More Optimization Stories

I have claimed in this book several times that optimization is around everywhere in nature and in all kinds of human endeavor. It is therefore impossible to cover in a book like this one all aspects of optimization. This final section serves as a pointer to further areas that have close connections to optimization but can only be treated peripherally.

Voronoi diagrams and Delaunay triangulations are examples of structures that can be defined by concepts of optimization theory. Today these are often considered as objects of computational geometry and play an important role in algorithm design. It is amazing to see how many other disciplines have arrived at these concepts from quite different initial questions.

Optimization is a field that employs ideas from many areas of mathematics. It is sometimes really surprising to see that results that may be viewed by some "hard core optimizers" as "esoteric pure mathematics" have significant bearing on optimization technology. One such example is Hilbert's 17^{th} problem that plays an important role in the representation of sets of feasible solutions by polynomials.

Optimization methods are also important tools in proofs. The correctness of a claim may depend on a large number of runs of optimization algorithms. Can we trust these results? A prime example is the proof of the Kepler conjecture that, in fact, gives rise to philosophical questions about mathematical proofs relying on computer runs.

The last two articles in this section build a bridge to economics. Optimizers usually assume that one objective function is given; but in reality there are often more goals that one wants to achieve – if possible simultaneously. Economists were the first to consider such issues and to formulate concepts of multi-criteria (or multi-objective) optimization.

The final article of this book touches upon several aspects not treated elsewhere in this book. One is stochastic optimization where optimization problems are considered for which information about a problem to be solved is partially unknown or insecure, or where only certain probabilities or distributions are known. The article starts with a game and "expected payoff", introduces utility functions (instead of objective functions) and ends with highly complex optimization questions in financial mathematics.

The relation of optimization with economics and management science is (for space reasons) underrepresented in this book. That is why I finish here with a few words about it.

Mathematicians have, for a long time, struggled mainly with the characterization of the solution set of equations. Economists have always considered questions such as the efficient allocation of scarce resources. The mathematical description of sets defined via the possible combination of resources under scarcity constraints naturally needs inequality constraints. That is one reason why the initial development of optimization in the middle of the twentieth century was strongly influenced by economists; and influential economists promoted the mathematical optimization approach to deal with such issues. Around the same time, game theory was developed (that should have also been treated in this book). The outstanding book by J. von Neumann and O. Morgenstern had a significant impact. The relations between questions and solution concepts in game theory to linear, nonlinear, and integer programming were worked out, and mutual significant influence became visible. The importance of linear programming for economics was recognized by the award of Nobel Prizes in Economic Sciences to L. V. Kantorovich and T. C. Koopmans in 1975. Several further Nobel Prizes recognizing contributions to game theory, auction theory, mechanism design theory and financial mathematics followed. All these areas have close connections to optimization.

Science is carried out to increase our understanding of the world and to use the information obtained to improve our well-being. I view the development of optimization theory and of its algorithmic methods as one of the most important contributions of mathematics to society in the 20$^{\text{th}}$ century. Today, for almost every good on the market and almost every service offered, some form of optimization has played a role in their production. This is not too well-known by the general public, and we optimizers should make attempts to make the importance of our field for all aspects of life more visible. History stories such as the ones presented in this book may help to generate attention and interest in our work.

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