On p-Adic Geometric Representations of $G_{\mathbb{Q}}$

To John Coates

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Abstract. A conjecture of Fontaine and Mazur states that a geometric odd irreducible p-adic representation ρ of the Galois group of $\mathbb{Q}$ comes from a modular form ([10]). Dieulefait proved that, under certain hypotheses, ρ is a member of a compatible system of ℓ-adic representations, as predicted by the conjecture ([9]). Thanks to recent results of Kisin ([15]), we are able to apply the method of Dieulefait under weaker hypotheses. This is useful in the proof of Serre's conjecture ([20]) given in [11], [14],[12],[13].

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1 Introduction.

Let $\overline{\mathbb{Q}}$ be an algebraic closure of $\mathbb{Q}$. For L a finite extension of $\mathbb{Q}$ contained in $\overline{\mathbb{Q}}$, we write G_L for the Galois group of $\overline{\mathbb{Q}}/L$. For ℓ a prime number, we write $\mathbb{Q}_\ell$ for the field of ℓ-adic numbers and $\overline{\mathbb{Q}_\ell}$ for an algebraic closure of $\mathbb{Q}_\ell$.
An ℓ-adic representation ρ of G_L of dimension d is a continuous morphism ρ from G_L to $\mathrm{GL}_d(\overline{\mathbb{Q}_\ell})$. In fact, ρ has values in $\mathrm{GL}_d(M)$, for M a finite extension of $\mathbb{Q}_\ell$ contained in $\overline{\mathbb{Q}_\ell}$ (lemma 2.2.1.1. of [6]). Such a representation ρ is said to be *geometric* if it satisfies the following two conditions ([10]):
- for $\mathcal{L}$ a prime of L above ℓ, the restriction of ρ to the decomposition subgroup $D_{\mathcal{L}}$ satisfies the potentially semi-stable condition of Fontaine's theory (exp. 8 of [1]) ;
- there exists a finite set S of primes of L such that ρ is unramified outside S and the primes above ℓ.
A geometric ℓ-adic Galois representation defines for each prime $\mathcal{L}$ of L an isomorphy class of representations of the Weil-Deligne group $\mathrm{WD}_{\mathcal{L}}$ in $\mathrm{GL}_d(\overline{\mathbb{Q}_\ell})$

([8], exp. 8 of [1], [10]). We call $r_{\mathcal{L}}(\rho)$ its F-semisimplification. It is attached to the restriction of ρ to the decomposition group $D_{\mathcal{L}}$. When $\mathcal{L}$ is of characteristic ℓ, in order to define $r_{\mathcal{L}}$, one needs to use the action of $\mathrm{WD}_{\mathcal{L}}$ on the filtered Dieudonné module attached to the restriction of ρ to $D_{\mathcal{L}}$ via Fontaine's theory (see remark 1 of section 4).

Let E be a finite extension of $\mathbb{Q}$ contained in $\overline{\mathbb{Q}}$. By a *compatible system of geometric representations of G_L with coefficients in* E of dimension d, we mean the following data :

- for each ℓ and for each embedding ι of E in $\overline{\mathbb{Q}_\ell}$, a geometric representation $\rho_\iota : G_L \to \mathrm{GL}_d(\overline{\mathbb{Q}_\ell})$,
- a finite set S of primes of L, and for each prime $\mathcal{L}$ of L, an F-semisimple representation $r_{\mathcal{L}}$ of $\mathrm{WD}_{\mathcal{L}}$ in $\mathrm{GL}_d(E)$, such that :
  - $r_{\mathcal{L}}$ is unramified if $\mathcal{L} \notin S$;
  - for each ι as above, $\iota \circ r_{\mathcal{L}}$ is isomorphic to $r_{\mathcal{L}}(\rho_\iota)$.

We fix a prime p. Let ρ be a p-adic geometric irreducible odd representation of dimension 2 of $G_{\mathbb{Q}}$. By odd, we mean that $\rho(c)$ has eigenvalues 1 et -1, for c a complex conjugation. We suppose that ρ has Hodge-Tate weights $(0, k-1)$, where k is an integer ≥ 2 : we shall say that ρ is of weight k. It is conjectured by Fontaine and Mazur that ρ comes from a modular form of weight k.

