Large Groups Actions on Manifolds

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Abstract. We shall survey some results concerning large groups actions on manifolds, with an emphasis on rigidity and geometric questions. By large groups actions, we, in short, mean actions of non free groups, with at least a dense orbit. Most of the results will concern lattices, but we shall present results and questions concerning other large groups.

Introduction

In this article, we shall be interested in large group actions on manifolds. Large actions will mean highly non proper actions such as topologically transitive (*i.e.* with a dense orbit), or volume preserving ergodic ones (*i.e.* every invariant subset is either of full or zero volume). Large group is a rather unprecise notion, of which we do not have a definition but examples. They are at least required to be finitely generated non free groups. An important and well studied class since R. Zimmer work [Z] is that of *higher rank lattices* : a *lattice* Γ in a unimodular real Lie group G is a discrete subgroup such that G/Γ has finite volume; it is *cocompact* if G/Γ is compact; using a restricted definition for the sake of simplicity, by *higher rank* we mean that G is simple of real rank greater than 2. A good class of examples of higher rank lattices is $SL(n,\mathbb{Z})$, with $n \geq 3$. However, we shall try not to restrict ourselves to this class and to present results and questions concerning other groups.

Obviously, due to the presence of relations among the elements of our groups, large group actions should be rare and difficult to construct. In particular, given an action of a large group Γ on a manifold M, one would like to answer the local rigidity question, whose answer turns out to be positive in many case.

Local Rigidity Question: *is any smooth action of Γ on M close enough to the original one, conjugate to it within the group of diffeomorphisms ?*

In the case of higher rank lattices, people are even more optimistic. The general belief, supported by Margulis-Zimmer superrigidity, is that every large action is essentially geometric because, in some sense, the lattice carries the geometry determined by its ambient group. This leads to the following precise question

Geometric Question: *does every smooth topologically transitive action of a higher rank lattice preserve a rigid geometric structure (see section* **1** *for definitions) on some open dense set ?*

This expected behaviour is in sharp contrast with actions of $\mathbb{Z}$, *i.e.* action generated by a diffeomorphism. Even for the best understood class, Anosov diffeomophims, the answer for the smooth rigidity question is no. In this context, a classical question related to the geometric one is the question to decide wether a diffeomorphism is linearizable smoothly in the neighbourhood of a fixed point. Once more the answer is no in the smooth category.

Another measure of the rarity of lattices actions is the following conjecture of Zimmer, which is still open, even for $n = 3$.

Zimmer's Conjecture: *there is no non trivial smooth volume preserving ergodic action of $SL(n, \mathbb{Z})$, with $n \geq 3$, on a manifold of dimension strictly less than n.*

Here, a non trivial action means an action which does not factor through a finite group.

Although the word survey is written in the abstract, this article has no pretention to be exhaustive. The references quoted here should therefore be considered as starting links to explore the subject rather than the definitive ones on this topic. I also have tried to write it in a way accessible to non experts. It follows that in less than 10 pages, I will have to omit important historical results. Worse than that, most of the results I will present, will only be special cases of the original theorems, thus restricting the generality and beauty of the work of many of my colleagues. Once and for all, I apologize here for all these outrageous omissions and simplifications.

The structure of this article is as follows: the first two sections (1. on rigid geometric structures, 2. on superrigidity) are introductory; we then present known examples of actions of lattices in section 3; section 4 is concerned with hyperbolic (in the dynamical sense) actions; section 5 deals with (non volume preserving) actions on boundary spaces, such as the action of $SL(n, \mathbb{Z})$ on the n-dimensional sphere; in 6, we will discuss analytic actions; section 7 exposes a result of topological nature; finally in section 8, we, at least, quit the realm of lattices and present results and questions on other types of groups.

Unless otherwise specified, all objects and concepts (manifolds, actions, conjugations, *etc*) will be C^∞.

