Equivariant Poincaré Series and Topology of Valuations

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Abstract. The equivariant with respect to a finite group action Poincaré series of a collection of r valuations was defined earlier as a power series in r variables with the coefficients from a modification of the Burnside ring of the group. Here we show that (modulo simple exceptions) the equivariant Poincaré series determines the equivariant topology of the collection of valuations.

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1 Introduction

A definition of the Poincaré series of a multi-index filtration was first given in [3] (for filtrations defined by collections of valuations). It is a formal power series in several variables with integer coefficients, i.e., an element of the ring $\mathbb{Z}[[t_1, \ldots, t_r]]$. In [1] it was shown that, for the filtration defined by the curve valuations corresponding to the irreducible components of a plane curve singularity, the Poincaré series coincides with the Alexander polynomial in several variables of the corresponding algebraic link: the intersection of the curve with a small sphere in $\mathbb{C}^2$ centred at the origin. This relation was obtained by a direct computation of the both sides in the same terms. Up to now there exist

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no conceptual proof of it. The Alexander polynomial in several variables of an algebraic link (and therefore the Poincaré series of the corresponding collection of valuations) determines the topological type of the corresponding plane curve singularity. In [2] the definition of the Poincaré series was reformulated in terms of an integral with respect to the Euler characteristics (over an infinite dimensional space).

The desire to understand deeper this relation led to attempts to find an equivariant version of it (for actions of a finite group G) and thus to define equivariant versions of the Poincaré series and of the Alexander polynomial. Some equivariant versions of the monodromy zeta-function (that is of the Alexander polynomial in one variable) were defined in [9] and [10]. Equivariant versions of the Poincaré series were defined in [4], [5] and [7].

In some constructions of equivariant analogues of invariants (especially those related to the Euler characteristic) the role of the ring of integers $\mathbb{Z}$ (where the Euler characteristic takes values) is played by the Burnside ring $A(G)$ of the group G. Therefore it would be attractive to define equivariant versions of the Poincaré series as elements of the ring $A(G)[[t_1, \ldots, t_r]]$ (or of a similar one). The equivariant versions of the monodromy zeta functions defined in [9] and [10] are formal power series with the coefficients from $A(G) \otimes \mathbb{Q}$ and $A(G)$ respectively.

In [4] the equivariant Poincaré series was defined as an element of the ring $R_1(G)[[t_1, \ldots, t_r]]$ of formal power series in $t_1, \ldots, t_r$ with the coefficients from the subring $R_1(G)$ of the ring $R(G)$ of complex representations of the group G generated by the one-dimensional representations. This Poincaré series turned out to be useful for some problems: see, e.g., [8], [11]. However, it seems to be rather "degenerate", especially for non-abelian groups.

In [5] the G-equivariant Poincaré series $P^G_{\{\nu_i\}}$ of a collection of valuations (or order functions) $\{\nu_i\}$ was not in fact a series, but an element of the Grothendieck ring of so called locally finite (G, r)-sets. This Grothendieck ring was rather big and complicated, the Poincaré series $P^G_{\{\nu_i\}}$ was rather complicated as well and contained a lot of information about the valuations and the G-action. In particular, for curve and divisorial valuations on the ring $\mathcal{O}_{\mathbb{C}^2,0}$ of functions in two variables the information contained in this Poincaré series was (almost) sufficient to restore the action of G on $\mathbb{C}^2$ and the G-equivariant topology of the set of valuations: [6].

In [7] the equivariant Poincaré series $P^G_{\{\nu_i\}}(t_1, \ldots, t_r)$ was defined as an element of the ring $\widetilde{A}(G)[[t_1, \ldots, t_r]]$ of formal power series in the variables $t_1, \ldots, t_r$ with the coefficients from a certain modification $\widetilde{A}(G)$ of the Burnside ring $A(G)$ of the group G. A simple reduction of this Poincaré series is an element of the ring $A(G)[[t_1, \ldots, t_r]]$. Thus it is somewhat close to the ("idealistic") model discussed above. However, in order to define the equivariant Poincaré series of this form, it was necessary to lose quite a lot of information about the individual valuations from the collection. (It is possible to say that one used averaging of the information over the group.) Thus it was not clear how much

information does it keep.
Equivariant topology of germs of plane curves seems to be much more involved than the usual (non-equivariant) one. For example, it is unclear whether the equivariant topology of a collection of curves always determines the equivariant Poincaré series of the collection. Here we discuss to which extent the G-equivariant Poincaré series from [7] determines the topology of a set of plane valuations. We show that the G-equivariant Poincaré series of a collection of divisorial valuations determines the equivariant topology of this collection (in a natural "weak" sense: see below). We also show that with some minor exceptions the equivariant Poincaré series of a collection of curve valuations determines the weak equivariant topology of the collection. (This answer resembles the one from [6]. However reasons for that (and thus the proofs) are quite different. The version of the equivariant Poincaré series considered in [6] is apriori a much more fine invariant than that considered here.)
The G-equivariant Poincaré series $P^G_{\{\nu_i\}}$ considered in [5] depends essentially on the set of valuations defining the filtration. In particular, the substitution of one of them (say, ν_i) by its shift $a^*\nu_i$, $a \in G$, changes the G-equivariant Poincaré series $P^G_{\{\nu_i\}}$. The Poincaré series $P^G_{\{\nu_i\}}(\underline{t})$ considered in [7] depends not on the valuations ν_i themselves, but on their G-orbits. The substitution of one of them by its shift does not change the G-equivariant Poincaré series $P^G_{\{\nu_i\}}(\underline{t})$. Therefore this series cannot determine the G-topology of a collection of divisorial and/or of curve valuations on $\mathcal{O}_{\mathbb{C}^2,0}$ in the form defined in [6]. One has to modify this notion a little bit.
Assume first that we consider sets of curve valuations. Let $\{C_i\}_{i=1}^r$ and $\{C'_i\}_{i=1}^r$ be two collections of branches (that is of irreducible plane curve singularities) in the complex plane $(\mathbb{C}^2, 0)$ with an action of a finite group G. We shall say that these collections are *weakly G-topological equivalent* if there exists a G-invariant germ of a homeomorphism $\psi : (\mathbb{C}^2, 0) \to (\mathbb{C}^2, 0)$ such that for each $i = 1, \dots, r$ one has $\psi(C_i) = a_i C'_i$ with an element $a_i \in G$ (i.e if the image of the G-orbit of the branch C_i coincides with the G-orbit of the branch C'_i). To formulate an analogue of this definition for collections of divisorial valuations, one can describe a divisorial valuation ν on $\mathcal{O}_{\mathbb{C}^2,0}$ by a pair of curvettes intersecting the corresponding divisor (transversally) at different points. The corresponding pair of curvettes allows to determine the divisor as the last one (and so the unique one with self-intersection equal to -1) appearing in the minimal embedded resolution of them. Two collections of divisorial valuations $\{\nu_i\}_{i=1}^r$ and $\{\nu'_i\}_{i=1}^r$ described by the corresponding collections of curvettes $\{L_{ij}\}_{i=1,j=1,2}^r$ and $\{L'_{ij}\}_{i=1,j=1,2}^r$ respectively are weakly G-topologically equivalent if there exists a G-invariant germ of a homeomorphism $\psi : (\mathbb{C}^2, 0) \to (\mathbb{C}^2, 0)$ such that for each $i = 1, \dots, r$ one has $\psi(L_{ij}) = a_i L'_{ij}$ for $j = 1, 2$ and an element $a_i \in G$.
One has an obvious analogue of Theorem 2.9 from [6]. This means that, for a fixed representation of the group G on $\mathbb{C}^2$, the weak topology of a collection of curve or/and divisorial valuations on $\mathcal{O}_{\mathbb{C}^2,0}$ is determined by the G-resolution graph Γ^G of the collection (where not individual branches or/and divisors, but their orbits are indicated) plus the correspondence between the tails of this

