, mentioned again in observation (2.3.1) on alternating links and Prop (5.2.16) on "states models" (from Schwärzler and Welsh (1993a)). §5.6, "Thistlethwaite's non-triviality criterion": the criterion depends on the signed graph.
§2.5, "The braid index and the Seifert index of a link", defines the Seifert graph, a signed graph based on splitting the link diagram. **(SGc, Knot)**
§5.7, "Link invariants and statistical mechanics", defines a relatively simple spin model for signed graphs, with an arbitrary finite number of possible spin values. The partition function is related to link diagrams.
§4.2, "The Ising model", introduces the basic concepts in mathematical terms. §6.4, "The complexity of the Ising model", "Computing ground states of spin systems", pp. 105–107, discusses finding a ground state of the Ising model. This is described as the min-weight cut problem with weights the negatives [this is an error] of the Ising bond interaction values: that is, the weighted frustration index problem in the negative [erroneous] of the Ising graph. It is the max-cut problem when the Ising graph is balanced (ferromagnetic) [should be antibalanced (antiferromagnetic)]. For external magnetic field, follows Barahona (1982a). **(sg: Fr, Phys)**
§3.6, "Ice models", counts "ice configurations" (certain graph orientations) via poise gains modulo 3, although the counting function is not gain-graphic. **(gg, N, Phys)**
§4.4: "The Ashkin–Teller–Potts model". This treatment of the Potts model has a different Hamiltonian from that of Fischer and Hertz (1991a). [It does not seem that Welsh intends to admit edge signs but if they are allowed then the Hamiltonian (without edge weights) is $-\sum \sigma(e_{ij})(\delta(s_i,s_j)-1)$. Up to halving and a constant term, this is Doreian and Mrvar's (1996a) clusterability measure $P(\pi)$, with $\alpha = .5$, of the vertex partition induced by the state.] [Also cf. Fischer and Hertz (1991a).] **(cl, Phys)**

1993b The complexity of knots. In: John Gimbel, John W. Kennedy and Louis V. Quintas, eds., *Quo Vadis, Graph Theory?*, pp. 159–171. Ann. Discrete Math., Vol. 55. North-Holland, Amsterdam, 1993. MR 94c:57021. Zbl. 801.68086.
Link diagrams $\leftrightarrow$ dual pairs of sign-colored plane graphs: based on Yajima and Kinoshita (1957a). Unsolved algorithmic problems about knots based on

link diagrams; in particular, triviality of diagrams is equivalent to *Problem* 4.2: A polynomial-time algorithm to decide whether the graphical Reidemeister moves can convert a given signed plane graph to one with edges all of one sign. (**SGc: D, Knot: Alg, Exp**)

1993c Knots and braids: some algorithmic questions. In: Neil Robertson and Paul Seymour, eds., *Graph Structure Theory* (Proc., Seattle, 1991), pp. 109–123. Contemp. Math., Vol. 147. Amer. Math. Soc., Providence, R.I., 1993. MR 94g:57014. Zbl. 792.05058.

§1 presents the sign-colored graph of a link diagram and §5, "Reidemeister graphs", describes Schwärtzler and Welsh (1993a). §3 defines the sign-colored Seifert graph. (**SGc. Sc(M): N, Alg, Knot: Exp**)

1997a Knots. In: Lowell W. Beineke and Robin J. Wilson, eds., *Graph Connections: Relationships between Graph Theory and other Areas of Mathematics*, Ch. 12, pp. 176–193. The Clarendon Press, Oxford, 1997. MR 99a:05001 (book). Zbl. 878.57001.

Mostly describes the signed graph of a link diagram and its relation to knot theory, including knot properties deducible directly from the signed graph, the Kauffman bracket and two-variable polynomials, etc. Similar to relevant parts of (1993a). (**SGc: Knot: N: Exp**)

D. de Werra

See C. Benzaken.

Arthur T. White

1984a *Graphs, Groups and Surfaces.* Completely revised and enlarged edn. North Holland Math. Stud., Vol. 8. North-Holland, Amsterdam, 1984. MR 86d:05047. Zbl. 551.05037.

Chapter 10: "Voltage graphs". (**GG: T, Cov**)

1994a An introduction to random topological graph theory. *Combinatorics, Probability and Computing* 3 (1994), 545–555. MR 95j:05083. Zbl. 815.05027.

Take a graph Γ with cyclomatic number k and randomly sign it so that each edge is negative with probability p. The probability that (Γ, σ) is balanced $= 2^{-k}$ if $p = \frac{1}{2}$ [obvious] and $\leq [\max(p, 1-p)]^k$ in general [not obvious] (this has an interesting asymptotic consequence due to Gimbel, given in this paper). (**SG: Rand, B**)

Neil L. White

See also A. Björner.

1986a A pruning theorem for linear count matroids. *Congressus Numerantium* 54 (1986), 259–264. MR 88c:05047. Zbl. 621.05009. (**Bic: Gen**)

Neil White and Walter Whiteley

1983a A class of matroids defined on graphs and hypergraphs by counting properties. Unpublished manuscript, 1983.

See Whiteley (1996a) for an exposition and extension. (**Bic: Gen**)

Walter Whiteley

See also N. White.

1996a Some matroids from discrete applied geometry. In: Joseph E. Bonin, James G. Oxley, and Brigitte Servatius, eds., *Matroid Theory* (Proc., Seattle, 1995), pp. 171–311. Contemp. Math., Vol. 197. Amer. Math. Soc., Providence, R.I., 1996.

Appendix: "Matroids from counts on graphs and hypergraphs", which expounds and extends Loréa (1979a), Schmidt (1979a), and especially White and Whiteley (1983a), describes matroids on the edge sets of graphs (and hypergraphs) that generalize the bicircular matroid. The definition: given $m \geq 0$ and $k \in \mathbb{Z}$, S is independent iff $\emptyset \subset S' \subseteq S$ implies $|S'| \leq m|V(S')| + k$. (**Bic: Gen**)(**Ref**)

Geoff Whittle

See also C. Semple.

