Euler and Variations

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When Euler came to Berlin in 1741, accepting the offer of the Prussian king Frederick II. to work at the Berlin academy of sciences, the king himself was at the first Silesian war with Austria. It was only the first of three wars that he waged. Two years later Euler bought a house in the centre of Berlin in the "Bärenstraße", today "Behrenstraße" number 21. There he lived up to 1766 when he left Berlin in order to return to St. Petersburg.

Yet already in those days, life was expensive in a city like Berlin. Hence in 1753 he bought an estate outside Berlin in the small village of Lietzow, belonging to the administrative district of Charlottenburg, that is to-day a part of a district of the city of Berlin. He paid 6000 Imperial Taler (Reichsthaler) for it. From then onward his large family lived on this estate, including his widowed mother, while he himself remained in Berlin.

Whenever he had Russian students of mathematics they too lived in the house in Berlin: from 1743 to 1744 Kirill Grigorevich Rasumovskii, later president of the Russian Academy of Sciences in St. Petersburg, and Grigorii Nikolaevich Teplov, from 1752 to 1756 Semen Kirillovich Kotelnikov, in 1754 Michail Sofronov, from 1754 to 1756 Stepan Yakovlevich Rumovskii. It did not happen by chance that 1756 was the year of departure. In 1756 Frederick II. began the Seven-Years War by penetrating into Saxony. His Prussian troops fought against the allied Russian, Saxon, and Austrian troops.

Euler carried on sending scientific manuscripts to St. Petersburg – that is, to Russia – and kept his good relations with the academy there. Yet he secretly helped the Prussian king with his knowledge of the Russian language by translating intercepted Russian messages. If the time did not suffice for a diligent translation he offered to summarize the content. For example in September 1758 a courier of the Russian guard was taken captive together with two Cossacks near to Neustettin. They carried seventy-nine letters for the Russian court. Euler's translation of the report of a Russian agent and of the statements of two Prussian deserters is still kept in the archives of the Berlin-Brandenburg Academy of Sciences and Humanities (`http://euler.bbaw.de/euleriana/ansicht.php?seite=216`).

The following years became very difficult for the Prussian king. In 1759 the allied Austrian and Russian troops defeated the troops of Frederick II. in the neighbourhood of Kunersdorf. On October 9, 1760 Russian and Saxon troops temporarily occupied Berlin and plundered the surrounding villages, especially Lietzow, and including Euler's estate. The command of the Russian Count Chernishef to spare this estate from plunder came too late.

Just nine days later, on October 18, 1760 Euler wrote to the historian Gerhard Friedrich Müller in St. Petersburg, since 1754 perpetual secretary of the Russian Academy of Sciences, in order to complain about this robbery and to make a claim for damages. "I have always wished that Berlin should be occupied by Russian troops if it should be ever occupied by foreign troops", he wrote, "yet the visit of the Russian officers entailed considerable damage." He told Müller that he had bought an estate for 6000 Imperial Taler in Charlottenburg that was well-known to Mr. Kotelnikov and to Mr. Rumovskii. On the occasion of that visit everything was removed or devastated. Then he enumerated the losses:

> I have lost four horses, twelve cows, many head of livestock, much oats and hay. All of the furniture of the house has been ruined. This damage is more than 1100 Imperial Taler according to an exact calculation...All in all the damage is at least 1200 roubles.

He asked Müller to inform his former student, then president of the Russian Academy, Count Rasumovskii, about his situation and to support his request. He was indeed amply recompensed by the Russian general and by the Russian tsarina Elisabeth.

By chance Euler's statements about his losses can be checked because the mayor of Charlottenburg elaborated a specification of damages for Lietzow and Charlottenburg that has been preserved in the Main Archives of the country Brandenburg of the Federal Republic of Germany in Potsdam. On October 24, 1760, the mayor sent a letter to the responsible Privy Councillor of War and of Domain (Geheimder Krieges und Domainen Rath) saying:

> As we have been ordered we have added and would like to most obediently submit the specification of money, grain, and cattle that the city of Charlottenburg has lost by the Russian invasion.
> [Anbefohlener Maßen haben Wir angeschlossen die Specification so wohl an baaren Gelde als an Getreyde und Vieh was die Stadt Charlottenburg durch die Russischen Invasion verlohren haben gehorsamst einreichen sollen.]