graph emerging from special points of the first component of the exceptional divisor with these special points (see below).

2 Equivariant Poincaré series

Let us briefly recall the definition of the G-equivariant Poincaré series $P^G_{\{\nu_i\}}(t_1, \ldots, t_r)$ of a collection of order functions on the ring $\mathcal{O}_{V,0}$ of germs of functions on $(V, 0)$ and the equation for it in terms of a G-equivariant resolution of curve or/and divisorial plane valuations which will be used here.

Definition: A finite *equipped G-set* is a pair $\widetilde{X} = (X, \alpha)$ where:

- X is a finite G-set;
- α associates to each point $x \in X$ a one-dimensional representation α_x of the isotropy subgroup $G_x = \{a \in G : ax = x\}$ of the point x so that, for $a \in G$, one has $\alpha_{ax}(b) = \alpha_x(a^{-1}ba)$, where $b \in G_{ax} = aG_xa^{-1}$.

Let $\widetilde{A}(G)$ be the Grothendieck group of finite equipped G-sets. The cartesian product defines a ring structure on it. The class of an equipped G-set $\widetilde{X}$ in the Grothendieck ring $\widetilde{A}(G)$ will be denoted by $[\widetilde{X}]$. As an abelian group $\widetilde{A}(G)$ is freely generated by the classes of the irreducible equipped G-sets $[G/H]_\alpha$ for all the conjugacy classes $[H]$ of subgroups of G and for all one-dimensional representations α of H (a representative of the conjugacy class $[H] \in \text{Conjsub}\, G$). There is a natural homomorphism ρ from the ring $\widetilde{A}(G)$ to the Burnside rings $A(G)$ of the group G defined by forgetting the one-dimensional representation corresponding to the points. The reduction $\hat{\rho} : \widetilde{A}(G) \to \mathbb{Z}$ is defined by forgetting the representations and the G-action. There are natural pre-λ-structure on a rings $A(G)$ and $\widetilde{A}(G)$ which give sense for the expressions of the form $(1-t)^{-[X]}$, $[X] \in A(G)$, and $(1-t)^{-[\widetilde{X}]}$, $[\widetilde{X}] \in \widetilde{A}(G)$ respectively: see [7]. Both ρ and $\hat{\rho}$ are homomorphisms of pre-λ-rings.

Let $(V, 0)$ be a germ of a complex analytic space with an action of a finite group G and let $\mathcal{O}_{V,0}$ be the ring of germs of functions on it. Without loss of generality we assume that the G-action on $(V, 0)$ is faithful. The group G acts on $\mathcal{O}_{V,0}$ by $a^*f(z) = f(a^{-1}z)$ ($z \in V$, $a \in G$). A valuation ν on the ring $\mathcal{O}_{V,0}$ is a function $\nu : \mathcal{O}_{V,0} \to \mathbb{Z}_{\geq 0} \cup \{+\infty\}$ such that:

1) $\nu(\lambda f) = \nu(f)$ for $\lambda \in \mathbb{C}^*$;
2) $\nu(f + g) \geq \min\{\nu(f), \nu(g)\}$;
3) $\nu(fg) = \nu(f) + \nu(g)$.

A function $\nu : \mathcal{O}_{V,0} \to \mathbb{Z}_{\geq 0} \cup \{+\infty\}$ which possesses the properties 1) and 2) is called an *order function*.

Let $\nu_1, \ldots, \nu_r$ be a collection of order functions on $\mathcal{O}_{V,0}$. It defines an r-index filtration on $\mathcal{O}_{V,0}$:

$$J(\underline{v}) = \{h \in \mathcal{O}_{V,0} : \underline{\nu}(h) \geq \underline{v}\}\ ,$$

where $\underline{v} = (v_1, \ldots, v_r) \in \mathbb{Z}^r_{\geq 0}$, $\underline{\nu}(h) = (\nu_1(h), \ldots, \nu_r(h))$ and $\underline{v}' = (v'_1, \ldots, v'_r) \geq \underline{v}'' = (v''_1, \ldots, v''_r)$ if and only if $v'_i \geq v''_i$ for all i.

Let $\omega_i : \mathcal{O}_{V,0} \to \mathbb{Z}_{\geq 0} \cup \{+\infty\}$ be defined by $\omega_i = \sum_{a\in G} a^*\nu_i$. The functions ω_i are G-invariant (they are not, in general, order functions). For an element $h \in \mathbb{P}\mathcal{O}_{V,0}$, that is for a function germ considered up to a constant factor, let G_h be the isotropy subgroup $G_h = \{a \in G : a^*h = \alpha_h(a)h\}$ and let $Gh \cong G/G_h$ be the orbit of h in $\mathbb{P}\mathcal{O}_{V,0}$. The correspondence $a \mapsto \alpha_h(a) \in \mathbb{C}^*$ determines a one-dimensional representation α_h of the subgroup G_h. Let $\widetilde{X}_h = [G/G_h]_{\alpha_h}$ be the element of the ring $\widetilde{A}(G)$ represented by the G-set Gh with the representation α_{a^*h} associated to the point $a^*h \in Gh$ $(a \in G)$. The correspondence $h \mapsto \widetilde{X}_h$ defines a function $(\widetilde{X})$ on $\mathbb{P}\mathcal{O}_{V,0}/G$ with values in $\widetilde{A}(G)$. The *equivariant Poincaré series* $P^G_{\{\nu_i\}}(\underline{t})$ of the collection $\{\nu_i\}$ is defined by the equation