1989a Dowling group geometries and the critical problem. *J. Combin. Theory Ser. B* 47 (1989), 80–92. MR 90g:51008. Zbl. 628.05018.

A Dowling-lattice version of Crapo and Rota's critical problem is developed. Some minimal matroids whose critical exponent is k (i.e., tangential k-blocks) are given, one being $G(\pm K_n^\circ)$. (**gg: M: N**)

Robin J. Wilson and John J. Watkins

1990a *Graphs: An Introductory Approach. A First Course in Discrete Mathematics.* Wiley, New York, 1990.

§3.2: "Social Sciences" (pp. 51–53) applies signed graphs. §5.1: "Signed digraphs" (pp. 96–98) discusses positive and negative feedback (i.e., positive and negative cycles) in applications. Based on Open University (1981a). (**SG, PsS, SD: Exp**)

Shmuel Winograd

See R.M. Karp.

Wayland H. Winstead

See J.R. Burns.

H.S. Witsenhausen

See Y. Gordon.

C. Witzgall and C.T. Zahn, Jr.

1965a Modification of Edmonds' maximum matching algorithm. *J. Res. Nat. Bur. Standards (U.S.A.) Sect. B* 69B (1965), 91–98. MR 32 #5548. Zbl. 141.21901. (**p: o**)

A. Wongseelashote

1976a An algebra for determining all path-values in a network with application to K-shortest-paths problems. *Networks* 6 (1976), 307–334. MR 56 #14628. Zbl. 375.90030. (**gg: Paths**)

Takeshi Yajima and Shin'ichi Kinoshita

1957a On the graphs of knots. *Osaka Math. J.* 9 (1957), 155–163. MR 20 #4845. Zbl. (e: 080.17002).

Examines the relationship between the two dual sign-colored graphs, Σ and Σ', of a link diagram (Bankwitz 1930a), translating the Reidemeister moves into graph operations and showing that they will convert Σ into Σ'. (**SGc: Knot**)

Jing-Ho Yan, Ko-Wei Lih, David Kuo, and Gerard J. Chang

1997a Signed degree sequences of signed graphs. *J. Graph Theory* 26 (1997), 111–117. MR 98i:05160. Zbl. 980.04848.

Net degree sequences of signed simple graphs. Theorem 2 improves the Havel–Hakimi-type theorem from Chartrand, Gavlas, Harary, and Schultz (1992a) by determining the length parameter. Theorem 7 characterizes the net degree sequences of signed trees. [There seems to be room to strengthen the characterization and generalize to weighted degree sequences: see notes on Chartrand et al.] (**SGw: N**)

Mihalis Yannakakis
See Esther M. Arkin and V.V. Vazirani.

Milhalis Yannakakis [Mihalis Yannakakis]
See Mihalis Yannakakis.

Yeong-Nan Yeh
See I. Gutman and S.-Y. Lee.

Anders Yeo
See G. Gutin.

J.W.T. Youngs

1968a Remarks on the Heawood conjecture (nonorientable case). *Bull. Amer. Math. Soc.* 74 (1968), 347–353. MR 36 #3675. Zbl. 161.43303.

Introducing "cascades": current graphs with bidirected edges. A "cascade" is a bidirected graph, not all positive, that is provided with both a rotation system (hence it is orientation embedded in a surface) and a current (which is a special kind of bidirected flow). Dictionary: "broken" means a negative edge. (**sg: O: Appl, Flows**)

1968b The nonorientable genus of K_n. *Bull. Amer. Math. Soc.* 74 (1968), 354–358. MR 36 #3676. Zbl. 161.43304.

"Cascades": see Youngs (1968b). (**sg: O: Appl**)

Cheng-Ching Yu
See C.-C. Chang.

Raphael Yuster and Uri Zwick

1994a Finding even cycles even faster. In: Serge Abiteboul and Eli Shamir, eds., *Automata, Languages and Programming* (Proc. 21st Internat. Colloq., ICALP 94, Jerusalem, 1994), pp. 532–543. Lect. Notes Computer Sci., Vol. 820. Springer-Verlag, Berlin, 1994. MR 96b:68002 (book). Zbl. 844.00024 (book).

Abbreviated version of (1997a). (**p: Cycles: Alg**)

1997a Finding even cycles even faster. *SIAM J. Discrete Math.* 10 (1997), 209–222. MR 98d:05137. Zbl. 867.05065.

For fixed *even* k, a very fast algorithm for finding a k-gon. Also, one for finding a shortest even polygon. [*Question.* Are these the all-negative cases of similarly fast algorithms to find positive k-gons, or shortest positive polygons, in signed graphs?] (**p: Cycles: Alg**)

C.T. Zahn, Jr.
See also C. Witzgall.

1973a Alternating Euler paths for packings and covers. *Amer. Math. Monthly* 80 (1973), 395–403. MR 51 #10137. Zbl. 274.05112. (**p: o**)

Robert B. Zajonc

1968a Cognitive theories in social psychology. In: Gardner Lindzey and Elliot Aronson, eds., *The Handbook of Social Psychology*, Second Edition, Vol. 1, Ch. 5, pp. 320–411. Addison-Wesley, Reading, Mass., 1968.

"Structural balance," pp. 338–353. "The congruity principle," pp. 353–359. (**PsS: SD, SG, B: Exp, Ref**)

Wenan Zang

1998a Coloring graphs with no odd-K_4. *Discrete Math.* 184 (1998), 205–212. MR 99e:05056.

An algorithm, based in part on Gerards (1994a), that, given an all-negative signed graph, finds a subdivided $-K_4$ subgraph or a 3-coloring of the underlying graph. *Question.* Is there a generalization to all signed graphs?
(sg: p: Col, Alg, Ref)

Thomas Zaslavsky

See also C. Greene, P. Hanlon, and P. Solé.