The list consists of nine columns. They enumerate the names of the twelve families concerned from the village of Lietzow and the robbery of cash currency, rye, barley and oat, hay, horses, cows, pigs, and sheep. The fourth line mentions Euler's losses reading:

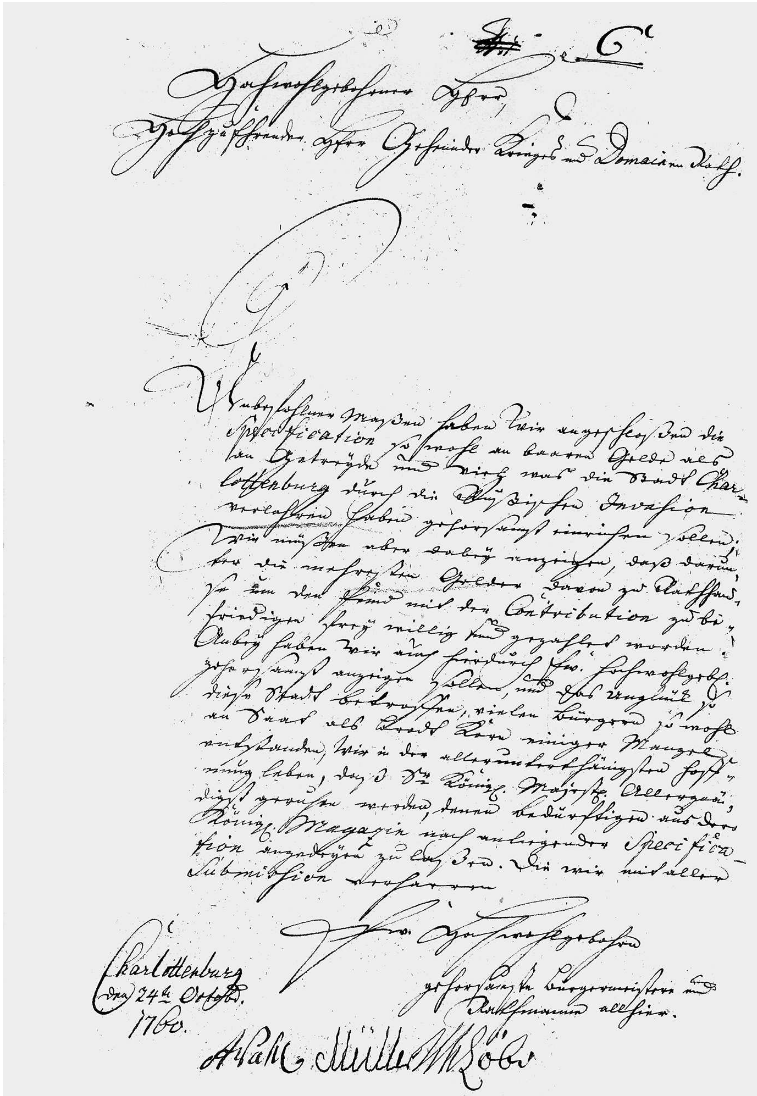
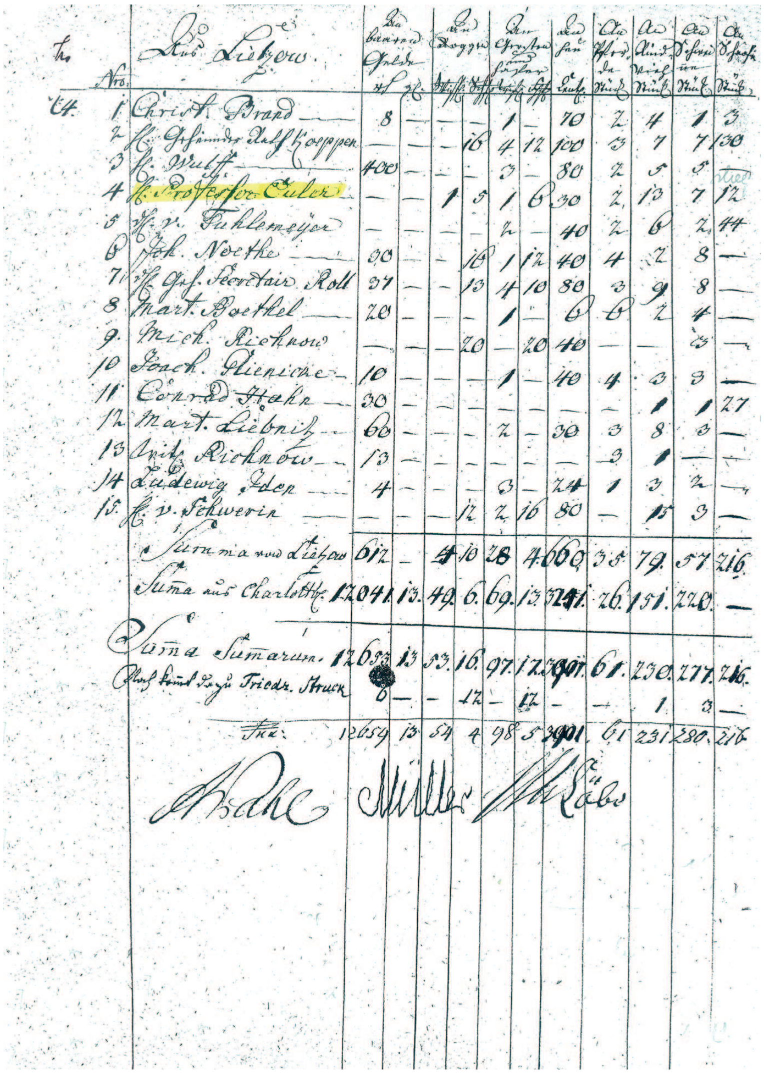


