Some Remarks on Morphisms Between Fano Threefolds: Erratum

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Ekaterina Amerik

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Abstract. It was pointed out to me by James McKernan that there is a mistake in the proof of Proposition 2.7. Namely, it is stated (and "used") there that the complement to a smooth conic in $\mathbb{P}^2$ is simply-connected. This is obviously false; indeed, the fundamental group is cyclic of order two. This mistake is, however, easy to deal with.

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This mistake is, however, easy to deal with. What I propose below is the corrected version of the end of the proof of Proposition 2.7, starting from the line 20. Instead of "But the latter is simply-connected..." and so forth, read:
"Remark that the fundamental group of the latter is cyclic of order two. In the case $deg(X) = 4$ and $m = 5$, the degree of p is 10; this means that H has at least 5 irreducible components. Each of them gives at least one conic through a general point of X. These conics are mapped to different lines on V_5, because they intersect. This is a contradiction because on V_5 there are only three lines through a general point.

In the other case $deg(X) = 10$ and $m = 2$, the degree of p is 4, and this means that H cannot be irreducible. To derive a contradiction, we need another easy observation, which is made for example in [HM] (Proposition 12). It says that (thanks to the fact that $Pic(X) = \mathbb{Z}$ and that the normal bundle of a general conic on X is trivial) for H_0 an irreducible component of H and $\mathcal{C}_0$ the universal family of conics over H_0, the evaluation map $\mathcal{C}_0 \to X$ cannot be generically one-to-one. So in fact each irreducible component of H provides at least two conics through a general point of X and therefore at least two lines through a general point of V_5; H must thus be irreducible, a contradiction."

References

[HM] J.M. Hwang, N. Mok, Finite morphisms onto Fano manifolds of Picard number 1 which have rational curves with trivial normal bundles, J. Algebraic Geom. 12 (2003), no. 4, 627–651.

Ekaterina Amerik
Université Paris-Sud
Laboratoire des Mathématiques
Bâtiment 425
91405 Orsay
France
Ekaterina.Amerik@math.u-psud.fr