1. Rigid geometric structures

Let M be a n-dimensional manifold. Its kth-frame bundle, noted $M^{(k)}$, is the bundle over M whose fiber at a point x is the set of all local diffeomorphisms of $\mathbb{R}^n$ into M sending 0 to x, up to the following equivalence relation: having the same derivatives up to order k. The structure group of this bundle is the group $G^{(k)}$ of k-jets of diffeomorphims of $\mathbb{R}^n$, fixing 0. A *geometric A-structure (of type V)* is a section of the bundle associated to an algebraic action of $G^{(k)}$ on an algebraic variety V. Usual geometric structures (*e.g.* affine, riemannian, symplectic, complex, conformal ...) are of this type.

It does make sense for a diffeomorphism to preserve a geometric A-structure, and such a diffeomorphism will be called an *isometry* of the structure. A geometric A-structure is called *rigid* (in the sense of M. Gromov) if there exists some integer l such that all derivatives of an isometry fixing a point is completely determined by its first l ones. For instance, connections, non degenerate metrics, projective structures are rigid, though complex and symplectic structures are not.

This notion has been introduced by M. Gromov in [Gr] where he also presents a proof (later corrected by Y. Benoist [Be]) of the following important result

Theorem (Gromov [Gr]) *If the pseudogroup of local isometries of a rigid geometric structure has a dense orbit, it has an open dense orbit.*

Basically, this theorem says that, up to a technical point, every geometric structure admitting a topologically transitive group of isometries is modelled on some locally homogeneous space on some open dense set. It follows that to prove an action preserves a rigid geometric structure is an important step (though not the last one) towards an explicit description of this action. This is why I wanted to emphasize the geometric question stated in the introduction. On the other hand, I cannot survey the vast field of actions preserving geometric structures and instead refer to [G], [d'A-G] and [Z2].

2. Lattices and superrigidity

Let Γ be a lattice in a group G. Let's first discuss the interplay between actions of Γ and of G. Obviously an action of G on M restricts to an action of Γ on M. On the other hand, from an ergodic volume preserving action of Γ on M, we get a an ergodic volume preserving action of G on $(M \times G)/\Gamma$. This latter action, called the *suspended action* carries most of the information about the action of Γ. This is why informations about actions of lattices are quite often immediately derived from results about actions of the ambient group and *vice versa*. In particular, one can prove that for topologically transitive actions the fact that the action of Γ preserves a rigid geometric structure on some open dense set is equivalent to the fact the suspended action of G preserves a rigid geometric structure on some open dense set.

The geometric data discussed in Zimmer's version of Margulis superrigidity are the following. First, we have a finite volume preserving ergodic action of Γ, or equivalently of G, on a manifold M. Second, we assume this action lifts to an action on a H-principal bundle and we wish to decribe the lifted action. A basic example, *ρ-twisted action*, is given by a representation ρ of G in H and the action of Γ on $M \times H$ given by $\gamma(m.h) = (\gamma m, \rho(\gamma)m)$.

Let us now fix a class of regularity, *e.g* measurable, continuous, smooth *etc.*, we then suppose that H is a semisimple Lie group and is minimal in the following sense: there is no Γ-invariant section of any associated H/L-bundle, for all semisimple subgroups L of H. Here, the section is required to be of the desired regularity class and defined on a set of full measure (in the case of measurable sections), or on an open dense set (in the continuous case). For the sake of simplicity, assume futhermore that H is simple and non compact.

The superrigidity question is to decide whether or not the action is equivalent (by a bundle isomorphim) to a ρ-twisted action over, maybe, a slightly smaller set (*i.e.* of full measure, or open dense, depending on the category).

Zimmer's version of Margulis superrigidity asserts that the answer is always yes in the measurable category. Topological or smooth superrigidity tries to figure out a decent extra hypothesis that would make the story work in the topological or smooth context.

