Marx, Mao and Mathematics: The Politics of Infinitesimals

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Abstract. The "Mathematical Manuscripts" of Karl Marx were first published (in part) in Russian in 1933, along with an analysis by S. A. Yanovskaya. Friedrich Engels was the first to call attention to the existence of these manuscripts in the preface to his Anti-Dühring [1885]. A more definitive edition of the "Manuscripts" was eventually published, under the direction of Yanovskaya, in 1968, and subsequently numerous translations have also appeared. Marx was interested in mathematics primarily because of its relation to his ideas on political economy, but he also saw the idea of variable magnitude as directly related to dialectical processes in nature. He regarded questions about the foundations of the differential calculus as a "touchstone of the application of the method of materialist dialectics to mathematics." Nearly a century later, Chinese mathematicians explicitly linked Marxist ideology and the foundations of mathematics through a new program interpreting calculus in terms of nonstandard analysis. During the Cultural Revolution (1966–1976), mathematics was suspect for being too abstract, aloof from the concerns of the common man and the struggle to meet the basic needs of daily life in a still largely agrarian society. But during the Cultural Revolution, when Chinese mathematicians discovered the mathematical manuscripts of Karl Marx, these seemed to offer fresh grounds for justifying abstract mathematics, especially concern for foundations and critical evaluation of the calculus. At least one study group in the Department of Mathematics at Chekiang Teachers College issued its own account of "The Brilliant Victory of Dialectics - Notes on Studying Marx's 'Mathematical Manuscripts'." Inspired by nonstandard analysis, introduced by Abraham Robinson only a few years previously, some Chinese mathematicians adapted the model Marx had laid down a century earlier in analyzing the calculus, and especially the nature of infinitesimals in mathematics, from a Marxist perspective. But they did so with new technical tools available thanks to Robinson but unknown to Marx when he began to study the calculus in the 1860s. As a result, considerable interest in nonstandard analysis has developed subsequently in China, and almost immediately after the Cultural Revolution was officially over in 1976, the first all-China conference on nonstandard analysis was held in Xinxiang, Henan Province, in 1978.

Chinese Versions of the "Mathematical Manuscripts" of Karl Marx

There were two editorial groups working in the early 1970s on Chinese translations of Marx's "Mathematical Manuscripts," one in Shanghai, the other in Beijing; the Shanghai group was the first to publish trial editions and then excerpts of Marx's "Mathematical Manuscripts." Working initially from a Japanese translation, the "Fu Dan University Scientific Reference Section" (复旦大学理科资料组) completed a first draft which was circulated for discussion in 1971. Two years later, with a copy of the Russian-German edition in hand (which provided transcriptions of the original manuscripts in German), a revised trial edition was printed and in 1974, translations of Marx's essays on derivatives, differentials, and the history of the calculus were published in two successive issues of the Shanghai journal, *Dialectics of Nature* (自然辩证法). A year later, the entire translation appeared as a special edition (专辑) of the *Journal of Fu Dan University* (复旦学报), along with a brief "Remark on the Translation" (译者的话). Meanwhile, in the same year that the Shanghai edition of the manuscripts was printed, a study group at Beijing University published its own translation of three of Marx's essays on the history of the differential calculus, interpreted specifically within a Marxist framework as a "stage in the development of history." When these appeared in the *Acta Mathematica Sinica* in 1975, they were preceded by a half-page of explanatory remarks from the "main editorial committee," wherein it was emphasized that this was a proletarian work, published by the People's Press (人民出版社), and meant to contribute to the socialist revolution and to socialist reconstruction:

> To promote the great campaign criticizing Lin Biao (林彪) and Confucius, the *Mathematical Manuscripts* of [Karl] Marx, who inspired the proletarian revolution, were translated and edited by the Mathematical Manuscripts Study Group of Beijing University, and published by the People's Press (人民出版社). This is a great event on our ideological battlefield.
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> Lenin pointed out that "with material dialectics to improve essentially the entire political economy, using dialectical materialism to elucidate history, natural science, philosophy, and the policies and strategies of the working class is the most important thing of concern to Marx and Engels, whereby they made their most important and novel contributions, and brilliantly took a giant step in revolutionary intellectual history."