Figure 1: Letter of the mayor of Charlottenburg dating from October 24, 1760 (By courtesy of the Brandenburgisches Landeshauptarchiv Potsdam, Rep. 2 Kurmärkische Kriegs- und Domänenkammer Nr. S 3498)

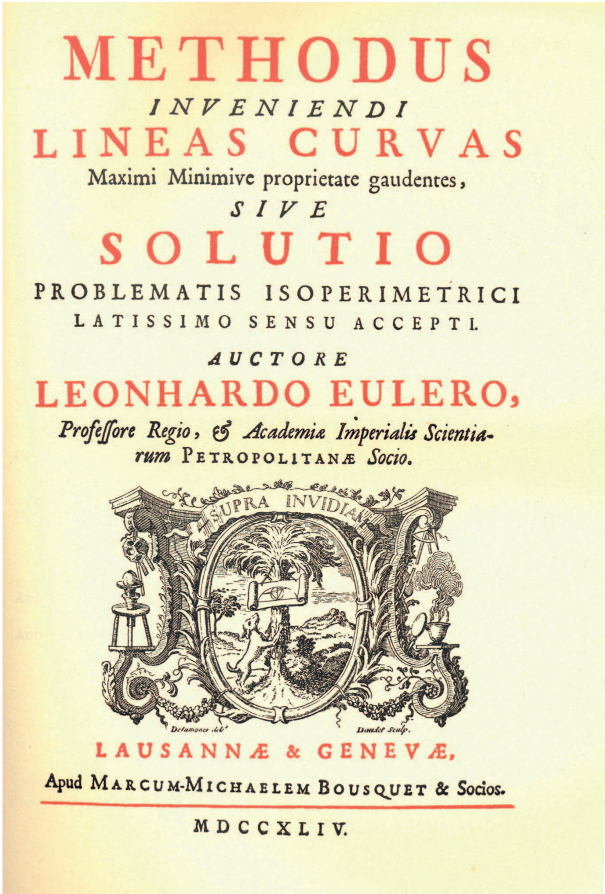
> Professor Euler: no cash currency; 1 Wispel, 5 Scheffel rye (1 Wispel = 24 Scheffel, 1 Scheffel = 54,73 litres); 1 Wispel, 6 Scheffel barley and oat; 30 metric hundred-weight of hay; two horses; thirteen cows; seven pigs; twelve sheep.