1977a Biased graphs. Unpublished manuscript, 1977.

Being published, greatly expanded, in (1989a, 1991a, 1995b, 20xxg) and more; as well as (but restricted to signed graphs) in (1982a, 1982b).
(GG: M)

1980a Voltage-graphic geometry and the forest lattice. In: *Report on the XVth Denison-O.S.U. Math. Conf.* (Granville, Ohio, 1980), pp. 85–89. Dept. of Math., The Ohio State Univ., Columbus, Ohio, 1980. **(GG: M, Bic)**

1981a The geometry of root systems and signed graphs. *Amer. Math. Monthly* 88 (1981), 88–105. MR 82g:05012. Zbl. 466.05058.

Signed graphs correspond to arrangements of hyperplanes in $\mathbb{R}^n$ of the forms $x_i = x_j$, $x_i = -x_j$, and $x_i = 0$. Consequently, one can compute the number of regions of the arrangement from graph theory, esp. for arrangements corresponding to "sign-symmetric" graphs, i.e., having both or none of each pair $x_i = \pm x_j$. Simplified account of parts of (1982a, 1982b, 1982c), emphasizing geometry. **(SG: M, G, N)**

1981b Characterizations of signed graphs. *J. Graph Theory* 5 (1981), 401–406. MR 83a:05122. Zbl. 471.05035.

Characterizes the sets of polygons that are the positive ones in some signing of a graph. **(SG: B)**

1981c Is there a theory of signed graph embedding? In: *Report on the XVIth Denison-O.S.U. Math. Conf.* (Granville, Ohio, 1981), pp. 79–82. Dept. of Math., The Ohio State Univ., Columbus, Ohio, 1981. **(SG: T, M)**

††1982a Signed graphs. *Discrete Appl. Math.* 4 (1982), 47–74. MR 84e:05095a. Zbl. 476.05080. Erratum. *Ibid.* 5 (1983), 248. MR 84e:05095b. Zbl. 503.05060.

Basic results on the bias matroid $G(\Sigma)$, the signed covering graph $\tilde{\sigma}$, the matrix-tree theorem [different from that of Murasugi (1989a)], and vector representation [as multisubsets of root systems $B_n \cup C_n$]. Examples. Conjectures about the interrelation between representability in characteristic 2 and unique representability in characteristic 0 [since answered by Geoff Whittle (A characterisation of the matroids representable over GF(3) and the rationals. *J. Combin. Theory Ser. B* 65 (1995), 222–261. MR 96m:05046. Zbl. 835.05015) as developed by Pagano (1998a, 20xxc)].
(SG, GG: M, B, Sw, Cov, I, G; EC, K)

††1982b Signed graph coloring. *Discrete Math.* 39 (1982), 215–228. MR 84h:05050a. Zbl. 487.05027.

A "proper k-coloring" of Σ partitions V into a special "zero" part, possibly void, that induces a stable subgraph, and up to k other parts (labelled from a set of k colors), each of which induces an antibalanced subgraph. A "zero-free proper k-coloring" is similar but without the "zero" part. [The suggestion is that the signed analog of a stable vertex set is one that induces an antibalanced subgraph. *Problem.* Use this insight to develop generalizations of stable-set notions, such as cliques and perfection. *Example.* Let $\alpha(\Sigma)$, the "antibalanced vertex set number", be the largest size of an antibalance-inducing vertex set. Then $\alpha(\Gamma) = \alpha(+\Gamma \cup -K_n.]$ One gets two related chromatic polynomials. The chromatic polynomial, $\chi_\Sigma(2k+1)$, counts all proper k-colorings; it is essentially the characteristic polynomial of the bias matroid. It can often be most easily computed via the zero-free chromatic polynomial, $\chi^*_\Sigma(2k)$, which counts proper zero-free colorings: see (1982c). (**SG, GG: M, Col, N, Cov, O, G**)

1982c Chromatic invariants of signed graphs. *Discrete Math.* 42 (1982), 287–312. MR 84h:05050b. Zbl. 498.05030.

Continuation of (1982b). The fundamental balanced expansion formulas, that express the chromatic polynomial in terms of the zero-free chromatic polynomial. Many special cases, treated in great detail: antibalanced graphs, signed graphs that contain $+K_n$ or $-K_n$, signed K_n's (a.k.a. two-graphs), etc. (**SG, GG: M, N, Col, Cov, O, G; EC, K**)

1982d Bicircular geometry and the lattice of forests of a graph. *Quart. J. Math. Oxford* (2) 33 (1982), 493–511. MR 84h:05050c. Zbl. 519.05020. (**GG: M, Bic, G, N**)

1982e Voltage-graphic matroids. In: Adriano Barlotti, ed., *Matroid Theory and Its Applications* (Proc. Session of C.I.M.E., Varenna, Italy, 1980), pp. 417–423. Liguore Editore, Naples, 1982. MR 87g:05003 (book). (**GG: M, EC, Bic, N**)

1984a How colorful the signed graph? *Discrete Math.* 52 (1984), 279–284. MR 86m:-05045. Zbl. 554.05026.

The zero-free chromatic number, and in particular that of a complete signed graph (possibly with parallel edges). (**SG: Col**)

1984b Multipartite togs (analogs of two-graphs) and regular bitogs. In: Proc. Fifteenth Southeastern Conf. on Combinatorics, Graph Theory and Computing (Baton Rouge, 1984), Vol. III. *Congressus Numer.* 45 (1984), 281–293. MR 86d:05109. Zbl. 625.05044. (**SG: TG: Gen: A, Sw**)

1984c Line graphs of switching classes. In: *Report of the XVIIIth O.S.U. Denison Maths Conference* (Granville, Ohio, 1984), pp. 2–4. Dept. of Math., Ohio State Univ., Columbus, Ohio, 1984.