In the particular case when the action is the lift of the action of Γ to $M^{(k)}$, one may think of the superrigidity question as a linearized version of the geometric question. A typical application of the ideas of superrigidity in the smooth context is the following result, we shall only state for actions of Lie groups for the sake of simplicity.

THEOREM (R. FERES-F. LABOURIE [F-L]) *Let G be a simple Lie group of real rank greater than 2 (e.g. $SL(n,\mathbb{R})$, $n \geq 3$), acting ergodically preserving a finite volume on a manifold M, assume that some real split element of G (e.g. a real diagonizable matrix different from the identity) preserves a rigid geometric structure on some open dense set, then the whole group preserves a rigid geometric structure on some open dense set.*

The books [Z] and [M] are the standard references on the subject. Notice that [F-L] explains a short and self contained proof of a special case of superrigidity, later expanded in [F]. This latter reference should be recommended for a first approach.

It may be useful to explain an important structural property of simple real Lie groups G of rank greater than 2 which make them very different to those of real rank 1. This property is easily seen in $SL(n,\mathbb{R})$, $n \geq 3$: given two real split matrices B and C there exist a finite sequence of matrices A_i, $i \in 1,\ldots,p$, such that A_i commutes with A_{i+1}, $A_1 = B$ and $A_p = C$. We shall say a group having such a property is *generated by a chain of centralizers*. Although it is not clear that higher rank lattices have this property, it is important that the ambient group have it. One of the major steps of superrigidity is to infer a property of the whole group from a property sastified by one element using chains of centralizers. In the measurable category, we can build, using the Kakutani-Markov theorem, a measurable objet invariant by a single element and from this, build something invariant for the whole group. In the other categories, the existence of an object invariant by a single element is far from granted.

3. EXAMPLES (AND COUNTER-EXAMPLES) OF ACTIONS OF LATTICES

One of the interests of lattices is that they possess many actions, mainly on locally homegeneous spaces.

(a) Isometric actions. This first class may be considered as the trivial case of the theory. Since higher rank lattices admit morphisms into compact groups, it follows that we can construct lots of smooth ergodic actions of a lattice preserving a riemaniann metric. Obviously these examples cannot be classified. They do however exhibit a rigidity property shown by J. Benveniste

THEOREM (J. BENVENISTE [B1]) *Every isometric action of a higher rank cocompact lattice on a compact manifold is locally rigid.*

(b) Volume preserving actions. The following two examples are sometimes called *standard actions.*

(1) This first example generalizes the action of $SL(n,\mathbb{Z})$ on the n-dimensional torus. Let N be a simply connected nilpotent group and Λ a lattice in N, take now a homomorphism of G in $Aut(N)$, such that a lattice Γ normalizes Λ. It follows that Γ acts on N/Λ.
(2) We can also take a morphism of G in a unimodular Lie group H, and take the corresponding left induced left action of Γ on H/Λ where Λ is a cocompact lattice in H. In some sense, this action generalizes the geodesic flow for rank 1 symetric spaces.

(c) Weakly hyperbolic standard actions. It is well known that both the action of $SL(n,\mathbb{Z})$ on the torus and the geodesic flow of negatively curved manifolds have some hyperbolic (or Anosov) properties from the point of view of dynamical system. Here are now subclasses of the above examples which exhibit some hyperbolic behaviour. I will give an algebraic description of these action, refereeing to [M-Q] for the much more useful (but longer) dynamical description.
(3) Start with an example like (1). We then have a natural morphism of Γ in $Aut(\mathcal{N})$ the automorphisms of the Lie algebra $\mathcal{N}$ of N. This morphism essentially comes from a representation π of G and we say the action is *weakly hyperbolic* if π does not contain a trivial representation.
(4) This time, start with a type 2 example. Such a standard action is weakly hyperbolic if the centralizer of $\pi(G)$ in H is discrete.