$$P^G_{\{\nu_i\}}(\underline{t}) = \int_{\mathbb{P}\mathcal{O}_{V,0}/G} \widetilde{X}_h \underline{t}^{\underline{\omega}(h)} d\chi \in \widetilde{A}(G)[[t_1, \ldots, t_r]]\,, \tag{1}$$

where $\underline{t} := (t_1, \ldots, t_r)$, $\underline{t}^{\underline{\omega}(h)} = t_1^{\omega_1(h)} \cdot \ldots \cdot t_r^{\omega_r(h)}$, $t_i^{+\infty}$ should be regarded as 0. The precise meaning of this integral see in [7].

Applying the reduction homomorphism $\rho : \widetilde{A}(G) \to A(G)$ to the Poincaré series $P^G_{\{\nu_i\}}(\underline{t})$, i.e. to its coefficients, one gets the series $\rho P^G_{\{\nu_i\}}(\underline{t}) \in A(G)[[t_1, \ldots, t_r]]$, i.e. a power series with the coefficients from the (usual) Burnside ring. Applying the homomorphism $\widehat{\rho} : \widetilde{A}(G) \to \mathbb{Z}$ one gets the series $\widehat{\rho} P^G_{\{\nu_i\}}(\underline{t}) \in \mathbb{Z}[[t_1, \ldots, t_r]]$. One has

$$\widehat{\rho} P^G_{\{\nu_i\}}(\underline{t}) = P_{\{a^*\nu_i\}}(t_1, \ldots, t_1, t_2, \ldots, t_2, \ldots, t_r, \ldots, t_r)\,,$$

where $P_{\{a^*\nu_i\}}(\bullet)$ is the usual (non-equivariant) Poincaré series of the collection of $|G|r$ order functions $\{a^*\nu_1, a^*\nu_2, \ldots, a^*\nu_r | a \in G\}$ (each group of equal variables in $P_{\{a^*\nu_i\}}$ consists of $|G|$ of them).

Now assume that a finite group G acts linearly on $(\mathbb{C}^2, 0)$ and let ν_i, $i = 1, \ldots, r$, be either a curve or a divisorial valuation on $\mathcal{O}_{\mathbb{C}^2,0}$. We shall write $I_0 = \{1, 2, \ldots, r\} = I' \sqcup I''$, where $i \in I'$ if and only if the corresponding valuation ν_i is a curve one. For $i \in I'$, let $(C_i, 0)$ be the plane curve defining the valuation ν_i.

A *G-equivariant resolution* (or a *G-resolution* for short) of the collection $\{\nu_i\}$ of valuations is a proper complex analytic map $\pi : (\mathcal{X}, \mathcal{D}) \to (\mathbb{C}^2, 0)$ from a smooth surface $\mathcal{X}$ with a G-action such that:

1) π is an isomorphism outside of the origin in $\mathbb{C}^2$;

2) π commutes with the G-actions on $\mathcal{X}$ and on $\mathbb{C}^2$;

3) the total transform $\pi^{-1}(\bigcup\limits_{i\in I',\, a\in G} aC_i)$ of the curve $GC = G(\bigcup\limits_{i\in I'} C_i)$ is a normal crossing divisor on $\mathcal{X}$ (in particular, the exceptional divisor $\mathcal{D} = \pi^{-1}(0)$ is a normal crossing divisor as well);

4) for each branch C_i, $i \in I'$, its strict transform $\widetilde{C}_i$ is a germ of a smooth curve transversal to the exceptional divisor $\mathcal{D}$ at a smooth point x of it and is invariant with respect to the isotropy subgroup $G_x = \{g \in G : gx = x\}$ of the point x;

5) for each $i \in I''$, the exceptional divisor $\mathcal{D} = \pi^{-1}(0)$ contains the divisor defining the divisorial valuation ν_i.

A G-resolution can be obtained by a G-invariant sequence of blow-ups of points. The action of the group G on the first component of the exceptional divisor can either be trivial (this may happen only if G is cyclic) or have fixed points of (proper) subgroups of G. (If G is abelian, these are the fixed points of G itself.) These points are called *special*.

Let $\overset{\circ}{\mathcal{D}}$ be the "smooth part" of the exceptional divisor $\mathcal{D}$ in the total transform $\pi^{-1}(GC)$ of the curve GC, i.e., $\mathcal{D}$ itself minus all the intersection points of its components and all the intersection points with the components of the strict transform of the curve GC. For $x \in \overset{\circ}{\mathcal{D}}$, let $\widetilde{L}_x$ be a germ of a smooth curve on $\mathcal{X}$ transversal to $\overset{\circ}{\mathcal{D}}$ at the point x and invariant with respect to the isotropy subgroup G_x of the point x. The image $L_x = \pi(\widetilde{L}_x) \subset (\mathbb{C}^2, 0)$ is called a *curvette* at the point x. Let the curvette L_x be given by an equation $h_x = 0$, $h_x \in \mathcal{O}_{\mathbb{C}^2,0}$. Without loss of generality one can assume that the function germ h_x is G_x-equivariant. Moreover we shall assume that the germs h_x associated to different points $x \in \overset{\circ}{\mathcal{D}}$ are choosen so that $h_{ax}(a^{-1}z)/h_x(z)$ is a constant (depending on a and x).

Let E_σ, $\sigma \in \Gamma$, be the set of all irreducible components of the exceptional divisor $\mathcal{D}$ (Γ is a G-set itself). For σ and δ from Γ, let $m_{\sigma\delta} := \nu_\sigma(h_x)$, where ν_σ is the corresponding divisorial valuation, h_x is the germ defining the curvette at a point $x \in E_\delta \cap \overset{\circ}{\mathcal{D}}$. One can show that the matrix $(m_{\sigma\delta})$ is minus the inverse matrix to the intersection matrix $(E_\sigma \circ E_\delta)$ of the irreducible components of the exceptional divisor $\mathcal{D}$. For $i = 1, \ldots, r$, let $m_{\sigma i} := m_{\sigma\delta}$, where E_δ is the component of $\mathcal{D}$ corresponding to the valuation ν_i, i.e. either the component defining the valuation ν_i if ν_i is a divisorial valuation (i.e. if $i \in I''$), or the component intersecting the strict transform of the corresponding irreducible curve C_i if ν_i is a curve valuation (i.e. if $i \in I'$). Let $\underline{m}_\sigma := (m_{\sigma 1}, \ldots, m_{\sigma r}) \in \mathbb{Z}^r_{\geq 0}$, $M_{\sigma i} := \sum_{a\in G} m_{(a\sigma)i}$, $\underline{M}_\sigma := (M_{\sigma 1}, \ldots, M_{\sigma r}) = \sum_{a\in G} \underline{m}_{a\sigma}$.