More precisely, let $k \geq 2$ and $N \geq 1$ be integers. Let $f = q + \ldots + a_n q^n + \ldots$ be a primitive modular form on $\Gamma_1(N)$ of weight k. Let $E(f)$ be its coefficient field, *i.e.* the field generated by the coefficients of f and the values of the character of f. The field $E(f)$ is a finite extension of $\mathbb{Q}$. It is classical that one can associate a p-adic representation $\rho(f)_\iota : G_{\mathbb{Q}} \to \mathrm{GL}_2(\overline{\mathbb{Q}_p})$ to f and an embedding ι of $E(f)$ in $\overline{\mathbb{Q}_p}$. The representation $\rho(f)_\iota$ is unramified at ℓ if ℓ is $\neq p$ and does not divide N and is characterized by :

$$\mathrm{tr}(\rho(f)_\iota(\mathrm{Frob}_\ell)) = \iota(a_\ell),$$

for these ℓ. Furthermore, $\rho(f)_\iota$ is absolutely irreductible, odd, geometric, of conductor N and of weight k (Hodge-Tate weights $(0, k-1)$). The conjecture of Fontaine and Mazur states that ρ is isomorphic to $\rho(f)_\iota$ for an f and a ι.

A consequence of the conjecture of Fontaine and Mazur is that ρ is a member of a compatible system of Galois representations. Dieulefait proved that it is the case under certain hypotheses ([9]). Using a recent result of Kisin ([15]), we give weaker hypotheses under which the result of Dieulefait is true.

The main tool of the proof is a theorem of Taylor ([26] and [25]). There exists a totally real number field F which is Galois over $\mathbb{Q}$ and such that $\rho_{|G_F}$ comes from an cuspidal automorphic representation π of $\mathrm{GL}_2(\mathbb{A}_F)$ of parallel weight k (or a Hilbert modular form for F). By Arthur-Clozel ([2]), for each F' such that the Galois group of F/F' is solvable, $\rho_{|G_{F'}}$ comes from an automorphic representation $\pi_{F'}$ for $\mathrm{GL}_2(\mathbb{A}_{F'})$. Using Brauer's theorem, we put together the compatible systems associated to the automorphic representations $\pi_{F'}$, and we obtain the compatible system of representations of $G_{\mathbb{Q}}$.

2 Taylor's theorem.

Let ρ be an odd irreducible geometric p-adic representation of $G_{\mathbb{Q}}$ of dimension 2 of weight k, k an integer ≥ 2.
We say that ρ is *potentially modular* if there exists a Galois totally real finite extension F of $\mathbb{Q}$ contained in $\overline{\mathbb{Q}}$ such that the restriction of ρ to G_F comes from a cuspidal automorphic representation π of $\mathrm{GL}_2(\mathbb{A}_F)$ of parallel weight k. The theorem of Taylor states in many cases that ρ is potentially modular. In fact, Taylor proves that the reduction $\overline{\rho}$ of ρ is potentially modular, with F unramified (resp. split) at p if the restriction of $\overline{\rho}$ to D_p is reducible (resp. irreducible). Then, the modularity of $\rho_{|G_F}$ follows from modularity theorems. According to which modularity theorem one applies, one get different statements. We write the following statement which is needed for our work with Khare on Serre's conjecture.

Théorème 1 *Let $\rho : G_{\mathbb{Q}} \to \mathrm{GL}_2(\overline{\mathbb{Q}_p})$ be a p-adic representation, absolutely irreducible, odd, unramified outside a finite set of primes. One supposes that the reduction $\overline{\rho}$ of ρ has non solvable image and, if $p \neq 2$, that $\overline{\rho}$ has Serre's weight $k(\overline{\rho})$ in the range $[2, p+1]$. Then ρ is potentially modular in the following cases :*
- a1) $p \neq 2$ and $\rho_{|D_p}$ is crystalline of weight $k = k(\overline{\rho})$;
- a2) $p = 2$, $k(\overline{\rho}) = 2$ and $\rho_{|D_2}$ is Barsotti-Tate ;
- b) $p \neq 2$ and $k(\overline{\rho}) \neq p+1$, $\rho_{|D_p}$ is potentially Barsotti-Tate, Barsotti-Tate after restriction to $\mathbb{Q}_p(\mu_p)$, and the restriction of the representation of the Weil-Deligne group WD_p to inertia is $(\omega_p^{k-2} \oplus 1)$, where ω_p is the Teichmuller lift of the cyclotomic character modulo p ;
- c) $p \neq 2$ and $k(\overline{\rho}) = p+1$ or $p = 2$ and $k(\overline{\rho}) = 4$ and $\rho_{|D_p}$ is semistable of weight 2.