(d) Actions on boundaries. A compact manifold M will be called a *boundary for* G if $M = G/P$, where P has finetely many components. The groups G and Γ act on boundaries, however these actions will never preserve any measure. Typical examples are te action of $SL(n,\mathbb{R})$ on spheres, projectives spaces, flag manifolds *etc.*

(e) Exotic examples. So far, all the above examples of actions preserve a rigid geometric structure everywhere on the manifold. The first "exotic" example is due to A. Katok and J. Lewis [K-L1]. Start with the action of $SL(n,\mathbb{Z})$ on the n-dimensional torus T^n. This action has a fixed point p_0. We can now blow up this point as algebraic geometers do, that is replace it by the projective space of its tangent plane. This new action will only preserve the original geometric structure on some open dense set. Exploring this idea, J. Benveniste [B2] has contructed a smooth family of ergodic actions of semisimple Lie groups, none of which are conjugate. This in particular implies that the answer to the rigidity question can not be always yes without any extra assumption. However, Beneveniste's examples preserve rigid geometric structures on some open dense set.

4. Hyperbolic actions

Since the original works of S. Hurder [H], A. Katok and J. Lewis [K-L2], actions of lattices with some hyperbolic behaviour have attracted a lot of attention.

Assume a 1-parameter group L_t acts on a space M and suppose its action lifts to a vector bundle E equipped with some metric. We say such an action is *Anosov* on E if we can find a continous splitting $E = E^+ \oplus E^-$, where

$$\exists A,\ B>0,\ s.t.\ \forall u^{\pm}\in E^{\pm},\ \forall t>0,\ \|L_{\pm t}(u^{\pm})\| \leq Ae^{-Bt}.$$

Now, we say that the action of a group on a compact manifold is *Anosov* if there exist a 1-parameter group L whose action is Anosov on TM/V, where V is the tangent bundle to the orbits of G. Next the action of a lattice is said to be *Anosov* if the suspended action is Anosov (with a little extra hypothesis, the definition makes sense even for non cocompact lattices). Of course the action of $SL(n, \mathbb{Z})$ on the n-dimensional torus is Anosov. Weak hyperbolicity is a generalisation of this hypothesis.

This is typically a situation where superrigidity will work. Taking the continuous splitting, $E = E^+ \oplus E^-$, as a starting point of the superrigidity procedure, will produce in good cases, after using a chain of centralizers, a rigid geometric structure on M.

In this situation, the following question seems to be within reach

CONJECTURE: *Every Anosov action of a lattice is smoothly conjugate to the standard action on a nilmanifold. More generally, every weakly hyperbolic action of a lattice on a compact manifold should be standard.*

I cannot state all the results on this subject, and I will underline two recent results. The first one is a definitive result on the local rigidity question.

THEOREM (G. MARGULIS - NANTIAN QIAN [M-Q]) *Standard weakly hyperbolic actions of higher rank lattices are smoothly rigid; that is, every smooth action close enough to the original one is smoothly conjugate to it.*

The second is a global result which make very weak topological assumption on the underlying manifold:

THEOREM (R. FERES - F. LABOURIE [F-L]) *Assume we have a Anosov volume preserving action of a lattice in $SL(n, \mathbb{R})$, $n \geq 3$ on some n-dimensional compact manifold M. Then M is a torus and the action preserves the connection of a flat metric on M.*

We should note the preceding results are valid only for higher rank lattices, and make therefore strong use of superrigidity.

Much more strikingly, A. Katok and R. Spatzier have obtained rigidity results for actions of $\mathbb{R}^k$ and $\mathbb{Z}^k$ for $k \geq 2$. The definition of standard actions makes also sense for these groups. Associate to these actions, are representations of $\mathbb{R}^k$ in $Aut(\mathcal{N})$ and $\mathcal{H}$ respectively where $\mathcal{N}$ and $\mathcal{H}$ are the lie algebras of N and H. We say the standard actions of $\mathbb{R}^k$ and $\mathbb{Z}^k$, have *semisimple linear part* if the corresponding representation of $\mathbb{R}^k$ is semisimple. An example of a standard action Anosov action with linear semisimple part is the left action of the group of real diagonalizable $n \times n$ matrices on $SL(n, \mathbb{R})/SL(n, \mathbb{Z})$. The result is then