Let $\widehat{\mathcal{D}}$ be the quotient $\overset{\circ}{\mathcal{D}}/G$ and let $p : \overset{\circ}{\mathcal{D}} \to \widehat{\mathcal{D}}$ be the factorization map. Let $\{\Xi\}$ be a stratification of the smooth curve $\widehat{\mathcal{D}}$ such that:

1) each stratum Ξ is connected;

2) for each point $\widehat{x} \in \Xi$ and for each point x from its pre-image $p^{-1}(\widehat{x})$, the conjugacy class of the isotropy subgroup G_x of the point x is the same, i.e., depends only on the stratum Ξ.

The condition 2) is equivalent to say that the factorization map $p : \overset{\circ}{\mathcal{D}} \to \widehat{\mathcal{D}}$ is a (non-ramified) covering over each stratum Ξ. The condition 1) implies that the inverse image in $\overset{\circ}{\mathcal{D}}$ of each stratum Ξ lies in the orbit of one component E_σ of the exceptional divisor. The element $\underline{M}_\sigma \in \mathbb{Z}^r_{\geq 0}$ depends only on the stratum Ξ and will be denoted by $\underline{M}_\Xi$.

For a point $x \in \overset{\circ}{\mathcal{D}}$, let $\widetilde{X}_x = [G/G_x]_{\alpha_{h_x}} \in \widetilde{A}(G)$. The equipped G-set $\widetilde{X}_x$ is one and the same for all points x from the preimage of a stratum Ξ and therefore it defines an element of $\widetilde{A}(G)$ which we shall denote by $[G/G_\Xi]_{\alpha_\Xi}$. In [7, Theorem 1] it was shown that

$$P^G_{\{\nu_i\}}(\underline{t}) = \prod_{\Xi} \left(1 - \underline{t}^{\underline{M}_\Xi}\right)^{-\chi(\Xi)[G/G_\Xi]_{\alpha_\Xi}} . \tag{2}$$

3 Topology of plane valuations

Let the complex plane $(\mathbb{C}^2, 0)$ be endowed by a faithful linear G-action and let $\{\nu_i\}_{i=1}^r$ be a collection of divisorial valuations on $\mathcal{O}_{\mathbb{C}^2,0}$.

Theorem 1 *The G-equivariant Poincaré series $P^G_{\{\nu_i\}}(\underline{t})$ of the collection $\{\nu_i\}$ of divisorial valuations determines the weak G-equivariant topology of this collection.*

Proof. One has to use the following “projection formula”. Let $I = \{i_1, \ldots, i_s\}$ be a subset of the set $\{1, \ldots, r\}$ of the indices numbering the valuations. Then one has

$$P^G_{\{\nu_i\}_{i\in I}}(t_{i_1}, \ldots, t_{i_s}) = P^G_{\{\nu_i\}_{i=1}^r}(t_1, \ldots, t_r)_{|t_i=1 \text{ for } i\notin I} ,$$

i.e. the (G-equivariant) Poincaré series for a subcollection of valuations is obtained from the one for the whole collection by substituting t_i by 1 for all i numbering the valuations which do not participate in the subcollection. (This equation is not valid for other types of valuations, say, for curve ones: see the proof of Theorem 2). The projection formula implies, in particular, that the G-equivariant Poincaré series $P^G_{\{\nu_i\}}(\underline{t})$ of a collection of divisorial valuations determines the G-equivariant Poincaré series (in one variable) of each individual valuation from it.

First we shall show that the Poincaré series $P^G_{\{\nu_i\}}(\underline{t})$ determines the G-resolution graph of the collection of valuations. It turns out that the necessary information about the G-equivariant resolution graph can be restored from the ρ-reduction $\rho P^G_\nu(t)$ of the G-equivariant Poincaré series $P^G_\nu(t)$ (i.e. the series from $A(G)[[t]]$ obtained by forgetting the one-dimensional representations associated with the G-orbits). Therefore we shall start with considering it.

First let us prove the statement for one divisorial valuation. The dual graph Γ^G of the minimal G-equivariant resolution of a divisorial valuation ν looks like in Fig. 1. This means the following.

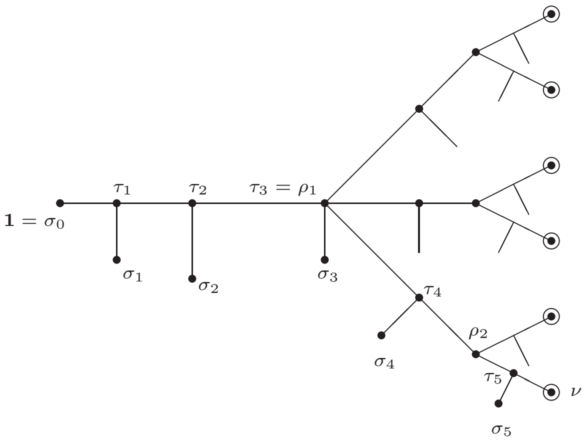


Figure 1: The dual equivariant resolution graph Γ^G of the valuation ν.

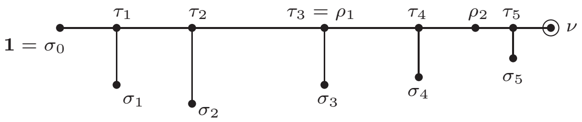


Figure 2: The dual resolution graph Γ of the valuation ν.