The theorem follows from the potential modularity of $\overline{\rho}$ ([26], [25]) and the modularity theorem stated in 8.3. of [13].
Remark. Using Skinner-Wiles modularity theorem ([22]), Taylor gives a variant of this statement in a lot of ordinary cases.

3 Field of coefficients of ρ.

Let $\rho : G_{\mathbb{Q}} \to \mathrm{GL}_2(\overline{\mathbb{Q}_p})$ be as in the preceeding section. Furthermore, we suppose that ρ is potentially modular.

Proposition 1 *There is a finite extension E of $\mathbb{Q}$ and an embedding $\iota_p : E \hookrightarrow \overline{\mathbb{Q}_p}$ and for each prime ℓ, a F-semisimple representation r_ℓ of the Weil-Deligne group WD_ℓ with values in $\mathrm{GL}_2(E)$ such that for each ℓ, the F-semisimplification $r_\ell(\rho)$ of the representation of the Weil-Deligne group WD_ℓ associated to ρ is isomorphic to $\iota_p \circ r_\ell$.*

Proof. Let F and π as in the theorem of Taylor. Let F' be a subfield of F such that F/F' has solvable Galois group. By Arthur and Clozel, we know that the restriction of ρ to $G_{F'}$ is also associated to a cuspidal representation $\pi_{F'}$ of $\mathrm{GL}_2(\mathbb{A}_{F'})$ ([2]). It follows that there exists a finite extension $E_{F'}$ of $\mathbb{Q}$ such that the F-semisimplification of the representation of the Weil-Deligne group $\mathrm{WD}_{\mathcal{L}}$ associated to the restriction of ρ to $G_{F'}$ can be realized in $E_{F'}$ for each prime $\mathcal{L}$ of F'. The rationality properties of $\pi_{F'}$ follows from Shimura for the unramified primes and from Rogawski-Tunnell for the ramified primes ([21], see also [19] ; [18]). The compatiblity of global and local Langlands correspondances follows for $\mathcal{L}$ of characteristic $\neq p$ from Carayol completed by Taylor ([7],[23]) and for $\mathcal{L}$ of characteristic p from Saito and Kisin ([19],[15]).
Take for E an extension of $\mathbb{Q}$ containing the images by all embeddings in $\overline{\mathbb{Q}}$ of the fields $E_{F'}$. Let $\mathcal{L}$ be a prime of F. Let $F'_{\mathcal{L}}$ be the subfield of F which is fixed by the decomposition subgroup of $\mathrm{Gal}(F/\mathbb{Q})$ for $\mathcal{L}$. Let $\mathcal{L}'$ be the restriction of $\mathcal{L}$ to $F'_{\mathcal{L}}$. The representation of the Weil-Deligne group $\mathrm{WD}_{\mathcal{L}'}$ defined by the restriction of ρ to $F'_{\mathcal{L}}$ can be realized in $E_{F'_{\mathcal{L}}}$. As the Weil-Deligne groups WD_ℓ and $\mathrm{WD}_{\mathcal{L}'}$ coincide, the proposition follows.
Remark. Particular cases of the compatibility between global and local Langlands correspondences for the primes dividing the characteristic follows from Breuil, Berger and Taylor ([5],[3],[24]).

4 Construction of the compatible system.

Théorème 2 *Let ρ be as in the preceeding section. Then, there exists a compatible system (ρ_ι) of geometric representations of $G_{\mathbb{Q}}$ with coefficients in a number field E such that there exists an embedding $\iota_p : E \hookrightarrow \overline{\mathbb{Q}_p}$ with ρ_{ι_p} isomorphic to ρ. The ρ_ι are irreducible, odd and of weight k.*

Proof. If ρ is induced from the p-adic representation associated to a Hecke's character Ψ of an imaginary quadratic field, then one takes for (ρ_ι) the compatible system induced from the one defined by the Hecke character. Otherwise, ρ remains absolutely irreducible after restriction to any open subgroup of $G_{\mathbb{Q}}$. We suppose this from now.
Let F, π, $E(\pi)$ and ι_p such that $\rho_{|G_F}$ is isomorphic to the Galois representation $\rho(\pi)_{\iota_p}$ attached to π, and the embedding ι_p of the coefficient field $E(\pi)$ of π in $\overline{\mathbb{Q}_p}$. As in Taylor's 5.3.3. of [27], one applies Brauer's theorem to the trivial representation of $\mathrm{Gal}(F/\mathbb{Q})$. There exist fields $F_i \subset F$, such that each F/F_i has a solvable Galois group, integers $m_i \in \mathbb{Z}$ and characters Ψ_i of $\mathrm{Gal}(F/F_i)$ such that the trivial representation of $\mathrm{Gal}(F/\mathbb{Q})$ equals :

