Hasse Invariant and Group Cohomology

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Abstract. Let $p \geq 5$ be a prime number. The Hasse invariant is a modular form modulo p that is often used to produce congruences between modular forms of different weights. We show how to produce such congruences between eigenforms of weights 2 and $p+1$, in terms of group cohomology. We also illustrate how our method works for inert primes $p \geq 5$ in the contexts of quadratic imaginary fields (where there is no Hasse invariant available) and Hilbert modular forms over totally real fields, cyclic and of even degree over the rationals.

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1 The puzzle

Let $p \geq 5$ be a prime number. In the theory of modular forms mod p (see [S] and [SwD]) a special role is played by the *Hasse invariant* and the Θ operator. We fix an embedding $\overline{\mathbf{Q}} \hookrightarrow \overline{\mathbf{Q}_p}$, and denote the corresponding place of $\overline{\mathbf{Q}}$ by $\wp$. We have a modular form E_{p-1} of weight $p-1$ in $M_{p-1}(SL_2(\mathbf{Z}), \mathbf{Z}_p)$, that is congruent to 1 mod $\wp$ (see [S] and [SwD]). By congruence we will mean a congruence of Fourier coefficients at almost all primes. The modular form E_{p-1} is the normalised form of the classical Eisenstein series, and has q-expansion

$$1 - 2(p-1)/B_{p-1} \sum \sigma_{p-2}(n) q^n,$$

and the congruence property of E_{p-1} is then a consequence of the divisibility of the denominator of B_{p-1} by p (the theorem of Clausen-von Staudt: see [S, §1.1]).

Multiplying by E_{p-1} gives the fact that for any positive integer N prime to p a weight 2 form in $S_2(\Gamma_1(N), \mathbf{Z}_p)$ is congruent mod p to a weight $p+1$ form in $S_{p+1}(\Gamma_1(N), \mathbf{Z}_p)$. It follows that a weight 2 normalized eigenform in $S_2(\Gamma_1(N), \overline{\mathbf{Z}_p})$ is congruent mod $\wp$ to a weight $p+1$ eigenform in $S_{p+1}(\Gamma_1(N), \overline{\mathbf{Z}_p})$ (see [DS, §6.10]).

For $N \geq 5$ prime to p, the Hasse invariant constructed geometrically (see [Kz]) is a global section of the coherent sheaf $\omega_{X_1(N)_{\mathbf{F}_p}}^{\otimes p-1}$ where $\omega_{X_1(N)_{\mathbf{F}_p}}$ is the pull back of the canonical sheaf $\Omega_{\mathcal{E}/X_1(N)_{\mathbf{F}_p}}$ by the zero section of the map $\mathcal{E}_{\mathbf{F}_p} \to X_1(N)_{\mathbf{F}_p}$, and with $\mathcal{E}_{\mathbf{F}_p}$ the universal generalised elliptic curve over $X_1(N)_{\mathbf{F}_p}$: E_{p-1} can be interpreted as a characteristic zero lift of the Hasse invariant (Deligne).
The Θ operator on modular forms mod p is defined by:

$$\Theta(\sum a_n q^n) = \sum n a_n q^n$$

where $a_n \in \overline{\mathbf{F}_p}$. It preserves levels, and increases weights by $p+1$, i.e., it gives maps:

$$M_k(\Gamma_1(N), \mathbf{F}_p) \longrightarrow M_{k+p+1}(\Gamma_1(N), \mathbf{F}_p),$$