The line graph of a switching class $[\Sigma]$ of signed graphs is a switching class of signed graphs; call it $[L'(\Sigma)]$. The reduced line graph L is formed from L' by deleting parallel pairs of oppositely signed edges. Then $A(L) = A(L') = 2I - MM^{\mathrm{T}}$, where M is the incidence matrix of Σ. Thm. 1: $A(L)$ has all eigenvalues ≤ 2. Examples: For an ordinary graph Γ, $L(-\Gamma) = -L(\Gamma)$. Taking $-\Gamma$ and attaching any number of pendant negative digons to each vertex yields (the negative of) Hoffman's generalized line graph. Additional results are claimed but there are no proofs. [See also 20xxb).] [This work is intimately related to that of Vijayakumar *et al.*, which was then unknown to the author, and to Cameron (1980a) and Cameron, Goethals, Seidel, and Shult (1976a).] (**SG: LG: Sw, A, I**)

1987a The biased graphs whose matroids are binary. *J. Combin. Theory Ser. B* 42 (1987), 337–347. MR 88h:05082. Zbl. 667.05015.

Forbidden-minor and structural characterizations. The latter for signed graphs is superseded by a result of Pagano (1998a). (**GG: M**)

1987b Balanced decompositions of a signed graph. *J. Combin. Theory Ser. B* 43 (1987),

1–13. MR 89c:05058. Zbl. 624.05056.

Decompose $E(\Sigma)$ into the fewest balanced subsets (generalizing the biparticity of an unsigned graph), or balanced connected subsets. These minimum numbers are δ_0 and δ_1. Thm. 1: $\delta_0 = \lceil \chi^* \rceil + 1$, where χ^* is the zero-free chromatic number of $-\Sigma$. Thm. 2: $\delta_0 = \delta_1$ if Σ is complete. *Conjecture* 1. Σ partitions into δ_0 balanced, connected, and spanning edge sets (whence $\delta_0 = \delta_1$) if it has δ_0 edge-disjoint spanning trees. [Solved and generalized to basepointed matroids by D. Slilaty.] *Conjecture* 2 is a formula for δ_1 in terms of δ_0 of subgraphs. [It has been thoroughly disproved by Slilaty.] (**SG: Fr**)

1987c Vertices of localized imbalance in a biased graph. *Proc. Amer. Math. Soc.* 101 (1987), 199–204. MR 88f:05103. Zbl. 622.05054.

Such a vertex (also, a "balancing vertex") is a vertex of an unbalanced graph whose removal leaves a balanced graph. Some elementary results. (**GG: Fr**)

1988a Togs (generalizations of two-graphs). In: M.N. Gopalan and G.A. Patwardhan, eds., *Optimization, Design of Experiments and Graph Theory* (Proc. Sympos., Bombay, 1986), pp. 314–334. Indian Inst. of Technology, Bombay, 1988. MR 90h:05112. Zbl. 689.05035.

An attempt to generalize two-graphs (here [alas?] called "unitogs") in a way similar to that of Cameron and Wells (1986a) although largely independent. The notable new example is "Johnson togs", based on the Johnson graph of k-subsets of a set. "Hamming togs" are based on a Hamming graph (that is, a Cartesian product of complete graphs) and generalize examples of Cameron and Wells. Other examples are as in (1984b). (**SG: TG: Gen**)

1988b The demigenus of a signed graph. In: *Report on the XXth Ohio State-Denison Mathematics Conference* (Granville, Ohio, 1988). Dept. of Math., Ohio State Univ., Columbus, Ohio, 1988. (**SG: T, M**)

1989a Biased graphs. I. Bias, balance, and gains. *J. Combin. Theory Ser. B* 47 (1989), 32–52. MR 90k:05138. Zbl. 714.05057.

Fundamental concepts and lemmas of biased graphs. Bias from gains; switching of gains; characterization of balance [for which see also Harary, Lindstrom, and Zetterstrom (1982a)]. (**GG: B, Sw**)

1990a Biased graphs whose matroids are special binary matroids. *Graphs Combin.* 6 (1990), 77–93. MR 91f:05097. Zbl. 786.05020. (**GG: M**)

††1991a Biased graphs. II. The three matroids. *J. Combin. Theory Ser. B* 51 (1991), 46–72. MR 91m:05056. Zbl. 763.05096.

Basic theory of the bias, lift, and complete lift matroids. Several questions and conjectures. (**GG: M**)

1991b Orientation of signed graphs. *European J. Combin.* 12 (1991), 361–375. MR 93a:05065. Zbl. 761.05095.

Oriented signed graph = bidirected graph. The oriented matroid of an oriented signed graph. A "cycle" in a bidirected graph is a bias circuit (a balanced polygon, or a handcuff with both circles negative) oriented to have no source or sink. Cycles in Σ are compared with those in its signed (i.e., derived) covering graph $\tilde{\Sigma}$. The correspondences among acyclic orientations of Σ and regions of the hyperplane arrangements of Σ and $\tilde{\Sigma}$, and dually the faces of the acyclotope of Σ. Thm. 4.1: the net degree vector $d(\tau)$ of an

orientation τ belongs to the face of the acyclotope that is determined by the union of all cycles. Cor. 5.3 (easy): a finite bidirected graph has a source or sink. **(SG: O, M, Cov, G)(SGw: N)**

1992a Orientation embedding of signed graphs. *J. Graph Theory* 16 (1992), 399–422. MR 93i:05056. Zbl. 778.05033.

Positive polygons preserve orientation, negative ones reverse it. The minimal embedding surface of a one-point amalgamation of signed graphs. The formula is almost additive. **(SG: T)**

1992b Strong Tutte functions of matroids and graphs. *Trans. Amer. Math. Soc.* 334 (1992), 317–347. MR 93a:05047. Zbl. 781.05012.