The standard (non-equivariant, minimal) dual resolution graph Γ of the valuation ν looks like in Fig. 2. The vertices σ_q, $q = 0, 1, \ldots, g$, are the dead ends of the graph (g is the number of the Puiseux pairs of a curvette corresponding to the valuation, $\sigma_0 = \mathbf{1}$ is the first component of the exceptional divisor), the vertices τ_i, $q = 1, \ldots, g$, are the rupture points, the vertex ν corresponds to the divisorial valuation under consideration. (The vertex ν may coincide with τ_g.) The set of vertices of the graph Γ is ordered according to the order of the birth of the corresponding components of the exceptional divisor. On $[\sigma_0, \nu]$ (the geodesic from $\sigma_0 = \mathbf{1}$ to ν) this order is the natural one: $\delta_1 < \delta_2$ if and only if the vertex δ_1 lies on $[\sigma_0, \delta_2]$.
The integers m_{σ_q}, $q = 0, 1, \ldots, g$, form the minimal set of generators of the semigroup of values of ν and are traditionally denoted by $\overline{\beta}_q$. One also uses the following notations. $e_q := \gcd(\overline{\beta}_0, \overline{\beta}_1, \ldots, \overline{\beta}_q)$,

$$N_q := \frac{e_{q-1}}{e_q} \left(= \frac{m_{\tau_q}}{m_{\sigma_q}} \right) .$$

The graph Γ^G of the minimal G-equivariant resolution consists of $|G|$ copies of graph Γ (numbered by the elements of G) glued together. The gluing is defined by a sequence

$$G = H_0 \supset H_1 \supset H_2 \supset \ldots \supset H_k$$

of subgroups of the group G such that all H_i with $i > 0$ are abelian and H_k is the isotropy group of the valuation ν ($\{a \in G : a^*\nu = \nu\}$) and by a sequence by vertices $\rho_1, \ldots, \rho_k$ of the graph Γ such that all of them lie on the geodesic from σ_0 to ν, $\rho_1 < \rho_2 < \ldots < \rho_k$. (Some of the vertices ρ_i may coincide with some of the vertices τ_j; the vertex ρ_1 may coincide with the initial vertex $\sigma_0 = \mathbf{1}$.) The copies of Γ numbered by the elements a_1 and a_2 from G are glued along the part preceeding ρ_ℓ (i.e., by identifying all the vertices smaller or equal to ρ_ℓ) if $a_1 a_2^{-1} \in H_{\ell-1}$. (In particular the initial vertices $\sigma_0 = \mathbf{1}$ of all the copies are identified.) Pay attention that some of the vertices ρ_i can be inbetween the vertices τ_g and ν. For $q = 1, 2, \ldots, g$, let $j(q)$ be defined by the condition $\rho_{j(q)} < \tau_q \le \rho_{j(q)+1}$.
For $\delta \in \Gamma^G$ (or for the corresponding $\delta \in \Gamma$), let $M_\delta := \sum_{a \in G} m_{a\delta}$. One can easily see that all the integers M_δ, $\delta \in \Gamma$, are different. (One has $M_{\delta_1} = M_{\delta_2}$ for δ_1 and δ_2 from Γ^G if and only if there exists $a \in G$ such that $\delta_2 = a\delta_1$.) One has $M_{\tau_q} = N_q M_{\sigma_q}$.
The series $\rho P^G_\nu(t)$ is given by the equation

$$\begin{aligned}\rho P^G_\nu(t) \quad &= \quad \prod_{q=0}^{g} \left(1 - t^{M_{\sigma_q}}\right)^{-[G/H_{j(q)}]} \cdot \prod_{q=1}^{g} \left(1 - t^{N_q M_{\sigma_q}}\right)^{[G/H_{j(q)}]} \times \\ &\times \quad \prod_{j=1}^{\ell} \left(1 - t^{M_{\rho_j}}\right)^{[G/H_j]-[G/H_{j-1}]} \cdot \left(1 - t^{M_\nu}\right)^{-[G/H_k]} .\end{aligned}$$

The fact that all the integers M_δ are different implies that the exponents M_{σ_q}, $q = 1, \ldots, g$, are among those which participate in the decomposition of the series $\rho P^G_\nu(t)$ with negative cardinalities of the multiplicities. (The multiplicity of a binomial $(1 - t^m)^{s_m}$, $s_m \in A(G)$, is s_m. Its cardinality is the (virtual) number of the points of it.) It is possible that the exponents of this sort include also M_ν corresponding to the divisorial valuation itself.
The subgroups $H_1 \supset H_2 \supset \ldots \supset H_k$ are defined by the multiplicities of all the factors in the decomposition of the series $\rho P^G_\nu(t)$ into the product of the binomials.
The vertex $\sigma_0 = \mathbf{1}$ coincides with ρ_1 if and only if the binomial with the smallest exponent in the decomposition of the series $\rho P^G_\nu(t)$ has a non-negative cardinality of the multiplicity. For $\sigma_q \le \rho_1$ one has $M_{\sigma_q} = |G| m_{\sigma_q}$ and $M_{\rho_1} = |G| m_{\rho_1}$. These equations give all the generators $\overline{\beta}_q$ of the semigroup of values with $\sigma_q \le \rho_1$ and also m_{ρ_1}.
For $\ell \ge 1$, let $\sigma_{q(\ell)}$ be the minimal dead end greater than ρ_ℓ (i.e. there are the dead ends $\sigma_{q(\ell)}, \ldots, \sigma_{q(\ell+1)-1}$ inbetween ρ_ℓ and $\rho_{\ell+1}$). Let us consider the dead ends σ_q such that $\rho_1 < \sigma_q < \rho_2$. One has

$$M_{\sigma_{q(1)}} = |H_1| m_{\sigma_{q(1)}} + (|G| - |H_1|) m_{\rho_1} = |H_1| m_{\sigma_{q(1)}} + (M_{\rho_1} - |H_1| m_{\rho_1}) .$$

The smallest multiple of the exponent $M_{\sigma_{q(1)}}$ in a binomial participating in the decomposition of the series $\rho P^G_\nu(t)$ is $M_{\tau_{q(1)}} = N_{q(1)} M_{\sigma_{q(1)}}$. Further, for $\rho_1 < \sigma_{q(1)} < \sigma_{q(1)+1} < \sigma_{q(1)+2} < \cdots \sigma_{q(2)-1} < \rho_2$, one has

$$\begin{aligned}
M_{\sigma_{q(1)+1}} &= |H_1| m_{\sigma_{q(1)+1}} + (M_{\rho_1} - |H_1| m_{\rho_1}) N_{q(1)}\,,\\
M_{\sigma_{q(1)+2}} &= |H_1| m_{\sigma_{q(1)}+2} + (M_{\rho_1} - |H_1| m_{\rho_1}) N_{q(1)} N_{q(1)+1}\,,\\
&\ldots\\
M_{\rho_2} &= |H_1| m_{\rho_2} + (M_{\rho_1} - |H_1| m_{\rho_1}) N_{q(1)} N_{q(1)+1} \cdot \ldots \cdot N_{q(2)-1}\,.
\end{aligned}$$

These equations give all the generators $\overline{\beta}_q$ of the semigroup of values with $\sigma_q < \rho_2$ and also m_{ρ_2}.