Marx, the preface points out, used dialectical materialism to evaluate the history of the calculus, and was especially critical of what he took to be its idealistic, metaphysical foundations. Chairman Mao himself emphasized repeatedly that dialectics was the key to proper understanding of the sciences. Dialectical materialism was the weapon, literally, that Mao expected Chinese revisionists to use-even revisionist mathematicians-to root out any bourgeois elements and advance mathematics down "Chairman Mao's revolutionary route." Mathematicians thus took their publication of the mathematical manuscripts of Karl Marx as the perfect blueprint showing how their own criticism of mathematics should proceed:

> The great leader, Chairman Mao, has written that "you who study the natural sciences should learn how to use dialectics." By studying Marx's *Mathematical Manuscripts*, our theoretical understanding will reach a higher level,

> and will help us to take hold of the perfect weapons, advancing criticism of revisionism and of bourgeois world outlooks, [thereby] joining the battlefield with Marxism. People who study or teach mathematics should study and use dialectical materialism, which is clarified in the *Mathematical Manuscripts* of [Karl] Marx, to guide their practice and conscientiously improve their world outlooks, pushing the study of mathematics very quickly along Chairman Mao's revolutionary route, making a greater contribution to the socialist revolution and socialist construction.

Within months the Beijing University study group was satisfied that its entire translation was ready for publication, and in July of 1975 issued its definitive edition which included photocopies of several pages from Marx's original manuscripts. Part II reproduced verbatim the sections already issued previously that year. Although the Beijing translation differs in choice of words from time to time from the Shanghai translation, what sets the Beijing edition apart is its inclusion of explanatory terms from the original German version from which the Beijing translation was made. For example, terms like *"Differentiation," "abgeleitete Funktion,"* and *"Grenzwert"* appear, parenthetically, to explain Chinese terminology when new terms/characters are first introduced.

FIRST REACTIONS TO PUBLICATION OF THE MATHEMATICAL MANUSCRIPTS

No sooner had the first two parts of the translation of the manuscripts by the Shanghai group appeared in print than the editors of the *Journal of the Dialectics* of Nature (自然辩证法杂志) began to receive letters from a wide variety of readers. The next number of the journal to appear contained a selection of these letters in a section entitled "Discussion of Problems Concerning Differentials and Limits" (关于微积分和极限问题的讨论). This began with a note from the editors explaining all of the mail the journal had received. Several letters were then published in their entirety, with excerpts from a number of others. The first letter was from a second-year student at Beijing Middle School No. 144, He Fang (何放), who asked "How Should the Concept of Limit be Understood?" (应当怎样认识极限?). The next contribution was from a worker at Factory No. 5703 in Shanghai, Fu Xi-tao (傅锡涛), who was interested in: "Trying to Say Something Concerning my Feelings About Improving Teaching of the Calculus Using Dialectics" (试用辩证法改革微积分教学的一点体会). Fu Xi-tao explained how dialectics could be applied to reform calculus teaching. A third letter came from Zheng Li-xing (郑礼星) of the Fujian Electrical Engineering School (福建机电学校) in Fuzhou, Fujian Province. Zheng took up one side of the debate over whether the differential dx was zero or not, arguing: "The Differential is Comparable to Zero," (微分是相对的零).

Along with their publication of "Selections from Manuscripts Received" (来稿摘登), the editors of *Dialectics of Nature* included excerpts from letters by readers who had studied the translation of the mathematical manuscripts published in the preceding two issues of the journal. The first was taken from a letter by Xu Ting-dong (徐庭栋), who identified himself as a young worker in the Qing-Hai Tractor Factory (西宁青海拖拉机厂). His comments were devoted to "The Differential is a Unity of Zero and Non-Zero" (微分是零和非零的统一),

and drew on similar dialectical criticism of the foundations of the calculus already raised by Zheng Li-xing. But Xu Ting-dong also considered the calculus applied to motion, and was especially interested in discussing acceleration and the derivative. The next letter, attributed to Wu Guang-xia (吳光夏) of Bao Tou Teachers School (包头师范学校) in Inner Mongolia, was also concerned with the zero/non-zero aspect of the differential. Another letter along these same lines came from Chen Ke-jian (陈克艰), a "knowledgeable youth" (知识青年) from Shang Shan Xia Xiang (上山下乡). Again, his analysis was devoted to considering the differential as "zero" and "non-zero," interpreting the calculus as it applied to motion and the paradoxes that arise from trying to consider a moving point as being in any "one" place.