preserving cusp forms. The analog in group cohomology of the Θ operator on mod p modular forms, can be found in [AS]. The aim of this note is to find a group theoretic substitute for the Hasse invariant.
Unlike as is done in [AS] in the case of Θ, for good reasons we cannot find an *element* in group cohomology that is an analog of the Hasse invariant. What we do find instead is a procedure for raising weights by $p-1$ of mod p Hecke eigenforms of weight two (preserving the level) that is one of the principal uses of the Hasse invariant.
Using the Eichler-Shimura isomorphism the relevant Hecke modules are $H^1(\Gamma_1(N), \mathbf{F}_p)$ and $H^1(\Gamma_1(N), \mathsf{Symm}^{p-1}(\mathbf{F}_p^2))$. From the viewpoint of group cohomology the above considerations give that a Hecke system of eigenvalues $(a_l)_{l \neq p}$ in the former also arises from the latter. This at first sight is puzzling as indeed the $p-1^{\mathbf{st}}$ symmetric power of the standard 2-dimensional representation of $SL_2(\mathbf{F}_p)$ is irreducible. In this short note we "resolve" this puzzle.
Indeed the solution to the puzzle is implicit in an earlier paper of one of us (cf. Remark 4 at the end of Section 3 of [K]) where the issue arose in trying to understand why the methods for studying Steinberg lifts of an irreducible modular Galois representation $\rho \colon G_{\mathbf{Q}} \to GL_2(\overline{\mathbf{F}_p})$ were qualitatively different from those for studying principal series and supercuspidal lifts. The buzzwords there were that the p-dimensional minimal K-type of a Steinberg representation of $GL_2(\mathbf{Q}_p)$ also arises in the restriction to $GL_2(\mathbf{Z}_p)$ of any unramified principal series representation of $GL_2(\mathbf{Q}_p)$.
The key to the solution of this puzzle (again) is a study of the degeneracy map $H^1(\Gamma_1(N), \mathbf{F}_p)^2 \to H^1(\Gamma_1(N) \cap \Gamma_0(p), \mathbf{F}_p)$.
In Section 3 we will give applications of our method in the situation of imaginary quadratic fields, where the "geometric Hasse invariant" perforce is not available. Furthermore modular forms in this setting do not have a multiplicative structure. We owe this observation, and indeed the suggestion that our methods should work in this case, to Ian Kiming. Our cohomological methods do work in this situation under the hypothesis that $p \geq 5$ is inert, but have the (inherent) defect that results are about characteristic p modular forms,

and may not be used directly to produce congruences between characteristic 0 eigenforms of different weights. This comes from the fact that in this situation torsion in cohomology can possibly occur even after localisation at "interesting" maximal ideals of the Hecke algebra (see the concluding remark).
In Section 4 we deal with the case of $p \geq 5$ inert in a totally real field that is cyclic and of even degree over $\mathbf{Q}$, and in Section 5 we spell out some consequences of [K] for raising of levels in higher weights.
It would be interesting to see if the other cases cases can be treated by extending these methods.

2 The solution to the puzzle

Let us recall the hypotheses: $p \geq 5$ is prime, and $N \geq 1$ is prime to p. Consider the cohomology groups $H^1(\Gamma_1(N), \mathbf{F}_p)$ and $H^1(\Gamma_1(N) \cap \Gamma_0(p), \mathbf{F}_p)$. We have the standard action of Hecke operators T_r on these cohomology groups. We recall that we only consider the action for $(r,p) = 1$. We have the degeneracy map

$$\alpha : H^1(\Gamma_1(N), \mathbf{F}_p)^2 \to H^1(\Gamma_1(N) \cap \Gamma_0(p), \mathbf{F}_p)$$

that is defined to be the sum $\alpha_1 + \alpha_2$ where α_1 is the restiction map, and α_2 the "twisted" restriction map, given by conjugation by

$$g := \begin{pmatrix} p & 0 \\ 0 & 1 \end{pmatrix}$$

followed by restriction. The map α is equivariant for the T_r's that we consider. We have the following variant of a lemma of Ihara and Ribet (see [R], and also [CDT, 6.3.1]).

Lemma 1 *The map* $\alpha: H^1(\Gamma_1(N), \mathbf{F}_p)^2 \to H^1(\Gamma_1(N) \cap \Gamma_0(p), \mathbf{F}_p)$ *is injective.*

Proof. Let Δ be the subgroup of $SL_2(\mathbf{Z}[1/p])$ of elements congruent to $\left(\begin{smallmatrix} 1 & * \\ 0 & 1 \end{smallmatrix}\right)$ modulo N. The arguments of [S2, II, §1.4] show that Δ is the amalgam of $\Gamma_1(N)$ and $g\Gamma_1(N)g^{-1}$ along their intersection $\Gamma_1(N) \cap \Gamma_0(p)$. The universal property of amalgams then implies that the kernel of α is $H^1(\Delta, \mathbf{F}_p)$ i.e., $Hom(\Delta, \mathbf{F}_p)$. By [S1], each subgroup of finite index of $SL_2(\mathbf{Z}[1/p])$ is a congruence subgroup, hence each morphism from Δ to $\mathbf{F}_p$ factors through the image Δ_n of Δ in some $SL_2(\mathbf{Z}/n\mathbf{Z})$ with n prime to p. The result follows, as p is at least 5 and does not divide N. (We use that $SL_2(\mathbf{Z})$ maps surjectively to $SL_2(\mathbf{Z}/n\mathbf{Z})$.)

