Erratum to the paper "Absolute Continuity of the Spectrum of a Schrödinger Operator with a Potential Which is Periodic in Some Directions and Decays in Others"

Documenta Math Vol. 9 (2004), 107-121

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Received: April 9, 2004

Communicated by Heinz Siedentop

Abstract. Lemma 6.1 and 6.2 in [1] are false as stated there. Below, we correct the proof of Theorem 6.1 accordingly.

2000 Mathematics Subject Classification: 35J10, 35Q40, 81C10

6 One fact from the theory of functions

Lemma 6.1. *Let U be an open subset of $\mathbb{R}^d$. Let f be a real-analytic function on the set $U \times (a, b)$, and pick $\Lambda \subset (a, b)$ such that* $\operatorname{mes} \Lambda = 0$. *Then*

$$\operatorname{mes}\{k \in U : \exists \lambda \in \Lambda \text{ s.t. } f(k, \lambda) = 0 \text{ and } \partial_{k_1} f(k, \lambda) \neq 0\} = 0. \tag{1}$$

Proof. The Implicit Function Theorem implies that, for any point (k^*, λ^*) such that $f(k^*, \lambda^*) = 0 \neq \partial_{k_1} f(k, \lambda^*)$, we can find rational numbers $\tilde{r} > 0$, $\tilde{\lambda}$, a vector $\tilde{k}$ with rational coordinates, and a real analytic function θ defined in $B_{\tilde{r}}(\tilde{k}', \tilde{\lambda})$ such that

1. $(k^*, \lambda^*) \in B_{\tilde{r}}(\tilde{k}, \tilde{\lambda})$;

[1] N.F.'s research was partially supported by the FNS 2000 "Programme Jeunes Chercheurs".

[2] F.K.'s research was partially supported by the program RIAC 160 at Université Paris 13 and by the FNS 2000 "Programme Jeunes Chercheurs".

2. $\theta((k^*)', \lambda) = k_1^*$;
3. $f(k, \lambda) = 0 \Leftrightarrow \theta(k', \lambda) = k_1$ if $(k, \lambda) \in B_{\tilde{r}}(\tilde{k}, \tilde{\lambda})$.

The Jacobian of the map

$$(k', \lambda) \mapsto (\theta(k', \lambda), k')$$

is bounded, so

$$\text{mes}\{(\theta(k', \lambda), k') : (k', \lambda) \in B_{\tilde{r}}(\tilde{k}', \tilde{\lambda}), \lambda \in \Lambda\} = 0,$$

and therefore,

$$\text{mes}\{k : \exists \lambda \in \Lambda \text{ s.t. } (k, \lambda) \in B_{\tilde{r}}(\tilde{k}, \tilde{\lambda}) \text{ and } f(k, \lambda) = 0\} = 0.$$

The set

$$\{(k, \lambda) : f(k, \lambda) = 0 \text{ and } \partial_{k_1} f(k, \lambda) \neq 0\}$$

can be covered by a countable number of balls $B_{\tilde{r}_i}(\tilde{k}_i, \tilde{\lambda}_i)$ constructed as above, hence the measure of the set in (1) is also equal to zero. □

Theorem 6.1. *Let U be a region in $\mathbb{R}^d$, Λ be a subset of an interval (a, b) such that* $\text{mes}\, \Lambda = 0$. *Let h be a real-analytic function defined on the set $U \times (a, b)$ and suppose that*

$$\forall \lambda \in \Lambda \quad \exists k \in U \quad \textit{such that} \quad h(k, \lambda) \neq 0. \tag{2}$$

Then,

$$\text{mes}\{k \in U : \exists \lambda \in \Lambda \textit{ s.t. } h(k, \lambda) = 0\} = 0.$$

Proof. The proof of Theorem 6.1 is that given in [1] except that one uses Lemma 6.1. □

References

[1] N. Filonov and F. Klopp. Absolute continuity of the spectrum of a Schrödinger operator with a potential which is periodic in some directions and decays in others. *Documenta Mathematica*, 9:107–121, 2004.

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