Assume that we have determined all the exponents m_{σ_q} for $q < q(\ell)$ and also the exponent m_{ρ_ℓ}. Let us consider the dead ends σ_q such that $\rho_\ell < \sigma_q < \rho_{\ell+1}$. One has

$$\begin{aligned}
M_{\sigma_{q(\ell)}} &= |H_\ell| m_{\sigma_{q(\ell)}} + (M_{\rho_\ell} - |H_\ell| m_{\rho_\ell})\,,\\
M_{\sigma_{q(\ell)+1}} &= |H_\ell| m_{\sigma_{q(\ell)+1}} + (M_{\rho_\ell} - |H_\ell| m_{\rho_\ell}) N_{q(\ell)}\,,\\
M_{\sigma_{q(\ell)+2}} &= |H_\ell| m_{\sigma_{q(\ell)+2}} + (M_{\rho_\ell} - |H_\ell| m_{\rho_\ell}) N_{q(\ell)} N_{q(\ell)+1}\,,\\
&\ldots\\
M_{\rho_{\ell+1}} &= |H_\ell| m_{\rho_\ell} + (M_{\rho_\ell} - |H_\ell| m_{\rho_\ell}) N_{q(\ell)} N_{q(\ell)+1} \cdot \ldots \cdot N_{q(\ell+1)-1}\,.
\end{aligned}$$

These equations give all the generators m_{σ_q} of the semigroup of values with $q < q(\ell+1)$ and also $m_{\rho_{\ell+1}}$.

The described procedure recovers m_{σ_q} for all $q \le g$. If, in the binomials of the decomposition of the series $\rho P^G_\nu(t)$, there are no exponents proportional to M_{σ_g}, one has $\nu = \tau_g$ and the resolution graph Γ is determined by the semigroup $\langle \overline{\beta}_0 \overline{\beta}_1, \ldots, \overline{\beta}_g \rangle$. Otherwise the described above procedure permits to determine the exponents m_{ρ_j} with $\rho_j \ge \tau_g$ and m_ν. This gives the G-equivariant resolution graph of one divisorial valuation.

Assume that we have a collection $\{\nu_i\}$ of divisorial valuations, $i = 1, 2, \ldots, r$. To restore the equivariant resolution graph Γ^G of the collection from the resolution graphs of each individual valuation ν_i, one has to determine the separation point δ_{ij} between each two valuations ν_i and ν_j (for simplicity let us assume that $i = 1$, $j = 2$). Let

$$\rho P^G_\nu(t_1, t_2, 1, \ldots, 1) = \prod (1 - t_1^{M_1} t_2^{M_2})^{s_{M_1 M_2}}\,, \tag{3}$$

$s_{M_1 M_2} \in \mathbb{Z}$, be the decomposition into the product of the binomials. The separation point δ_{12} corresponds to the maximal exponent in the decomposition (3) with

$$\frac{M_{\delta 1}}{M_{\delta 2}} = \frac{M_{\sigma_0 1}}{M_{\sigma_0 2}}\,.$$

This proves that the reduction $\rho P^G_{\{\nu_i\}}(\underline{t}) \in A(G)[[t_1, \ldots, t_r]]$ of the G-equivariant Poincaré series $P^G_{\{\nu_i\}}(\underline{t})$ determines the minimal G-resolution graph of the set $\{\nu_i\}$ of divisorial valuations.

In order to prove that one can also determine the weak G-topology of the collection of valuations, one has to show how is it possible to restore the representation of the group G on $\mathbb{C}^2$ and the correspondence between (some) tails of the (minimal) G-resolution graph and the special points on the first component of the exceptional divisor. For that one should use the non-reduced Poincaré series $P^G_{\{\nu_i\}}(\underline{t}) \in \widetilde{A}(G)[[t_1, \ldots, t_r]]$ itself. (If there are no special points on the first component of the exceptional divisor (this can happen only if G is cyclic), only the representation of G on $\mathbb{C}^2$ has to be determined.) We follow the scheme described in [6].

Let us consider the case of an abelian group G first. If there are no special points on the first component $E_{\mathbf{1}}$ of the exceptional divisor, all points of $E_{\mathbf{1}}$ are fixed with respect to the group G, the group G is cyclic and the representation is a scalar one. This (one dimensional) representation is dual to the representation of the group G on the one-dimensional space generated by any linear function. The case when there are no more components in $\mathcal{D}$, i.e. if the resolution is achieved by the first blow-up, is trivial. Otherwise let us consider a maximal component E_σ among those components E_τ of the exceptional divisor for which $G_\tau = G$ and the corresponding curvette is smooth. (The last condition can be easily detected from the resolution graph.) The smooth part $\overset{\bullet}{E}_\sigma$ of this component contains a special point x with $G_x = G$ (or all the points of $\overset{\bullet}{E}_\sigma$ are such that $G_x = G$). The point(s) from $\overset{\bullet}{E}_\sigma$ with $G_x = G$ bring(s) into the decomposition of the Poincaré series $P^G_{\{\nu_i\}}(\underline{t})$ a factor of the form $(1 - \underline{t}^{\underline{M}})^{-[G/G]_\alpha}$. The ($G$-equivariant) curvette L at the described special point of the divisor is smooth. Therefore the representation of G on the one-dimensional space generated by a G-equivariant equation of L coincides with the representation on the space generated by a linear function. Let us take all factors of the form $(1 - \underline{t}^{\underline{M}})^{-[G/G]_\alpha}$ in the decomposition of the Poincaré series $P^G_{\{v_i\}}$. For each of them, the exponent $\underline{M}$ determines the corresponding component of the exceptional divisor and therefore the topological type of the corresponding curvettes. The factor which corresponds to a component with a smooth curvette gives us the representation α on the space generated by a linear function.

Now assume that there are two special points on the first component of the resolution. Without loss of generality we can assume that they correspond to the coordinate axis $\{x = 0\}$ and $\{y = 0\}$. The representation of the group G on $\mathbb{C}^2$ is defined by its action on the linear functions x and y. For each of them this action can be recovered from a factor of the form described above just in the same way. Moreover, a factor, which determines the action of the group G on the function x, corresponds to a component of the exceptional divisor from the tail emerging from the point $\{x = 0\}$.