Suppose that a function of matroids with labelled points is defined that is multiplicative on direct sums and satisfies a Tutte-Grothendieck recurrence with coefficients (the "parameters") that depend on the element being deleted and contracted, but not on the particular minor from which it is deleted and contracted: specifically, $F(M) = a_e F(M \setminus e) + b_e F(M/e)$ if e is not a loop or coloop in M. Thm. 2.1 completely characterizes such "strong Tutte functions" for each possible choice of parameters: there is one general type, defined by a rank generating polynomial $R_M(a, b; u, v)$ (the "parametrized rank generating polynomial") involving the parameters $a = (a_e)$, $b = (b_e)$ and the variables u, v, and there are a few special types that exist only for degenerate parameters. All have a Tutte-style basis expansion; indeed, a function has such an expansion iff it is a strong Tutte function (Thms. 7.1, 7.2). The Tutte expansion is a polynomial within each type. If the points are colored and the parameters of a point depend only on the color, one has a multicolored matroid generalization of Kauffman's (1989a) Tutte polynomial of a sign-colored graph. Kauffman's particular choices of parameters are shown to be related to matroid and color duality.

For a graph the "parametrized dichromatic polynomial" $Q_\Gamma = u^{\beta_0(\Gamma)} R_{G(\Gamma)}$, where G = graphic matroid and β_0 = number of connected components. A "portable strong Tutte function" of graphs is multiplicative on disjoint unions, satisfies the parametrized Tutte-Grothendieck recurrence, and has value independent of the vertex set. Thm. 10.1: Such a function either equals Q_Γ or is one of two degenerate exceptions. Prop. 11.1: Kauffman's (1989a) polynomial of a sign-colored graph equals $R_{G(|\Sigma|),\sigma}(a, b; d, d)$ for connected Σ, where $a_+ = b_- = B$ and $a_- = b_+ = A$. [Cf. Traldi 1989a).]

[This paper differs from other generalizations of Kauffman's polynomial, by Przytycka and Przytycki (1988a) and Traldi (1989a) (and partially anticipated by Fortuin and Kasteleyn (1972a)), who also develop the parametrized dichromatic polynomial of a graph, principally in that it characterizes *all* strong Tutte functions; also in generalizing to matroids and in having little to say about knots. Schwärzler and Welsh (1993a) generalize to signed matroids (and characterize their strong Tutte functions) but not to arbitrary colors.] **(Sc(M), SGc: Gen: N, D, Knot)**

1993a The projective-planar signed graphs. *Discrete Math.* 113 (1993), 223–247. MR 94d:05047. Zbl. 779.05018.

Characterized by six forbidden minors or eight forbidden topological subgraphs, all small. A close analog of Kuratowski's theorem; the proof even has much of the spirit of the Dirac-Schuster proof of the latter, and all but one of the forbidden graphs are simply derived from the Kuratowski graphs.

[Paul Seymour showed me an alternative proof from Kuratowski's theorem that explains this; but it uses sophisticated results, as yet unpublished, of Robertson, Seymour, and Shih.] (**SG: T**)

[Related: "projective outer-planarity" (POP): embeddable in the projective plane with all vertices on a common face. I have found most of the 40 or so forbidden topological subgraphs for POP of signed graphs (finding the rest will be routine); the proof is long and tedious and will probably not be published. *Problem.* Find a reasonable proof.] (**SG: T**)

1994a Frame matroids and biased graphs. *European J. Combin.* 15 (1994), 303–307. MR 95a:05021. Zbl. 797.05027.

A simple matroidal characterization of the bias matroids of biased graphs. (**GG: M**)

1995a The signed chromatic number of the projective plane and Klein bottle and antipodal graph coloring. *J. Combin. Theory Ser. B* 63 (1995), 136–145. MR 95j:05099. Zbl. 822.05028.

Introducing the signed Heawood problem: what is the largest signed, or zero-free signed, chromatic number of any signed graph that orientation embeds in the sphere with h crosscaps? Solved for $h = 1, 2$. (**SG: T, Col**)

††1995b Biased graphs. III. Chromatic and dichromatic invariants. *J. Combin. Theory Ser. B* 64 (1995), 17–88. MR 96g:05139. Zbl. 950.25778.

Polynomials of gain and biased graphs: the fundamental object is a four-variable polynomial, the "polychromial" ("polychromatic polynomial"), that specializes to the chromatic, dichromatic, and Whitney-number polynomials. The polynomials come in two flavors: unrestricted and balanced, depending on the edge sets that appear in their defining sums. (They can be defined in the even greater abstraction of "two-ideal graphs", which clarifies the most basic properties.)

§4: "Gain-graph coloring". In $\Phi = (\Gamma, \varphi, \mathfrak{G})$, a "zero-free k-coloring" is a mapping $f : V \to [k] \times \mathfrak{G}$; it is "proper" if, when $e{:}vw$ is a link or loop and $f(v) = (i, g), f(w) = (i, h)$, then $h \neq g\varphi(e; v, w)$. A "k-coloring" is similar but the color set is enlarged by inclusion of a color 0; propriety requires the additional restriction that $f(v)$ and $f(w)$ are not both 0 (and $f(v) \neq 0$ if v supports a half edge). In particular, a "group-coloring" of Φ is a zero-free 1-coloring (ignoring the irrelevant numerical part of the color). A "partial group-coloring" is a group-coloring of an induced subgraph [which can only be proper if the uncolored vertices form a stable set]. The unrestricted and balanced chromatic polynomials count, respectively, unrestricted and zero-free proper k-colorings; the two Whitney-number polynomials count all colorings, proper and improper, by their improper edge sets.

§5: "The matroid connection". The various polynomials are, in essence, bias matroid invariants and closely related to corresponding lift matroid and extended lift matroid invariants.

Almost infinitely many identities, some of them (esp., the balanced expansion formulas in §6) essential. Innumerable examples worked in detail. [The first half, to the middle of §6, is fundamental. The rest is more or less ornamental. Most of the results are, intentionally, generalizations of properties of ordinary graphs.] (**GG: N, M, Col**)

1996a The order upper bound on parity embedding of a graph. *J. Combin. Theory Ser. B* 68 (1996), 149–160. MR 98f:05055. Zbl. 856.05030.