By Shapiro's lemma we see that $H^1(\Gamma_1(N) \cap \Gamma_0(p), \mathbf{F}_p)$ is isomorphic (as a Hecke module) to $H^1(\Gamma_1(N), \mathbf{F}_p[\mathbf{P}^1(\mathbf{F}_p)])$. Using an easy computation of Brauer characters we deduce that the semisimplification of $\mathbf{F}_p[\mathbf{P}^1(\mathbf{F}_p)]$ under the natural action of $\Gamma_1(N)$ (that factors through $\Gamma_1(N)/\Gamma_1(N) \cap \Gamma(p)$) is $\mathsf{id} \oplus \mathsf{Symm}^{p-1}(\mathbf{F}_p^2)$. In fact as the cardinality of $\mathbf{P}^1(\mathbf{F}_p)$ is prime to p we deduce that this is indeed true even before semisimplification, i.e., $\mathbf{F}_p[\mathbf{P}^1(\mathbf{F}_p)]$ is semisimple as a $SL_2(\mathbf{F}_p)$-module. The submodule id is identified with the constant functions, with complement the functions with zero average. Thus we identify $H^1(\Gamma_1(N)\cap\Gamma_0(p),\mathbf{F}_p)$ with $H^1(\Gamma_1(N),\mathbf{F}_p)\oplus H^1(\Gamma_1(N),\mathsf{Symm}^{p-1}(\mathbf{F}_p^2))$. The degeneracy map α takes the form:

$$H^1(\Gamma_1(N),\mathbf{F}_p)^2 \to H^1(\Gamma_1(N),\mathbf{F}_p)\oplus H^1(\Gamma_1(N),\mathsf{Symm}^{p-1}(\mathbf{F}_p^2)).$$

Lemma 2 *The map:*

$$\beta\colon H^1(\Gamma_1(N),\mathbf{F}_p) \longrightarrow H^1(\Gamma_1(N),\mathsf{Symm}^{p-1}(\mathbf{F}_p^2)),$$

that is the composition of α_2 with the projection of $H^1(\Gamma_1(N)\cap\Gamma_0(p),\mathbf{F}_p)$ to $H^1(\Gamma_1(N),\mathsf{Symm}^{p-1}(\mathbf{F}_p^2))$, is injective.

Proof. This is an immediate consequence of Lemma 1, and the fact that $\alpha_1\colon H^1(\Gamma_1(N),\mathbf{F}_p)\to H^1(\Gamma_1(N)\cap\Gamma_0(p),\mathbf{F}_p)$ has image exactly the first summand of $H^1(\Gamma_1(N),\mathbf{F}_p)\oplus H^1(\Gamma_1(N),\mathsf{Symm}^{p-1}(\mathbf{F}_p^2))$.

In view of the Eichler-Shimura isomorphism (see [DI, §12]), we have a new proof by a purely group cohomological method of the following well-known result.

Corollary 1 *Suppose moreover that $N\geq 5$. A semi-simple representation $\rho\colon G_{\mathbf{Q}}\to GL_2(\overline{\mathbf{F}_p})$ that arises from $S_2(\Gamma_1(N),\overline{\mathbf{Q}_p})$ also arises from $S_{p+1}(\Gamma_1(N),\overline{\mathbf{Q}_p})$.*

Proof. This follows from the above lemma, together with the following facts.

1. The degeneracy map β is Hecke equivariant for the T_r's that we consider, and it sends $H^1_{\mathrm{par}}(\Gamma_1(N),\mathbf{F}_p)$ to $H^1_{\mathrm{par}}(\Gamma_1(N),\mathsf{Symm}^{p-1}(\mathbf{F}_p^2))$. The subscript "par" denotes parabolic cohomology, i.e., the intersection of the kernels of the restriction maps to the cohomology of the unipotent subgroups of $\Gamma_1(N)$.