From Harbin Industrial University (哈尔滨工业大学), Shen Tian-ji (沈天骥) wrote to suggest that "The Differential Reflects Quantitative Change from (Two) Different Points of View" (微分反映量变在不同层次的关节点). Here the two different points of view were of Δx versus δx, and the difference between non-zero and zero, as well as the meaning of $\delta y = f(x)\Delta x$. The last letter in this collection of differing points of view prompted by publication of the "Mathematical Manuscripts" was from a young worker at a Shanghai machine packing plant, Chen Li-qin (陈利钦), who insisted that "The Differential Must be Considered as Zero" (微分应当归结为零). Chen's argument was based on his understanding of the limit: $\lim_{\Delta x \to 0} = \frac{\Delta y}{\Delta x} = \frac{dy}{dy} = f'(x)$.

MATHEMATICIANS BEGIN TO RESPOND

Thus in 1975, two definitive editions of Marx's "Mathematical Manuscripts" appeared in Chinese. The Beijing edition differed only slightly from the Shanghai version, and in some cases they paralleled each other *verbatim* in the Chinese. But with the entire collection of Marx's "Mathematical Manuscripts" now at their disposal, it was not only high school students and factory workers who took an interest, but so too professional mathematicians. For example, writing in the *Journal of Beijing Normal University*, Zhi Zhou (郅洲) of the Philosophy Department explained "How to Understand Derivatives (导函数)—Notes on studying Marx's Mathematical Manuscripts" (怎样理解导函数—学习马克思≪数学手稿≫笔记). In his introduction, Zhi Zhou explained that whereas the calculus as a scientific subject came into being at the end of the 17th century, it did not develop into a satisfactory theory until the middle of the 19th century. As a result of work done in the 17th and 18th centuries, when metaphysical concepts dominated the natural sciences, the fundamental concept of the calculus, namely the derivative, was also subjected to strong metaphysical influences. In the 1870s, the revolutionary teachings of Marx severely criticized such metaphysical foundations for the derivative, and advocated a correct interpretation on the basis of dialectical materialism. In the first section of his paper, Zhi Zhou examined the history of the derivative, and explained how Newton and Leibniz had introduced the concept as a ratio of differentials. He added that according to Bishop Berkeley (大主教), a representative of "English subjective idealism" (英国的主观唯心主义), the differential dx was literally, in Chinese, exactly what Berkeley had said it was in English, "the ghost of a departed quantity" (逝去量的鬼魂) [Zhi Zhou 1975, p. 19]. Zhi Zhou also considered both d'Alembert's approach to the calculus in terms of differences Δt and differentials dt, as well as Lagrange's approach expanding functions in terms of their Taylor series, for which the derivative was taken as the coefficient of the linear term of the infinitesimal h, i.e.:

$$f(x+h) = f(x) + hp_1(x) + h^2 p_2(x)/2! + h^3 p_3(x)/3! + \dots$$

where $p_1(x)$ was taken to define the derivative, i.e. $f'(x) = p_1(x)$. But whereas Marx stopped with his analysis of the historical development of the calculus at this point, Zhi Zhou went further to consider the contributions made by the French mathematician Cauchy (哥西), specifically the definition of the limit that Cauchy gave in his *Cours d'analyse* of 1821 (分析教程). This was all discussed expressly in terms that Engels had used in 1830 in his *Dialectics of Nature*. Although Zhi Zhou was aware of the fact that Marx never expressly mentioned Cauchy by name, he could not believe that Marx was unaware of the basic ideas used by Cauchy, since Cauchy's point of view was represented in many of the most popular scientific books of his day. Zhi Zhou, who asks why Cauchy did not permit the variable x to actually reach or attain the limit $x = 0$, explained that it was because he feared this would lead to the "monster" (怪物) 0/0. He notes that later, in the 1850s, the $\epsilon - \delta$ method of proof appeared, and a few decades later, in the 1870s, this was linked to a thorough critique of the real numbers (实数理论). Nevertheless, the first person to "strip away the appearances" and submit the concept of the derivative (导函数) to a thorough metaphysical analysis was, in Zhi Zhou's opinion, none other than Karl Marx. Zhi Zhou devoted the second part of his paper to describing Marx's analysis of the derivative, especially the differential quotient dy/dx in terms of the paradoxical nature of 0/0, which strictly speaking was undefined, or could represent any value at all. He concluded his essay by returning to the founders of the calculus, to Newton and Leibniz. The characterization was classic Marxism: "Newton's and Leibniz's contributions to the calculus, are great pioneering works in the development of mathematics, but due to the constraints of metaphysical ideology, their works could not avoid being colored by mysticism (神秘主义)." From Newton until the time of Marx, although the calculus underwent considerable development, and despite the fact that the concept of the derivative also had a rich dialectical context, it was still trapped in a web of metaphysical ideology. Owing to the constraints of metaphysics, and even though they raised their voices against "old fashioned orthodox schools of thought," mathematicians could find no alternatives:

> (Our) revolutionary leader Marx, because of his deep grasp of the method of dialectical materialism, thus focused on the idea of the derivative and advanced a series of brilliant dialectical thoughts, even though over the past 200 years mathematicians have been working but have not yet been able to make a great contribution.
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> Marx's "Mathematical Manuscripts" are one part of a brilliant, monumental mathematical work, and are a precious scientific legacy Marx has left to us. It is not only part of the mathematical writings, but is also a part

> of his philosophy which uses the methods of dialectics as a model for studying mathematics. Teaching and studying the "mathematical manuscripts" is necessary for today's revolution in education, and is needed in the battle to conquer mathematics.
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> Scientists and especially mathematicians, from the study and research of the "Mathematical Manuscripts," [have] a powerful ideological weapon to transform directly the old mathematical system and to reform the study and teaching of mathematics.

JOURNAL OF FU DAN UNIVERSITY (复旦学报)—1975

Meanwhile, in Shanghai the editors of the *Journal of Fu Dan University* continued to publish new manuscripts submitted in the wake of their publication of the *Mathematical Manuscripts of Karl Marx*. In the second number of 1975, for example, Ou Yang Guang-zhong (欧阳光中) and Zhu Xue-yan (朱学炎) offered a "Discussion on some Ways of Looking at the Calculus of Functions of Several Variables" (谈谈对多元函数微积分的一些看法). Ou Yang was a prominent mathematician who published a considerable amount during the Cultural Revolution; the article begins with strong praise for Marx:

> One hundred years ago the great revolutionary teacher Marx wrote his mathematical manuscripts, and although in the course of these hundred years mathematics has undergone tremendous development, Marx's mathematical manuscripts nevertheless still shine with a brilliant radiance. Marx in his mathematical manuscripts used the special materialist dialectics of Marxism to criticize every shade of idealist metaphysics, tearing the mysterious veil from the deceptive derivatives and differentials, and bringing to light their true essence, thereby setting a brilliant example for us.

In addition to political rhetoric, Ou Yang provided some sophisticated mathematics as well. This was more technical than anything written to this point in connection with the mathematical manuscripts of Karl Marx, for Ou Yang considered vector analysis, potential differences, gradients, the Poisson integral, triple integrals, and a host of related subjects. On a more elementary level, in the next issue of the *Journal of Fu Dan University*, a mathematician by the name of Shu Zuo (舒左) offered a paper meant to serve as "A Starting Point for Calculating with Differentials," (微分演算的出发). Here the name "Shu Zuo" was not only a pseudonym, but also a play on words and characters, for the characters 舒左 ("Shu Zuo") literally mean "Unfold the Left," but with a slightly different change in tone, the characters 数作 (also "Shu Zuo") mean "Do Mathematics":

> In the midst of the movement to study the theory of the dictatorship of the proletariat now surging forward with great momentum, publication of the translation of Marx's *Mathematical Manuscripts* is of great significance. "The proletariat must include in its superstructure (上层建筑) all areas of culture to exercise its dictatorship in every respect over the bourgeoisie." While in practice the domain of natural sciences is opposed to the universal dictatorship of the bourgeoisie, it is necessary to submit

> the development of the domain of the [sciences] to the great revolutionary criticism. Marx's mathematical manuscripts constitute a brilliant model for our great revolutionary criticism of the development of the domain of this subject.