The smallest surface that holds K_n with loops, if odd polygons reverse orientation, even ones preserve it (this is parity embedding). That is, the demigenus $d(-K_n^\circ)$. **(P: T)**

1997a Is there a matroid theory of signed graph embedding? *Ars Combinatoria* 45 (1997), 129–141. MR 97m:05084. **(SG: M, T)**

1997b The largest parity demigenus of a simple graph. *J. Combin. Theory Ser. B* 70 (1997), 325–345. MR 99e:05043. Zbl. 970.37744.

Like (1996a), but without loops. *Conjecture* 1. The minimal surface for parity embedding K_n is sufficient for orientation embedding of any signed K_n. *Conjectures* 3–4. The minimal surfaces of $\pm K_n^\circ$ and $\pm K_n$ are the smallest permitted by the lower bound obtained from Euler's polyhedral formula. **(P: K: T)**

1998a Signed analogs of bipartite graphs. *Discrete Math.* 179 (1998), 205–216. Zbl. 980.06737

Basically, they are the antibalanced and bipartite signed graphs; but the exact description depends on the characterization one chooses for biparticity: whether it is evenness of polygons, closed walks, face boundaries in surface embeddings, etc. Characterization by chromatic number leads to a slightly more different list of analogs. **(SG: Str, T)**

1998b A mathematical bibliography of signed and gain graphs and allied areas. *Electronic J. Combin.*, Dynamic Surveys in Combinatorics (1998), No. DS8. Zbl. 898.05001.

Complete and annotated—or as nearly so as I can make it. In preparation in perpetuum. Hurry, hurry, write an article!

(SG, O, GG, GN, SD, VS, TG, PsS, ...)

Published edns.: Edn. 6a (Edition 6, Revision a), 1998 July 20 (iv + 124 pp.).

1998c Glossary of signed and gain graphs and allied areas. *Electronic J. Combin.*, Dynamic Surveys in Combinatorics (1998), No. DS9. Zbl. 898.05002.

A complete (or so it is intended) terminological dictionary of signed, gain, and biased graphs and related topics; including necessary special terminology from ordinary graph theory and mathematical interpretations of the special terminology of applications.

(SG, O, GG, GN, SD, VS, TG, ... , Chem, Phys, PsS, Appl)

Published edns.: 1998 July 21 (25 pp.). Second edn. 1998 September 18 (41 pp.).

1997p Avoiding the identity. Problem 10606, *Amer. Math. Monthly* 104, No. 7 (Aug.–Sept., 1997), 664. Solution by Stephen M. Gagola, *ibid.* 106 (6) (June-July 1999), 590-591.

[The solution implies that $(*)$ $f_0(m) \leq \lceil 2^{m-1}(m-1)!\sqrt{e}\rceil$, where $f_0(m) =$ the smallest r such that every group of order $\geq r$ is a possible gain group for every contrabalanced gain graph of cyclomatic number equal to m. *Problem* 1. Find a good upper bound on f_0. ($(*)$ is probably weak.) *Problem* 2. Find a good lower bound. *Problem* 3. Estimate f_0) asymptotically.] ("Avoiding the identity" concerns not f_0 but a larger function f corresponding to a simplified question.) **(GG)**

20xxa The largest demigenus of a bipartite signed graph. Submitted.

The smallest surface for orientation embedding of $\pm K_{r,s}$. **(SG: T)**

20xxb Line graphs of signed graphs and digraphs. In preparation. (See: Abstract 768-05-3, Line graphs of digraphs. *Notices Amer. Math. Soc.* 26, No. 5 (August, 1979), A-448.)

Line graphs of signed graphs are, fundamentally, (bidirected) line graphs of bidirected graphs. Then the line graph of a signed graph is a polar graph, i.e., a switching class of bidirected graphs; the line graph of a polar graph is a signed graph; and the line graph of a sign-biased graph, i.e., of a switching class of signed graphs, is a sign-biased graph. In particular, the line graph of an antibalanced switching class is an antibalanced switching class. (Partly for this reason, ordinary graphs should usually be regarded as antibalanced, i.e., all negative, in line graph theory.) Since a digraph is an oriented all-positive signed graph, its line graph is a bidirected graph whose positive part is the Harary-Norman line digraph. Among the line graphs of signed graphs, some reduce by cancellation of parallel but oppositely signed edges to all-negative graphs; these are precisely Hoffman's generalized line graph of ordinary graphs, a fact which explains their line-graph-like behavior. [Attempts at a completely descriptive line graph of a digraph were Muracchini and Ghirlanda (1965a) and Hemminger and Klerlein (1979a). The geometry of line graphs and signed graphs has been developed by Vijayakumar *et al.* See also (1984c).] (**SG: LG: O, I, A(LG). Sw**)

20xxc Perpendicular dissections of space. In preparation. (**GG: M, G**)

20xxd Geometric lattices of structured partitions: I. Gain-graphic matroids and group-valued partitions. Manuscript, 1985 et seq. (**GG: M, N, col**)

20xxe Geometric lattices of structured partitions: II. Lattices of group-valued partitions based on graphs and sets. Manuscript, 1985 et seq. (**GG: M, N, col**)

††20xxf Biased graphs. IV. Geometrical realizations. *J. Combin. Theory Ser. B.* (to appear). (**GG: M, G, N**)

20xxg Universal and topological gains for biased graphs. In preparation. (**GG: T**)

20xxh Supersolvable frame-matroid and graphic-lift lattices. *European J. Combin.* (to appear).