$$\sum_i m_i \mathrm{Ind}_{G_{F_i}}^{G_{\mathbb{Q}}} \Psi_i.$$

One has :

$$\rho = \sum_i m_i \mathrm{Ind}_{G_{F_i}}^{G_{\mathbb{Q}}} (\rho_{|G_{F_i}} \otimes \Psi_i).$$

As in the proof of proposition 1, it follows from the theorems of Taylor and Arthur-Clozel that $\rho_{|G_{F_i}}$ is the Galois representation $\rho(\pi_i)$ attached to an automorphic representation π_i of $\mathrm{GL}_2(\mathbb{A}_{F_i})$ whose coefficient field is embedded in E.

Let ι be an embedding of E in $\overline{\mathbb{Q}_q}$ for a prime q. We enlarge E such that it contains the values of the characters Ψ_i. One defines the virtual representation R_ι in the Grothendieck group of irreducible representations of $G_{\mathbb{Q}}$ with coefficients in $\overline{\mathbb{Q}_q}$ by :

$$R_\iota = \sum_i m_i \mathrm{Ind}_{G_{F_i}}^{G_{\mathbb{Q}}}(\rho(\pi_i)_\iota \otimes \Psi_i).$$

Let us prove that R_ι is a true representation. For i and j, let $\{\tau_k\}$, $\tau_k \in G_{\mathbb{Q}}$ be a set of representatives of the double classes $G_{F_i}\backslash G_{\mathbb{Q}}/G_{F_j}$. Let us call F_{ijk} the compositum of F_i and $\tau_k(F_j)$. One has :

$$\mathrm{Ind}_{G_{F_j}}^{G_{\mathbb{Q}}}(\rho(\pi_j)_\iota \otimes \Psi_j)_{|G_{F_i}} = \sum_k \mathrm{Ind}_{G_{F_{ijk}}}^{G_{F_i}}\left(((\rho(\pi_j)_\iota \otimes \Psi_j) \circ \mathrm{int}(\tau_k^{-1}))_{|G_{F_{ijk}}}\right).$$

It follows that the scalar product $< R_\iota, R_\iota >$ in the Grothendieck group is equal to the sum over i, j, k of :

$$m_i m_j \ < \left((\rho(\pi_j)_\iota \otimes \Psi_j) \circ \mathrm{int}(\tau_k^{-1})\right)_{|G_{F_{ijk}}}, (\rho(\pi_i)_\iota \otimes \Psi_i)_{|G_{F_{ijk}}} > .$$

We see that the scalar product of R_ι with itself is $\sum_{i,j,k} m_i m_j t_{ijk}$ with $t_{ijk} = 1$ or 0 depending whether

$$\left((\rho(\pi_j)_\iota \otimes \Psi_j) \circ \mathrm{int}(\tau_k^{-1})\right)_{|G_{F_{ijk}}} \simeq (\rho(\pi_i)_\iota \otimes \Psi_i)_{|G_{F_{ijk}}}$$

or not. One has a similar calculation for the scalar product of ρ with itself in the Grothendieck group of irreducible representations of $G_{\mathbb{Q}}$ with coefficients in $\overline{\mathbb{Q}_p}$. The calculation gives $\sum_{ijk} m_i m_j t'_{ijk}$, with $t'_{ijk} = 1$ or 0 depending whether

$$\left((\rho \otimes \Psi_j) \circ \mathrm{int}(\tau_k^{-1})\right)_{|G_{F_{ijk}}} \simeq (\rho \otimes \Psi_i)_{|G_{F_{ijk}}}$$

or not. As $\rho(\pi_i)_\iota$ and $\rho_{|G_{F_i}}$ are irreducible and have the same characteristic polynomial of Frobenius outside a finite set of primes, one has $t_{ijk} = t'_{ijk}$. As $< \rho, \rho >= 1$, it follows that the scalar product of R_ι with itself is 1. As the dimensions of R_ι and ρ are both $\sum 2m_i[G_{\mathbb{Q}} : G_{F_i}]$, we have $\dim(R_\iota) = 2$. We see that R_ι is a true representation of dimension 2. We call it ρ_ι.