2. For V any $\Gamma_1(N)$-module that is free of finite rank over $\mathbf{Z}$, and such that $H^0(\Gamma_1(N),\mathbf{F}_p\otimes V^\vee)=0$, the map:

$$H^1_{\mathrm{par}}(\Gamma_1(N),V)\to H^1_{\mathrm{par}}(\Gamma_1(N),\mathbf{F}_p\otimes V)$$

is surjective (one uses that $H^1_{\mathrm{par}}(\Gamma_1(N),\mathbf{F}_p\otimes V)$ is a quotient of $H^1_c(Y_1(N),\mathbf{F}_p\otimes\mathcal{F}_V)$, with $\mathcal{F}_V$ the sheaf given corresponding to V, and that $H^2_c(Y_1(N),\mathbf{F}_p\otimes\mathcal{F}_V)=0$ by Poincaré duality).

Remarks.

1. One can ask the converse question as to which maximal ideals m of the Hecke algebra acting on $H^1(\Gamma_1(N),\mathsf{Symm}^{p-1}(\mathbf{F}_p^2))$ are pull backs of maximal ideals of the Hecke algebra acting on $H^1(\Gamma_1(N),\mathbf{F}_p)$. Then, for non-Eisenstein m, the answer is in terms of the Galois representation ρ_{m}: a necessary and sufficient condition is that ρ_{m} be finite flat at p (see [R1, Thm. 3.1]). Perhaps one does not expect to have a group cohomological approach to such a subtle phenomenon.

2. Let ρ be an irreducible 2-dimensional mod p representation of $G_{\mathbf{Q}}$. We just saw that if ρ arises from $S_2(\Gamma_1(N))$, then it also does so from $S_{p+1}(\Gamma_1(N))$, using only group cohomology. See [He] for the case of higher weights, when the raising of weights by $p-1$ is explained more directly in terms of the Jordan-Hölder series of the $\mathsf{Symm}^n(\mathbf{F}_p^2)$. On the other hand the method here being cohomological cannot be used to raise weights from 1 to p, while multiplication by the Hasse invariant can be used for this (see [DS]).

3 Imaginary quadratic fields

Let $p \geq 5$ be a prime number, K an imaginary quadratic field in which p is inert, and $\mathcal{N}$ a non-zero ideal in the ring of integers $\mathcal{O}_K$ not containing p.
Let $\Gamma_1(\mathcal{N})$ be the congruence subgroup of $SL_2(\mathcal{O}_K)$ of level $\mathcal{N}$. As before we have the degeneracy map:

$$\alpha\colon H^1(\Gamma_1(\mathcal{N}), \mathbf{F}_p)^2 \to H^1(\Gamma_1(\mathcal{N}) \cap \Gamma_0(p), \mathbf{F}_p)$$

that is defined to be the sum $\alpha_1 \oplus \alpha_2$ where α_1 is the restriction map, and α_2 the "twisted" restriction map, given by "conjugation" by

$$g := \begin{pmatrix} p & 0 \\ 0 & 1 \end{pmatrix}$$

followed by restriction. Then we again have:

Lemma 3 *The map: $\alpha\colon H^1(\Gamma_1(\mathcal{N}), \mathbf{F}_p)^2 \to H^1(\Gamma_1(\mathcal{N}) \cap \Gamma_0(p), \mathbf{F}_p)$ is injective.*

Proof. One replaces $\mathbf{Z}$ by $\mathcal{O}_K$ and N by $\mathcal{N}$ in the proof of Lemma 1. Strong approximation (see [PR]) guarantees that the reduction map from $SL_2(\mathcal{O}_K)$ to $SL_2(\mathcal{O}_K/n\mathcal{O}_K)$ is surjective for all $n \geq 1$.