Supersolvable biased-graph matroids, characterized by a form of simplicial vertex ordering (that is, reverse perfect vertex elimination scheme)—but with a few exceptions (it's combinatorics!). Later sections treat examples. §4: "Near-Dowling and dowling lift lattices". §5: "Group expansions and biased expansions". §6: "An extension of Edelman and Reiner's theorem" to general gain groups (see Edelman and Reiner (1994a)). §7: "Composed partitions and circular n-permutation polynomials": the lattice of k-composed partial partitions and the meet subsemilattice of k-composed partitions. §8: "Bicircular matroids". (**GG, SG: M, G**)

20xxi Big flats in a box. In preparation.

The naive approach to characteristic polynomials via lattice point counting (in characteristic 0) and Möbius inversion (as in Blass and Sagan (1998a)) can only work when one expects it to. [This is a theorem!] (**GG: G, M, N, col**)

Morris Zelditch, Jr.

See J. Berger.

Bohdan Zelinka

See also R.L. Hemminger.

1973a Polare und polarisierte Graphen. In: *XVIII. Internat. Wiss. Kolloqu.* (Ilmenau, 1973), Vol. 2, Vortragsreihe "Theorie der Graphen und Netzwerke", pp. 27–28. Technische Hochschule, Ilmenau, 1973. Zbl. 272.05102.

See (1976a). [This appears to be a very brief abstract of a lecture.] **(sg: O, sw)**

1973b Quasigroups and factorisation of complete digraphs. *Mat. Časopis* 23 (1973), 333–341. MR 50 #12799. Zbl. 271.20039.

Establishes correspondences between quasigroups, algebraic loops, and groups on one hand, and 1-factored complete digraphs on the other, and between automorphisms of the latter and autotopies of the former. **(GG: Aut)**

1974a Polar graphs and railway traffic. *Aplikace Mat.* 19 (1974), 169–176. MR 49 #12066. Zbl. 283.05116.

See (1976a) for definitions. Railway tracks and switches modeled by edges and vertices of a polar graph. Forming its derived graph (see (1976d)), thence a digraph obtained therefrom by splitting vertices into two copies and adjusting arcs, the time for a train to go from one segment to another is found by a shortest path calculation in the digraph. A similar method is used to solve the problem for several trains. **(sg: O, sw: LG: Appl)**

1976a Isomorphisms of polar and polarized graphs. *Czechoslovak Math. J.* 26 (101) (1976), 339–351. MR 58 #16429. Zbl. 341.05121.

Basic definitions (Zítek (1972a)): "Polarized graph" B = bidirected graph (with no negative loops and no parallel edges sharing the same bidirection). "Polar graph" $P \cong$ switching class of bidirected graphs (that is, we forget which direction at a vertex is in and which is out—here called "north" and "south" poles—but we remember that they are different).

Thms. 1–6. Elementary results about automorphisms, including finding the automorphism groups of the "complete polarized" and polar graphs. (The "complete polarized graph" has every possible bidirected link and positive loop, without repetition.) Thm. 7: With small exceptions, any (ordinary) graph can be made polar as, say, P so that $\operatorname{Aut} P$ is trivial.

Thms. 8–10. Analogs of Whitney's theorem that the line graph almost always determines the graph. The "pole graph" B^* of B or $[B]$: Split each vertex into an "in" copy and an "out" copy and connect the edges appropriately. [Generalizes splitting a digraph into a bipartite graph. It appears to be a "twisted" signed double covering graph.] Thm. 8. The pole graph is determined, with two exceptions, by the edge relation $e \sim_1 f$ if both enter or both leave a common vertex. (A trivial consequence of Whitney's theorem.) Thm. 9. A polar graph $[B]$ with enough edges going in and out at each vertex is determined by the edge relation $e \sim_2 f$ if one enters and the other exits a common vertex. (Examples show that too few edges going in and out leave $[B]$ undetermined.) Thm. 10. Knowing $\sim_1$, $\sim_2$, and which edges are parallel with the same sign, and if no component of the simplified underlying graph of B is one of twelve forbidden graphs, then $[B]$ is determined. [*Problem* 1. Improve Thm. 10 to a complete characterization of the bidirected graphs that are reconstructible from their line graphs (which are to be taken as bidirected; see Zaslavsky (1984c, 20xxb)). In connection with this, see results on characterizing line graphs of bidirected (or signed) graphs by Vijayakumar (1987a). *Problem* 2. It would be interesting to improve Thm. 9.] **(sg: O, sw: Aut, lg)**

1976b Analoga of Menger's theorem for polar and polarized graphs. *Czechoslovak Math. J.* 26 (101) (1976), 352–360. MR 58 #16430. Zbl. 341.05122.

See (1976a) for basic definitions. Here is the framework of the 8 theorems. Given a bidirected or polar graph, B or P, vertices a and b, and a type X of walk, let s_X $[s'_X]$ = the fewest vertices [edges] whose deletion eliminates all (a,b) walks of type X, and let d_X $[d'_X]$ = maximum number of suitably pairwise internally vertex-disjoint [or, suitably pairwise edge-disjoint] walks of type X from a to b. [My notation.] By "suitably" I mean that a common internal vertex or edge is allowed in P (but not in B) if it is used oppositely by the two walks using it. (See the paper for details.) Thms. 1–4_1 (there are two Theorems 4) concern all-positive and all-introverted walks in a bidirected ("polarized") graph, and are simply the vertex and edge Menger theorems applied to the positive and introverted subgraphs. Thms. 4_2–7 concern polar graphs and have the form $s_X \le d_X \le 2s_X$ $[s'_X \le d'_X \le 2s'_X]$, which is best possible. Thms. 4_2–5 concern type "heteropolar" (equivalently, directed walks in a bidirected graph). The proofs depend on Menger's theorems in the double covering graph of the polar graph. [Since this has 2 vertices for each 1 in the polar graph, the range of d_X $[d'_X]$ is explained.] Thms. 6–7 concern type "homopolar" (i.e., antidirected walks). The proofs employ the pole graph (see (1976a)). **(sg: O, sw: Paths)**

1976c Eulerian polar graphs. *Czechoslovak Math. J.* 26 (101) (1976), 361–364. MR 58 #21869. Zbl. 341.05123.