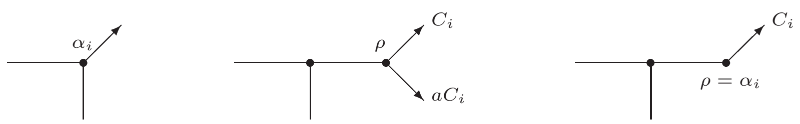
Now let G be an arbitrary (not necessarily abelian) group. For an element $g \in G$ consider the action of the cyclic group $\langle g\rangle$ generated by g on $\mathbb{C}^2$. One can see that the G-equivariant Poincaré series $P^G_{\{v_i\}}(\underline{t})$ determines the $\langle g\rangle$-

Poincaré series $P^{\langle g\rangle}_{\{v_i\}}(\underline{t})$ just like in [5, Proposition 2]. This implies that the G-equivariant Poincaré series determines the representation of the subgroup $\langle g\rangle$. (Another way is to repeat the arguments above adjusting them to the subgroup $\langle g\rangle$.) Therefore the G-Poincaré series $P^{G}_{\{v_i\}}(\underline{t})$ determines the value of the character of the G-representation on $\mathbb{C}^2$ for each element $g \in G$ and thus the representation itself. Special points of the G-action on the first component $E_{\mathbf{1}}$ of the exceptional divisor correspond to some abelian subgroups H of G. For each such subgroup H there are two special points corresponding to different one-dimensional representations of H. Again the construction above for an abelian group permits to identify tails of the dual resolution graph with these two points. $\square$

Let $\{C_i\}$, $i = 1, \ldots, r$, be a collection of irreducible curve singularities in $(\mathbb{C}^2, 0)$ such that it does not contain curves from the same G-orbit and it does not contain a smooth curve invariant with respect to a non-trivial element of G whose action on $\mathbb{C}^2$ is not a scalar one. Let $\{\nu_i\}$ be the corresponding collection of valuations. Let $G_i \subset G$ be the isotropy group of the branch C_i, $1 \le i \le r$.

Theorem 2 *The G-equivariant Poincaré series $P^{G}_{\{\nu_i\}}(\underline{t})$ of the collection $\{\nu_i\}$ determines the weak G-equivariant topology of the collection $\{\nu_i\}$ of curve valuations.*

Proof. The minimal resolution graph Γ of the plane curve singularity $C = \bigcup_{i=1}^{r} C_i$ is essentially the same as the graph of the divisorial valuations defined by the set of irreducible components $\{E_{\alpha_i}\}$ of the exceptional divisor such that the strict transform of C_i intersects the component E_{α_i}. Instead of the mark used for the divisor E_{α_i} (like in Figures 1 and 2 for one valuation) one puts an arrow corresponding to C_i connected to the vertex α_i. Notice that there can be several arrows connected to the same vertex, i.e. $\alpha_i = \alpha_j$ for different branches C_i, C_j. In the case of one branch the graph looks like the one in Figure 2 but the vertex marked by ν coincides with τ_g and there is an arrow connected with τ_g. The number g is equal to the number of Puiseux pairs of the curve and $m_{\sigma_i} = \bar{\beta}_i$, $0 \le i \le g$, are the elements of the minimal set of generators of the semigroup of the branch. (In particular they determine the minimal resolution graph of the curve.)



(a) Γ (b) Γ^G (c) Γ enlarged

Figure 3: The graphs Γ, Γ^G and Γ enlarged.

The same rules apply for the graph Γ^G. However Γ^G corresponds to the embedded resolution of the union of all the orbits of the branches of C. So, it is possible that, in order to achieve the minimal equivariant resolution (i.e. in order to separate all the conjugates of each one of the branches C_i), one has to add some additional blow-ups starting at the point α_i. Note that in this case some of the vertices ρ (see the notations in the proof of Theorem 1 and Figures 1 and 2) do not appear in Γ. In order to preserve the scheme and the notations from the proof of the case of divisorial valuations it is better to enlarge Γ in such a way that the new one (also denoted by Γ) is the minimal one in which all the vertices ρ are present (see Figure 3). Note that $aE_{\alpha_i} = E_{a\alpha_i}$ for $a \in G$, so in this way the (new) resolution graph Γ is just the quotient of Γ^G by the obvious action of G on Γ^G.
As in the case of divisorial valuations, for each $\delta \in \Gamma^G$ let $h_\delta = 0$, $h_\delta \in \mathcal{O}_{\mathbb{C}^2,0}$, be the equation of a curvette at the component E_δ, $m_{\delta i}$ be the value $\nu_i(h_\delta)$, $M_{\delta i} = \sum_{a\in G} m_{(a\delta)i} = \sum_{a\in G}(a^*\nu_i)(h_\delta)$ and $\underline{M}_\delta = (M_{\delta 1}, \ldots, M_{\delta r}) \in \mathbb{Z}^r_{\geq 0}$. All the $\underline{M}_\sigma$, $\sigma \in \Gamma$, are different and for $\sigma, \tau \in \Gamma^G$ $\underline{M}_\sigma = \underline{M}_\tau$ if and only if $E_\tau = aE_\sigma$ for some $a \in G$. Let $G_i \subset G$ be the isotropy group of the branch C_i, $1 \leq i \leq r$.
For $i, j \in \{1, \ldots, r\}$, $m_{\alpha_i j}$ is just the intersection multiplicity between C_i and C_j and

$$M_{\alpha_i j} = \sum_{a\in G} m_{(a\alpha_i)j} = \sum_{a\in G}(a^*\nu_j)(h_{\alpha_i}) = (C_i, \bigcup_{a\in G} aC_j) = (C_j, \bigcup_{a\in G} aC_i) = M_{\alpha_j i}\,.$$

In contrast with the case of divisorial valuations the projection formula is different from the one for divisorial valuations formulated at the beginning of the proof of Theorem 1. Instead of it one has the following one: For $i_0 \in \{1, \ldots, r\}$ one has

$$P^G_{\{\nu_i\}}(\underline{t})_{|_{t_{i_0}=1}} = (1 - \underline{t}^{\underline{M}_{\alpha_{i_0} i_0}})^{[G/G_{i_0}]_{\alpha_{i_0}}}_{|_{t_{i_0}=1}} P^G_{\{\nu_i\}_{i\neq i_0}}(t_1, \ldots, t_{i_0-1}, t_{i_0+1}, \ldots, t_r)\,. \quad (4)$$

(This can be easily deduced from (2).) Using (4) repeatedly one also has:

$$P^G_{\{\nu_i\}}(\underline{t})_{|_{t_i=1, i\neq i_0}} = \prod_{i\neq i_0}(1 - t_{i_0}^{M_{\alpha_i i_0}})^{[G/G_i]_{\alpha_i}} P^G_{\nu_{i_0}}(t_{i_0})\,. \quad (5)$$