By Shapiro's lemma we see that $H^1(\Gamma_1(\mathcal{N}) \cap \Gamma_0(p), \mathbf{F}_p)$ is isomorphic to $H^1(\Gamma_1(\mathcal{N}), \mathbf{F}_p[\mathbf{P}^1(\mathbf{F}_\wp)])$, where $\mathbf{F}_\wp = \mathcal{O}_K/\wp$. Using an easy computation of Brauer characters we deduce that the semisimplification of $\mathbf{F}_\wp[\mathbf{P}^1(\mathbf{F}_\wp)]$ under the natural action of $\Gamma_1(\mathcal{N})$ (that factors through $\Gamma_1(\mathcal{N})/\Gamma_1(\mathcal{N}) \cap \Gamma(\wp)$) is $\mathsf{id} \oplus \mathsf{Symm}^{p-1}(\mathbf{F}_\wp^2) \otimes \mathsf{Symm}^{p-1}(\mathbf{F}_\wp^2)^\sigma$, with σ the non-trivial automorphism of $\mathbf{F}_\wp$, and the superscript denotes that the action has been twisted by σ. Note that $\mathsf{Symm}^{p-1}(\mathbf{F}_\wp^2) \otimes \mathsf{Symm}^{p-1}(\mathbf{F}_\wp^2)^\sigma$ is irreducible as a $SL_2(\mathbf{F}_\wp)$-module: this is a particular case of the well-known tensor product theorem of Steinberg (see [St]). In fact as the cardinality of $\mathbf{P}^1(\mathbf{F}_\wp)$ is prime to p we deduce as before that this is indeed true even before semisimplification, i.e., $\mathbf{F}_p[\mathbf{P}^1(\mathbf{F}_\wp)]$ is semisimple as a $SL_2(\mathbf{F}_\wp)$-module.
Thus α maps $H^1(\Gamma_1(\mathcal{N}), \mathbf{F}_\wp)^2$ into the direct sum of $H^1(\Gamma_1(\mathcal{N}), \mathbf{F}_\wp)$ and $H^1(\Gamma_1(\mathcal{N}), \mathsf{Symm}^{p-1}(\mathbf{F}_\wp^2) \otimes \mathsf{Symm}^{p-1}(\mathbf{F}_\wp^2)^\sigma)$, and composing with the projection to the second term gives a map:

$$\beta\colon H^1(\Gamma_1(\mathcal{N}), \mathbf{F}_\wp) \to H^1(\Gamma_1(\mathcal{N}), \mathsf{Symm}^{p-1}(\mathbf{F}_\wp^2) \otimes \mathsf{Symm}^{p-1}(\mathbf{F}_\wp^2)^\sigma)$$

Lemma 4 *The map β is injective.*

Proof. After the above discussion, this is an immediate consequence of Lemma 3 as before.

The map β is equivariant for the the action of all Hecke operators outside p (i.e., induced by elements of $SL_2(\mathbf{Q}_l \otimes K)$ for $l \neq p$). Thus we have proved:

Corollary 2 *Each system of Hecke eigenvalues in $\overline{\mathbf{F}_p}$ that arises from $H^1(\Gamma_1(\mathcal{N}), \mathbf{F}_p)$ also arises from $H^1(\Gamma_1(\mathcal{N}), \mathsf{Symm}^{p-1}(\mathbf{F}_\wp^2) \otimes \mathsf{Symm}^{p-1}(\mathbf{F}_\wp^2)^\sigma)$.*

Remark. This result as it stands does not yield any information about congruences of systems of Hecke eigenvalues occurring in characteristic zero, as there is a problem with lifting. More precisely, the obstruction is in the p-torsion of $H^2(\Gamma_1(\mathcal{N}), \mathsf{Symm}^{p-1}(\mathcal{O}^2) \otimes \mathsf{Symm}^{p-1}(\mathcal{O}^2)^\sigma)$.