Nro.	Aus Lietzow											
1	Christ. Brand	8	—	—	—	1	—	70	2	4	1	3
2	H. Geheimder Rath Koeppen	—	—	—	16	4	12	100	3	7	7	130
3	H. Wulff	400	—	—	—	3	—	80	2	5	5	[illegible]
4	H. Professor Euler	—	—	1	5	1	6	30	2	13	7	12
5	H. v. Fuhlemeyer	—	—	—	—	2	—	40	2	6	2	44
6	Joh. Noethe	90	—	—	16	1	12	40	4	2	8	—
7	H. Geh. Secretair Roll	37	—	—	13	4	10	80	3	9	8	—
8	Mart. Boethel	20	—	—	—	1	"	6	6	2	4	—
9	Mich. Richnow	—	—	—	20	—	20	40	—	—	3	—
10	Joach. Plienicke	10	—	—	—	1	—	40	4	3	3	—
11	Conrad Hahn	30	—	—	—	—	—	—	—	1	1	27
12	Mart. Liebnitz	60	—	—	—	2	—	30	3	8	3	—
13	Fritz Richnow	13	—	—	—	—	—	—	3	1	—	—
14	Ludewig Iden	4	—	—	—	3	—	24	1	3	2	—
15	H. v. Schwerin	—	—	—	12	2	16	80	—	15	3	—
	Summa von Lietzow	612	—	4	10	28	4	660	35	79	57	216
	Suma aus Charlottbg.	12041	13	49	6	69	13	3241	26	151	220	—
	Suma Sumarum	12653	13	53	16	97	17	3901	61	230	277	216
	[illegible] Friedr. Struck	6	—	—	12	—	12	—	—	1	3	—
	Thl:	12659	13	54	4	98	5	3901	61	231	280	216

Figure 2: List of damages regarding the village Lietzow (By courtesy of the Brandenburgisches Landeshauptarchiv Potsdam, Rep. 2 Kurmärlische Kriegs- und Domänenkammer Nr. S 3498)

The astonished reader notices at once that Euler has doubled the number of stolen horses. In 1763 he had already negotiated with the Russian Academy of Sciences for his return to St. Petersburg, which indeed took place in 1766. For that reason he sold his estate in Charlottenburg for 8500 Imperial Taler, that is, at a profit of more than forty per cent, thus practising again his private calculus of variations. All in all he made a good profit out of his estate.



METHODUS
INVENIENDI
LINEAS CURVAS
Maximi Minimive proprietate gaudentes,
SIVE
SOLUTIO
PROBLEMATIS ISOPERIMETRICI
LATISSIMO SENSU ACCEPTI.
AUCTORE
LEONHARDO EULERO,
Professore Regio, & Academiæ Imperialis Scientiarum PETROPOLITANÆ Socio.
SUPRA INVIDIAM
LAUSANNÆ & GENEVÆ,
Apud MARCUM-MICHAELEM BOUSQUET & Socios.
MDCCXLIV.

Figure 3: Title page of Euler's book on the calculus of variations (L. Euler, Opera omnia, series I, vol. 24, Bern 1952, p. 1)

Thanks to a letter from Euler to the president Maupertuis of the Berlin Academy of Sciences and Fine Arts from March 14, 1746 we know that Euler had written his official, famous book on the calculus of variations, his Method of finding curves with an extreme property or the solution of the isoperimetric problem understood in the broadest sense, already in St. Petersburg, that is, in spring 1741 at the latest. It appeared in Lausanne in 1744 including the appendix II with Euler's explanation of the principle of least action. Constantin Carathéodory called the book one of the most beautiful mathematical works that has ever been written. But that is another story.

References

[1] Eberhard Knobloch: Leonhard Euler 1707–1783, Zum 300. Geburtstag eines langjährigen Wahlberliners. In: Mitteilungen der Deutschen Mathematiker-Vereinigung 15 (2007), 276–288.

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