Equations (4) and (5) imply that in order to describe inductively the minimal G-resolution graph Γ^G one has to detect the binomial $(1-\underline{t}^{\underline{M}_{\alpha_{i_0}}})$ corresponding to some i_0 from the G-equivariant Poincaré series and also the intersection multiplicities of C_{i_0} with the other branches of C. As in the divisorial case, the necessary information about the G-equivariant resolution graph can be restored from the ρ-reduction $\rho P^G_{\{\nu_i\}}(\underline{t})$ of the Poincaré series $P^G_{\{\nu_i\}}(\underline{t})$ to the ring $A(G)[[t_1, \ldots, t_r]]$. From the factorization given in (2) one can write $\rho P^G_{\{\nu_i\}}(\underline{t}) = \prod_{\sigma\in\Gamma}(1-\underline{t}^{\underline{M}_\sigma})^{s_\sigma}$, where $s_\sigma \in A(G)$. Note that the multiplicity s_σ may be equal to zero, i.e. the binomial factor corresponding to σ may be absent.

The determination of the G-equivariant resolution graph from the series $\rho P^G_\nu(t)$ for one branch almost repeats the one described for one divisorial valuation, e.g. the semigroup is the same as the one of the divisorial valuation defined by the component E_{τ_g} of the exceptional divisor. So, let us assume $r > 1$ and let us fix $j, k \in \{1, \ldots, r\}$. The separation point $s(\alpha_j, \alpha_k) \in \Gamma^G$ of α_j and α_k is defined by the condition $[\mathbf{1}, \alpha_j] \cap [\mathbf{1}, \alpha_k] = [\mathbf{1}, s(\alpha_j, \alpha_k)]$. Here $[\mathbf{1}, \sigma]$ is the geodesic in the dual graph Γ^G joining the first vertex $\mathbf{1}$ with the vertex σ. Now, let us define the separation vertex $s(\alpha_j, k)$ of C_j and GC_k as the maximun of $s(\alpha_j, a\alpha_k)$ for $a \in G$. Note that, if $a \in G$ then $s(a\alpha_j, k) = as(\alpha_j, k) \in \Gamma^G$ so $s(j, k) = s(\alpha_j, k)$ is a well defined vertex of the graph Γ. We refer to it as the separation vertex of C_i and C_j in Γ.
The ratio $M_{\sigma j}/M_{\sigma k}$ is constant for σ in $[\mathbf{1}, s(j, k)]$ and is a strictly increasing function for $\sigma \in [s(i, j), \alpha_j] \subset \Gamma$ as well as in the geodesic $[as(j, k), a\alpha_j] \subset \Gamma^G$ for $a \in G$. Notice that for $\sigma \notin \bigcup_{a \in G} ([\mathbf{1}, a\alpha_j] \cup [\mathbf{1}, a\alpha_k])$ the ratio $M_{\sigma j}/M_{\sigma k}$ is equal to $M_{\sigma' j}/M_{\sigma' k}$ where σ' is the vertex such that

$$[\mathbf{1}, \sigma'] = \max_{a \in G} \{([\mathbf{1}, a\alpha_j] \cup [\mathbf{1}, a\alpha_k]) \cap [\mathbf{1}, \sigma]\} \ .$$

Let $\sigma \in \Gamma$ be such that the exponent $\underline{M}_\sigma$ is a maximal one among the set of exponents $\underline{M}_\tau$ appearing in the factorization

$$\rho P^G_{\{\nu_i\}}(\underline{t}) = \prod_{\tau \in \Gamma\ ,\ s_\tau \neq 0} (1 - \underline{t}^{\underline{M}_\tau})^{s_\tau} \ . \tag{6}$$

(Here we use the partial order $\underline{M} = (M_1, \ldots, M_r) \leq \underline{M}' = (M'_1, \ldots, M'_r)$ if and only if $M_i \leq M'_i$ for all $i = 1, \ldots, r$.) Note that in this case the corresponding factor has positive cardinality and there exists an index $j \in \{1, \ldots, r\}$ such that $\alpha_j = \sigma$.
Let $A \subset \{1, \ldots, r\}$ be the set of indices j such that $M_{\sigma j}/M_{\sigma k} \geq M_{\tau j}/M_{\tau k}$ for all $k \in \{1, \ldots, r\}$ and all $\tau \in \Gamma^G$ such that the binomial $(1 - \underline{t}^{\underline{M}_\tau})$ appears in (6), i.e. $s_\tau \neq 0$. From the comments above it is clear that all indices j such that $\alpha_j = \sigma$ belong to A, however A could contain some other indices ℓ such that $\alpha_\ell \neq \sigma$.
Let us assume that there exists $\ell \in A$ such that $\alpha_\ell \neq \sigma$. The behaviour of the ratios $M_{\tau\ell}/M_{\tau k}$ along $[1, \alpha_\ell]$ described above implies that $\sigma \in [1, \alpha_\ell]$. By definition of the set A, for any $\tau \in [\sigma, \alpha_\ell]$, $\tau \neq \sigma$, the binomial $(1 - \underline{t}^{\underline{M}_\tau})$ does not appear in (6), i.e. $s_\tau = 0$, in particular $\chi(\overset{\circ}{E}_\tau) = 0$. As a consequence, $\alpha_\ell < \sigma$ and α_ℓ is the end point σ_g on the dual graph of C_j (here $j \in A$ such that $\alpha_j = \sigma$). In this case one has $M_{\sigma\ell} < M_{\sigma j}$ and one can distinguish ℓ by this condition. Note that if such an $\ell \in A$ exists then it is unique.
Let $i_0 \in A$ be such that $M_{\sigma i_0} \geq M_{\sigma j}$ for all $j \in A$. Then $\alpha_{i_0} = \sigma$ and the factor $(1 - \underline{t}^{\underline{M}_{\alpha_{i_0}}})^{[G/G_{i_0}]}$ appears in the factorization (6). Thus, the projection formula permits to recover the G-equivariant resolution graph by induction.
As in Theorem 1 one has to show that the Poincaré series $P^G_{\{\nu_i\}}(\underline{t})$ determines the representation of G on $\mathbb{C}^2$, and the correspondence between "tails" of the

resolution graph. The proof in this case does not differ from the one made in Theorem 1 for divisorial valuations since the collection $\{C_i\}$ does not contains smooth curves invariant with respect to a non-trivial element of G whose action is not a scalar one. □

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