See (1976a) for basic definitions. An Eulerian trail in a bidirected graph is a directed trail containing every edge. [Equivalently, a heteropolar trail that contains all the edges in the corresponding polar graph.] It is closed if the endpoints coincide and the trail enters at one end and departs at the other. The fewest directed trails needed to cover a connected bidirected graph is $\frac{1}{2}$ the total of the absolute differences between in-degrees and out-degrees at all vertices, or 1 if in-degree = out-degree everywhere. **(sg: O, sw: Paths)**

1976d Self-derived polar graphs. *Czechoslovak Math. J.* 26 (101) (1976), 365–370. MR 58 #16431. Zbl. 341.05124.

See (1976a) for basic definitions. The "derived graph" of a bidirected graph [this is equivalent to the author's terminology] is essentially the positive part of the bidirected line graph. The theorem can be restated, somewhat simplified: A finite connected bidirected graph B is isomorphic to its derived graph iff B is balanced and contains exactly one polygon. **(sg: O, sw: LG)**

1976e Groups and polar graphs. *Časopis Pěst. Mat.* 101 (1976), 2–6. MR 58 #21790. Zbl. 319.05118.

See (1976a) for basic definitions. A polar graph $PG(\mathfrak{G},A)$ of a group and a subset A is defined. [It is the Cayley digraph.] In bidirected language: a (bi)directed graph is "homogeneous" if it has automorphisms that are transitive on vertices, both preserving and reversing the orientations of edges, and that induce an arbitrary permutation of the incoming edges at any given vertex, and similarly for outgoing edges. It is shown that the Cayley digraph $PG(\mathfrak{G},A)$, where $\mathfrak{G}$ is a group and A is a set of generators, is homogeneous if A is both arbitrarily permutable and invertible by Aut $\mathfrak{G}$. [Bidirection—i.e., the polarity—seems to play no part here.] **(sg: O, sw: Aut)**

1982a On double covers of graphs. *Math. Slovaca* 32 (1982), 49–54. MR 83b:05072. Zbl.

483.05057.

Is a simple graph Γ a double cover of some signing of a simple graph? An elementary answer in terms of involutions of Γ. Further: if there are two such involutions α_0, α_1 that commute, then Γ/α_i has involution induced by α_{1-i}, so is a double cover of $\Gamma/\langle\alpha_0, \alpha_1\rangle$, which is not necessarily simple. [No properties of particular interest for signed covering are treated.] **(sg: Cov)**

1983a Double covers and logics of graphs. *Czechoslovak Math. J.* 33 (108) (1983), 354–360. MR 85k:05098a. Zbl. 537.05070.

The double covers here are those of all-negative simple graphs (hence are bipartite; denote them by $B(\Gamma)$. Some properties of these double covers are proved, then connections with a certain lattice (the "logic") of a graph. **(p: Cov: Aut)**

1983b Double covers and logics of graphs II. *Math. Slovaca* 33 (1983), 329–334. MR 85k:05098b. Zbl. 524.05058.

The second half of (1983a). **(p: Cov: Aut)**

1988a A remark on signed posets and signed graphs. *Czechoslovak Math. J.* 38 (113) (1988), 673–676. MR 90g:05157. Zbl. 679.05067 (q.v.).

Harary and Sagan (1983a) asked: which signed graphs have the form $S(P)$ for some poset P? Zelinka gives a rather complicated answer for all-negative signed graphs, which has interesting corollaries. For instance, Cor. 3: If $S(P)$ is all negative, and P has $\hat{0}$ or $\hat{1}$, then $S(P)$ is a tree. **(SG, S)**

Hans-Olov Zetterström

See Harary, Lindstrom, and Zetterström (1982a).

G.M. Ziegler

See A. Björner and L. Lovász.

Ping Zhang

1997a The characteristic polynomials of subarrangements of Coxeter arrangements. *Discrete Math.* 177 (1997), 245–248. MR 98i:52016. Zbl. 980.06614.

Blass and Sagan's (1998a) geometrical form of signed-graph coloring is used to calculate (I) characteristic polynomials of several versions of k-equal subspace arrangements (these are the main results) and (II) [also in Zhang (20xxa)] the chromatic polynomials (in geometrical guise) of ordinary graphs extending K_n by one vertex, signed graphs extending $\pm K_n^\circ$ by one vertex, and $\pm K_n$ with any number of negative loops adjoined. **(sg: N, G, col)**

20xxa The characteristic polynomials of interpolations between Coxeter arrangements. Submitted.

Uses signed-graph coloring (in geometrical guise) to evaluate the chromatic polynomials (in geometrical guise) of all signed graphs interpolating between (1) $+K_n$ and $+K_{n+1}$ [i.e., ordinary graphs extending a complete graph by one vertex]; (2) $\pm K_{n-1}^\circ$ and $\pm K_n^\circ$; (3) $\pm K_n$ and $\pm K_n^\circ$ [known already by several methods, including this one]; (4a) $\pm K_{n-1}$ and $\pm K_{n-1} \cup +K_n$; (4b) $\pm K_{n-1} \cup +K_n$ and $\pm K_n$; and certain signed graphs interpolating (by adding negative edges one vertex at a time, or working down and removing them one vertex at a time) between (5) $+K_n$ and $\pm K_n^\circ$; (6) $+K_n$ and $\pm K_n$. In cases (1)–(3) the chromatic polynomial depends only on how many edges are added [which is obvious from the coloring procedure]. **(sg: N, col, G)**

Xiankun Zhang

See H.-J. Lai.

F. Zítek

1972a Polarisované grafy. [Polarized graphs.] Lecture at the Czechoslovak Conference on Graph Theory, Štiřín, May, 1972.

For definitions see Zelinka (1976a). For work on these objects see many papers of Zelinka. **(sg: O, sw)**

Uri Zwick

See R. Yuster.

Ondřej Zýka

See J. Kratochvíl.