THEOREM (A. KATOK-R. SPATZIER [K-S]) *Every standard Anosov action of $\mathbb{R}^k$ or $\mathbb{Z}^k$ with semisimple linear part is smoothly rigid.*

5. ACTIONS ON BOUNDARIES

Actions on S^1. Actions of groups on the circle is a subject by itself. I am therefore going to single out for the moment only results concerning lattices. Any time you have a hyperbolic structure on a compact surface S, this defines on action of $\pi_1(S)$ on the boundary at infinity of the hyperbolic plane, and this action factors through the standard $PSL(2,\mathbb{R})$ action, and in particular preserves a projective structure on the circle. Of course, since we can deform hyperbolic metrics on the surface, such an action is not locally rigid, but nevertheless these actions can be characterized. Let's first remark that to every action of $\pi_1(S)$ on the circle we can associate a number, namely the Euler class of the associated circle bundle on the surface. For the actions I just described, this Euler number is maximal (*i.e.* equal to the euler number of the surface). E. Ghys has proved

THEOREM (E. GHYS [GH]) *Every smooth action of the fundamental group of a compact surface with maximal Euler number factors through an action of $PSL(2,\mathbb{R})$ and in particular preserves a real projective structure on the circle.*

Very recently, E. Ghys and independently, M. Burger and N. Monod have announced results that tend to prove the following conjecture

CONJECTURE: *There is no non trivial smooth action of a higher rank lattice on the circle.*

This is to compare with the following result of D. Witte

THEOREM (D. WITTE [W]) *There is no non trivial continuous action of a higher rank lattice of $\mathbb{Q}$-rank greater than 2 on the circle.*

Notice that the smoothness (actually C^1) hypothesis is extremely restrictive, and that the proofs of Ghys, Burger and Monod do not adapt to the continuous case. In both cases, non trivial means actions that do not factor through a finite group.

Higher dimensional boundaries. Since actions on boundaries do not preserve measure superrigidity will not work nicely. However, a standard observation, already used by E. Ghys, is that there is some correspondance between the actions on Γ on G/P and the action of L on G/Γ where L is the reductive part of P. Using this idea and exploiting their results for the rigidity of abelian actions, A. Katok and R. Spatzier have proved the following almost definitive result

THEOREM (A. KATOK, R. SPATZIER [K-S]) *Let Γ be a higher rank cocompact lattice of a simple group G, then the action of Γ on a boundary for G is smoothly rigid.*

This greatly generalizes a previous result of M. Kanai [K] whose completely different proof relied on stochastic calculus.

6. ANALYTIC ACTIONS

It is a classical question to determine wether a diffeomorphim is linearizable on the neighbourhood of a fixed point, which, in geometric terms, means to preserve a flat connection. For analytic actions, E. Ghys and G. Cairns have shown

THEOREM (G. CAIRNS, E. GHYS [C-GH]) *Every higher rank lattice acting analytically on $\mathbb{R}^n$ fixing 0 is linearizable.*

On the other hand, the same authors have shown that there exist smooth actions of $SL(n, \mathbb{R})$ having a fixed point and which are not linearizable. An immediate application of the preceding theorem is the

COROLLARY *Every topologically transitive analytic action of a higher rank lattice having a fixed point preserves a flat connection on some open dense set.*

Quite recently, amongst other results, B. Farb and P. Shalen have shown the following version of Zimmer's conjecture

THEOREM (B. FARB, P. SHALEN [F-S]) *Any analytic action of $SL(n, \mathbb{Z})$, $n \geq 5$ on a compact surface, other than the torus and the Klein bottle, factors through a finite group.*

To prove this, they start with a fixed point for some element of the lattice, then, using the fact the lattice itself is generated by chain of centralizers, they reduce the situation to a 1-dimensional question which is settled by Witte's Theorem.