4 Totally real fields

The method is also applicable at inert primes $p \geq 5$ in the case of Hilbert modular forms over cyclic totally real fields of even degree. We quickly sketch the approach which is similar to that of the previous two sections. Let $F/\mathbf{Q}$ be a totally real, cyclic extension of even degree, $\mathrm{Gal}(F/\mathbf{Q}) = \langle \sigma \rangle$, $p \geq 5$ an inert prime, with $\wp$ the unique prime of F above it, $\mathbf{F}_\wp$ the residue field at $\wp$, and $\mathcal{N}$ an ideal of the ring of integers of F that is prime to p.
Consider the quaternion algebra D over F ramified at all infinite places and unramified at all finite places, and for any F-algebra R, set $B(R) = (D \otimes_F R)^*$. Let $\mathbf{A}$ be the adeles of F, and $U_1(\mathcal{N})$ the standard open compact (mod centre) subgroup of $B(\mathbf{A})$. The space of mod p weight 2 modular forms $S(\mathcal{N})$ (resp., $S(\mathcal{N}, \wp)$) for $U_1(\mathcal{N})$ (resp., $U_1(\mathcal{N}) \cap U_0(\wp)$) in this case consists of functions $B(\mathbf{A}) \to \mathbf{F}_p$ that are left and right invariant under $B(F)$ and $U_1(\mathcal{N})$ (resp., $U_1(\mathcal{N}) \cap U_0(\wp)$) respectively, modulo the space of functions that factor through the norm. These spaces come equipped with Hecke actions. This time controlling the kernel of the degeneracy map $S(\mathcal{N})^2 \to S(\mathcal{N}, \wp)$, i.e., analog of Lemmas 1 and 3, is easier and follows from strong approximation. Note again that the representation $\mathsf{Symm}^{p-1}(\mathbf{F}_\wp^2) \otimes \mathsf{Symm}^{p-1}(\mathbf{F}_\wp^2)^\sigma \otimes \cdots \otimes \mathsf{Symm}^{p-1}(\mathbf{F}_\wp^2)^{\sigma^{[F:\mathbf{Q}]-1}}$ of $GL_2(\mathbf{F}_\wp)$ (which again is a direct summand, with complement the trivial representation, of the induction of the trivial representation from the Borel subgroup of $GL_2(\mathbf{F}_\wp)$ to $GL_2(\mathbf{F}_\wp)$) is irreducible as a consequence of Steinberg's tensor product theorem.
Now following the method of the previous section, and invoking the Jacquet-Langlands correspondence yields the following result.

Proposition 1 *With notation as above, suppose that an irreducible representation $\rho: G_F \to GL_2(\overline{\mathbf{F}_p})$ arises from a Hilbert modular form on $\Gamma_1(\mathcal{N})$ of weight $(2, \ldots, 2)$. Then it also arises from a a Hilbert modular form on $\Gamma_1(\mathcal{N})$ of weight $(p+1, \ldots, p+1)$.*

Remarks: It will be of interest to work out some of the Hasse invariants with non-parallel weights obtained by E. Goren from the viewpoint of this paper (see [G]).

5 Congruences between forms of level N and Np for weights ≥ 2

We take this opportunity to write down a level raising criterion from level N to level Np for all weights that is easily deduced from Corollary 9 of [K], and list some errata to [K].

Proposition 2 *Let f be a newform in $S_k(\Gamma_1(N))$ for an integer $k \geq 2$, such that the mod $\wp$ representation corresponding to it is irreducible. Then:*

- *If $k = 2$, f is congruent to a p-new form in $S_k(\Gamma_1(N) \cap \Gamma_0(p))$ if and only if $a_p(f)^2 = \varepsilon_f(p)$ mod $\wp$ where ε_f is the nebentypus of f.*
- *If $2 < k \leq p+1$, f is congruent to a p-new form in $S_k(\Gamma_1(N) \cap \Gamma_0(p))$ if and only if $a_p(f)$ is 0 mod $\wp$.*
- *If $k > p+1$, f is always congruent to a p-new form in $S_k(\Gamma_1(N) \cap \Gamma_0(p))$.*

Errata to [K]

One of us (C.K.) would like to point out some typos in [K]:

1. Lines 12 and 19 of Definition 10 page 143 of [K] replace
$$f\colon D(\mathbf{Q})\backslash D(\mathbf{A}^\infty)/V \longrightarrow Hom_{\mathcal{O}}(M, Symm^{k-2}(\mathcal{O})).$$
by
$$f\colon D(\mathbf{Q})\backslash D(\mathbf{A}^\infty) \longrightarrow Hom_{\mathcal{O}}(M, Symm^{k-2}(\mathcal{O})).$$
2. On line 12, page 146 of [K] replace $V_1(N)^p$ by $V_1(N)^p \times V_p$.

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