After a study of the derivative, the very popular example $y = x^3$, and the problem of how to interpret dy/dx as $0/0$, Shu Zuo's paper draws to a close with a citation from the well-known letter Engels wrote to Marx on November 21, 1882, in which he discussed the meaning of $x + h$ as a point moving from position x to x_1. The paper ends on a typically Marxist note:

> We certainly must take this sharp weapon of dialectical materialism, to develop the great revolutionary criticism of the domain of our subject, dare to revolt, and know how to revolt! We are full of confidence that the mysterious veil of every shade enshrouding the natural sciences will certainly be torn away completely, and the domination of the natural sciences by idealism and metaphysics of the past systems will be thoroughly smashed, and the red flag of Marxism, Leninism and the thoughts of Mao Zedong will flutter high above the front position of the natural sciences. This is the universal truth that enlightens us by studying Marx's "Mathematical Manuscripts."

Shu Zuo's paper was immediately followed by another concerned with the "Mathematical Manuscripts," this one by Wu Wen-jing (吴文京) who identified himself as a worker at the Birch Woods Rubber Factory in Mu Dan Jiang (牡丹江桦林橡胶厂), a town in the North-East of China. Wu Wen-jing's paper was devoted to "The Differential and Dialectics" (微分和辩证法), and interpreted dialectics in terms of change, translated into an analysis of the mathematics of motion, a favorite Maoist theme among Marxist mathematicians. The paper discusses velocity and acceleration in terms of derivatives. Through a proper application of dialectical materialism, Wu Wen-jing insisted that a critical evaluation of the calculus would reveal its true essence. He also introduced another familiar theme as well, that it was the forces of production in society that spurred development of the natural sciences, in the course of which mathematics changed from a study of constants to variables, from static situations to ones that were dynamic and constantly changing. The last paper to be discussed here that was devoted to Marx's "Mathematical Manuscripts" in the 1975 issue of the *Journal of Fu Dan University* was a contribution by Yan Shao-zong (严绍宗), who presented his thoughts on "Basing the Concept of the Derivative on the Law of Opposites" (对立统一法则支配≪导函数概念≫ 的建立). Here "对立统一," the popular Maoist expression meaning "the unity of opposites," was nothing other than the familiar Hegelian or Marxist doctrine of the dialectical polarities of antithesis/synthesis. Yan asked the usual question, "What is to be understood by dy/dx?" Yan also cited Marx and the problem of interpreting $0/0$, and then took up the ubiquitous analysis of the equation $y = x^3$, in terms of which he discussed derivatives and distinguished between quotients of differences $\Delta y/\Delta x$ and differentials dy/dx.

CHAIRMAN MAO SPEAKS!

In 1975 a special issue of the journal *Practice and Understanding of Mathematics* (数学的实践与认识) opened with two slogans from *The Collected Sayings of Chairman Mao* (毛主席语录):

> From a certain point of view, the most talented and able soldier is one who has had the most practical experience.
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> Our improvement is improvement on the basis of popularization; our popularization is popularization under the guidance of improvement.

These slogans were meant to reflect the ideology of the journal, as well as the articles in an issue devoted to popularizing mathematics while emphasizing its practical applications. The opening contribution was by the pseudonymous Shu Zuo (舒左), who also contributed a paper that year to the *Journal of Fu Dan University*. This time his article was devoted to a report of a meeting held to study the mathematical manuscripts of Karl Marx, in the spirit of popularization that Chairman Mao himself had admonished everyone to pursue, all of which was reflected directly in the aphorism at the head of the journal. Another attempt to present the basic ideas found in Marx's "Mathematical Manuscripts" to a wider audience was a series of lectures devoted to "Studying Marx's Mathematical Manuscripts" that appeared in the popular journal, *Chinese Science* (中国科学). The first of these was written by Shu Li (舒立) from Beijing University, and was devoted to "Using Marxism to Conquer the Battlefield of Mathematics" (用马克思主义占领数学阵阵地). The allusion to conquering the battlefield was a rhetorical flourish drawing on language Mao himself often used in referring to the struggles China had to face on all fronts. In this case, the point was to advance the battle using dialectical materialism to criticize and revise the foundations of mathematics.