7. A TOPOLOGICAL RESULT

Even though we cannot for the moment classify actions of higher rank lattices, it is interesting to have some restrictions on the topology of the underlying manifold. Let's start with a definition. Assume a simply connected group acts ergodically on a manifold M preserving a volume form, and notice that the action of G lifts to any finite cover P of M. The action is said to be *totally engaging* is there is no measurable G-invariant section of $P \to M$ for any finite cover P of M. For a simple group of rank greater than 2, the action of G on G/Γ, for Γ a cocompact lattice, is totally engaging. The following result of A. Lubotzky and R. Zimmer sheds some light on the topological structure of the manifold M.

THEOREM (A. LUBOTZKY, R. ZIMMER [L-Z]) *Suppose that the action of a simply connected simple Lie group G of real rank greater than 2 is totally engaging on M, then for all finite dimensional representation σ of $\pi_1(M)$ in $GL(V)$, $\sigma(\pi_1(M))$ is an arithmetic lattice. In fact $\sigma(\pi_1(M))$ is commensurable to $H_{\mathbb{Z}}$, where H is a linear $\mathbb{Q}$-group in which contains a quotient of G.*

Let's try to explain the last sentence. It means first that there exists a group H which is a subgroup of some linear group $Gl(\mathbb{Q}^N)$ and which is defined by polynomial equations with rational coefficients. $H_{\mathbb{Z}}$ consist then of the matrices in H with integer entries. *Being commensurable* is the relation of equivalence generated by the relation *being of finite index in.* This theorem can be thought of as a generalization of Margulis's Arithemeticity Theorems [M]. Again let's notice that Benveniste's examples [B2] are not totally engaging and do not satisfy the conclusion of the above theorem.

8. OTHER GROUPS AND QUESTIONS

So far, we have stayed in the realm of higher rank lattices with a brief excursion in the kingdom of surfaces and abelian groups. Our reasons for that were the following: first, we have lots of examples of actions of lattices; second, by using the supension procedure we can turn questions about lattices into question about real Lie groups whose structure is quite well known; third the superrigidity method yelds interesting results.

What are now the other candidates for being large groups ? A typical property we would like them to satisfy is that they are generated by chains of centralizers, at least virtually, like lattices. Another important property used in superrigidity is that the centralizers that appear in the chain is non amenable.

Even though we do not have a precise definition, we have examples of groups that are good candidates for being large groups. For instance, E. Ghys and V. Sergiescu advocate the case of the Thomson group [Gh-S] which present a rigidity property. This group is known to be generated by chain of centralizers though its non amenability is not known.

For the moment, the best candidates for being large groups are the mapping class groups $\mathcal{M}(g)$ for surfaces S of genus g greater than 2, which are believed to share many properties of higher rank lattices. E. Ghys has for instance announced

Theorem (E. Ghys) *There is no non trivial actions of the mapping class group on the circle.*

On the other hand, we have lots of examples of actions of the mapping class groups, namely on the space $X(G,g)$ of representations of $\pi_1(S)$ in a compact Lie group G. These actions are known to be volume preserving (actually they are symplectic) and W. Goldman has shown

Theorem (W. Goldman [Go]) *The action of $\mathcal{M}(g)$ on $X(SU(2),g)$ is ergodic.*

Forgetting for a brief moment that $X(G,g)$ are not manifolds, it is tempting to ask whether these actions are rigid. Here is a simpler version of this question. In the case the compact group is S^1, the moduli space is the jacobian torus

$$X(S^1,g) = H^1(S,\mathbb{R})/H^1(S,\mathbb{Z}),$$

and the action factors through a lattice action which is known to be locally rigid. Let's now ask the

Test Question *Is the action of $\mathcal{M}(g)$ locally rigid on $X(S^1,g)$?*

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