It follows from the formula defining R_ι that the restriction of ρ_ι to G_F is associated to π. By Blasius-Rogawski ([4]), $(\rho_\iota)_{|G_F}$ comes from a motive, except perhaps if $k = 2$. It then follows by Tsuji that the restriction of ρ_ι to the decomposition group for the characteristic q of ι is potentially semi-stable of weight k ([28]). The case $k = 2$ and ρ_ι is constructed as a limit of q-adic representations attached to automorphic forms with one local component discrete series is taken care by Kisin ([23],[15]).

The F-semisimple representation of the Weil-Deligne group WD_ℓ on ρ_ι is isomorphic to :

$$\sum_i m_i \left(\sum_{\mathcal{L}} \mathrm{Ind}_{D_{\mathcal{L}}}^{D_\ell} (r_{\mathcal{L}}(\pi_i) \otimes \Psi_i) \right),$$

where $\mathcal{L}$ describes the set of primes of F_i over ℓ. The compatibility follows from the fact that $\pi_i \mapsto \rho(\pi_i)$ is compatibility with local Langlands correspondance (see the references quoted in the proof of proposition 1).

By an argument of Ribet, it follows from compatibility that ρ_ι is absolutely irreducible ([17]). As the restriction of ρ_ι to G_F is associated to π, it is odd and ρ_ι is odd. This finishes the proof of the theorem.

Remarks.

1) Let M be a finite extension of $\mathbb{Q}_p$ contained in $\overline{\mathbb{Q}_p}$ and let $\gamma : G_M \to \mathrm{GL}_d(E)$ be a potentially semistable representation of the Galois group G_M with coefficients in a finite extension E of $\mathbb{Q}_p$. Let WD_M be the Weil-Deligne group. Let M_0 be the maximal unramified extension of $\mathbb{Q}_p$ contained in M. Fontaine has defined a representation of WD_M on the filtered Dieudonné D module attached to γ (exp. 8 of [1]). Let us recall how it defines, up to conjugacy, a representation r of WD_M in $\mathrm{GL}_d(\overline{\mathbb{Q}_p})$. The filtered Dieudonné module D is a $L \otimes_{\mathbb{Q}_p} E$-module D, L a finite unramified extension of M_0 in $\overline{\mathbb{Q}_p}$, with an action of WD_M commuting with the action of $L \otimes_{\mathbb{Q}_p} E$. One knows that the $E \otimes_{\mathbb{Q}_p} L$-module D is free. Let us briefly recall why. Let us choose such an embedding of E in $\overline{\mathbb{Q}_p}$, and let us call $E_1 = E \cap L$. For each element τ of the Galois group of $E_1/\mathbb{Q}_p$, let D_τ be the sub-module of the elements x of D such that $(e \otimes 1)x = (1 \otimes \tau(e))x$ for every $e \in E_1$. As the Frobenius ϕ of D acts semi-linearly relatively to the action of L and commutes with the action of E, ϕ transitively permutes the D_τ, and the D_τ have the same dimension. This implies the freeness. As the action of the Weil-Deligne group WD_M on D commutes with the action of $E \otimes_{\mathbb{Q}_p} L$, it follows that WD_M acts on each D_τ. One defines r as the F-simplification of the action of WD_M on D_{id}.

2) One can describe the projective representation associated to ρ_ι as in [29]. Let F and π as in Taylor's theorem. Let ρ_ι the Galois-representation associated to π and ι. The multiplicity one theorem ([16]) implies that for $\sigma \in G_{\mathbb{Q}}$, the automorphic representations π and ${}^\sigma\pi$ are isomorphic. It follows that the Galois representations ρ_ι and $\rho_\iota \circ \mathrm{int}(\sigma)$ are isomorphic. That means that there exists $\overline{g_\sigma} \in \mathrm{PGL}_2(\overline{\mathbb{Q}}_q)$ such that :

$$\rho_\iota \circ \mathrm{int}(\sigma) \simeq \mathrm{int}(\overline{g_\sigma}) \circ \rho_\iota.$$

This characterizes $\overline{g_\sigma}$ as $\rho_{F,q}$ is absolutely irreducible. Then, $\sigma \mapsto g_\sigma$ defines a projective representation which is the projective representation associated to ρ_ι. As in [29], one can show directly that this projective representation lifts to a representation in $\mathrm{GL}_2(\overline{\mathbb{Q}}_q)$.

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