1976—YEAR OF THE DRAGON

On January 8, 1976, Premier Zhou Enlai died. Six months later, on July 28, the industrial and mining city of T'ang-Shan was destroyed by a major earthquake, killing 655,000 people and leaving more than a million people homeless. The third cataclysmic event that year occurred on September 9, when Chairman Mao died. The cover of the journal *Practice and Understanding of Mathematics* (数学的实践与认识) immediately carried a portrait of the Chairman, adorned with the slogan "Eternal Glory to the Mighty Leader and Teacher Chairman Mao Ze-Dong!" (伟大的领袖和导师毛泽东主席永垂不朽!).

The opening paper in this memorial issue commemorating Chairman Mao was a joint work from the study group for Marx's "Mathematical Manuscripts" in the Department of Mathematics at Beijing Normal Teacher's College. The article, "Studying Different World Outlooks from Two Different Mathematical Approaches" (从两种不同的数学推导看不同的世界观), contrasted d'Alembert's approach to the calculus with the foundations advocated by Marx. Admittedly a preliminary study, it was based on a "first reading" of the "Mathematical Manuscripts," but nevertheless reflected a remarkably sophisticated view of the historical differences between d'Alembert's theory of limits and the critical views of foundations of the calculus held by Marx. The same slogan—"Eternal Glory to the Mighty Leader and Teacher Chairman Mao Ze-Dong!"—also ran across the cover of the third number of the *Journal of Central China Industrial College* (华中工学院学报), atop its third issue for 1976, along with the same portrait of Chairman Mao that appeared virtually everywhere throughout China. Inside, however, a paper said to have been written by Shu Xuan (舒煊) in Wuhan was devoted to "Continuing to Use Marxism to Study Nonstandard Analysis" (坚持用马克思主义考察非标准分析). Although this article does not go into the technicalities of nonstandard analysis with actual applications to mathematics, pure or applied, it does try to develop the value of using nonstandard analysis in a spirit of evaluation and criticism of mathematics compatible with the views of Marx and Engels, both of whom are cited extensively in the article. The main point Shu Xuan makes here is that despite its suspect ideology, nonstandard analysis is nevertheless an important tool in reevaluating calculus along lines inspired by Marx and Engels.

Serious notice of nonstandard analysis

1976, the Year of the Dragon, was also the first in which a serious attempt was made in China to relate the technical details of Abraham Robinson's nonstandard analysis to proper understanding of the calculus. Written under a pseudonym, Shu Ji (舒基), an article appeared in the *Journal of North-West University* (西北大学学报) devoted to: "Discussing the Physical Origins of the Mathematical Structure of *R" (谈谈数学结构*R的现实原型). The major point of this paper was to introduce the nonstandard continuum *R, which included both infinitesimals and transfinite numbers as legitimate real numbers. Shu Ji sought to justify these, as well as nonstandard analysis in general, in terms of Marxist dialectical materialism. Once the theory was on firm ideological ground, the article proceed with deeper technical discussion of nonstandard analysis on its own terms. The article itself, and the views it introduced concerning nonstandard analysis, were prompted, Shu Ji notes, by opinions formed "after studying the dialectics of nature and Marx's mathematical manuscripts."

Shu Ji (舒基) devotes an entire section of his article to arguing that "the infinitely small (large) really are real numbers" (无限小(大)量是实在的数), where "really are real" means that the real numbers are ontologically real, concrete—in physical, material terms. After quoting from Marx's mathematical manuscripts, Engels' *Dialectics of Nature*, and Chairman Mao's "On the Correct Handling of Contradictions Among the People" (关于正确处理人民内部矛盾的问题), Shu Ji claims that Robinson himself recognized that nonstandard analysis was grounded in a concrete, material way in so far as the usefulness of infinitesimals was best seen in applications to real-world problems.

1977

In 1977 the first draft of a course of lectures given at Beijing Normal University were published by Huang Shun-Ji (黄顺基) and Wu Yan-Fu (吴延涪) in the journal *Understanding and Practice of Mathematics*. The opening lecture began with an introduction to studying the "Mathematical Manuscripts," noting that these constituted a "brilliant document" using dialectical materialism to analyze mathematics, and were a "treasure trove" (宝库) of dialectics. The first lecture follows Marx very closely in offering a critical analysis of the foundations of the calculus through its historical development. The authors point out that studying the "Mathematical Manuscripts" confirms what Engels said at Marx's graveside: that Marx had a special interest in mathematics and made fundamental contributions of his own to the subject. The contributions were primarily in applications to Marx's theory of surplus value, and in applications revealing the special laws of change underlying the evolution of capitalism and patterns of development reflected in modern society. As Huang and Wu emphasized in their introduction:

> The times we are facing today "are times when everything is turned upside down, to which nothing in past history can compare." To strengthen and reinforce the dictatorship of the proletariat, using Marxist-Leninism, the thoughts of Mao Ze-dong have taken command of every position, pioneered study of the manuscripts and research of very important practical significance.

The authors' introductory lecture is divided into four parts, the first devoted to describing the aims Marx had in mind when he wrote the manuscripts. Then comes a section devoted to the major contents and basic ideas of the manuscripts, followed by a third section explaining the process of writing and publishing the manuscripts. The last and most interesting part of this introduction to Marx's mathematical manuscripts considers their practical significance. Here Huang and Wu list a number of major practical results that follow from study of the "Mathematical Manuscripts." Above all, they note that in every branch of science the manuscripts may be used as "a pioneering weapon of revolutionary criticism."

The final article to be discussed here was published by Zhou Guan-xiong (周冠雄) in 1977: "Using the Philosophy of Marxism to Evaluate Nonstandard Analysis" (以马克思主义哲学为指导评价非标准分析). This appeared in the *Journal of Central China Industrial College*, and summarized its main argument as follows:

> The study and discussion of Marx's "Mathematical Manuscripts" are of real and profound value in helping us to understand dialectical materialism, and in studying mathematics using Marxism.... Chairman Mao's directive identifies how we should approach our study of foreign things, how the accounts of Marx, Engels, Lenin and Mao of the infinite and of higher mathematics supply theoretical weapons for evaluating nonstandard analysis. In his "Mathematical Manuscripts," Marx traced the history of the calculus from Newton to Lagrange, acknowledging their contributions and pointing out their idealistic and metaphysical errors.
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> Marx also analyzed the concepts of derivative, differential, differential operations, etc. Using his own philosophy, Marx outlines a series of very important results, which constitute a glorious model for examining nonstandard analysis....

> The core of nonstandard analysis provides a foundation for higher mathematics [with] infinitesimals. In [his] nonstandard analysis, [Abraham] Robinson (鲁滨逊) shows there is a certain infinitesimal between zero and any positive number using the methods of mathematical logic. The entire theory of nonstandard analysis constructs a mathematical system based on infinitesimals. The system provides another interpretation for the [viability] of the calculus, and another (mathematical) method distinct from the method of limits. We should accept the contributions Robinson has made, but object to the influence of Robinson's formalism, which in a system of natural science has its limitations. We must criticize Robinson's idealism as it appears in his works.

Conclusion

Since the founding of the People's Republic of China in 1949, Chinese scholars have produced a series of studies meant to explain, popularize and establish the methods and philosophy of dialectical materialism in virtually every field of study. In the sciences this has led to criticism, if not condemnation, of Mendelian genetics, of physics in both its Newtonian and Einsteinian interpretations, and in mathematics, of Euclidean geometry and—as has been described in some detail here—of the infinitesimal calculus. But unlike many of their colleagues in the Soviet Union, the Chinese avoided the disastrous consequences of Lysenko's triumph over Mendel by allowing that successful scientists, despite faulty philosophies, nevertheless unconsciously must have used dialectical materialism in guiding their research.

Throughout the Cultural Revolution (1966-1976), Mao Ze-dong promoted Marxism and dialectics to encourage reforms in all fields of endeavor, including the sciences. In mathematics, this encouraged, as it had Marx, an appreciation (with criticism) of the infinitesimal calculus. For Chinese mathematicians, application of Abraham Robinson's newly created nonstandard analysis not only rehabilitated infinitesimals in a technical sense, but (when understood within an appropriate materialist framework), could be used to justify and promote two new fields of study in China—model theory and nonstandard analysis.

[A complete text of this paper, including notes and bibliography, is available upon request from the author].

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