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More on Generalized Homeomorphisms in Topological Spaces

Más Sobre Homeomorfismos Generalizados en Espacios Topológicos

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Abstract

The aim of this paper is to continue the study of generalized homeomorphisms. For this we define three new classes of maps, namely generalized Λ_s-open, generalized Λ_s^c-homeomorphisms and generalized Λ_s^I-homeomorphisms, by using $g.\Lambda_s$-sets, which are generalizations of semi-open maps and generalizations of homeomorphisms.
Key words and phrases: Topological spaces, semi-closed sets, semi-open sets, semi-homeomorphisms, semi-closed maps, irresolute maps.

Resumen

El objetivo de este trabajo es continuar el estudio de los homeomorfismos generalizados. Para esto definimos tres nuevas clases de aplicaciones, denominadas Λ_s-abiertas generalizadas, Λ_s^c-homeomofismos generalizados y Λ_s^I-homeomorfismos generalizados, haciendo uso de los conjuntos $g.\Lambda_s$, los cuales son generalizaciones de las funciones semi-abiertas y generalizaciones de homeomorfismos.
Palabras y frases clave: Espacios topológicos, conjuntos semi-cerrados, conjuntos semi-abiertos, semi-homeomorfismos, aplicaciones semi-cerradas, aplicaciones irresolutas.

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1 Introduction

Recently in 1998, as an analogy of Maki [10], Caldas and Dontchev [5] introduced the Λ_s-sets (resp. V_s-sets) which are intersections of semi-open (resp. union of semi-closed) sets. In this paper we shall introduce three classes of maps called generalized Λ_s-open, generalized Λ_s^c-homeomorphisms and generalized Λ_s^I-homeomorphisms, which are generalizations of semi-open maps, generalizations of homeomorphisms, semi-homeomorphisms due to Biswas [2] and semi-homeomorphisms due to Crossley and Hildebrand [7] and we investigate some properties of generalized Λ_s^c-homeomorphisms and generalized Λ_s^I-homeomorphisms from the quotient space to other spaces.

Throughout this paper we adopt the notations and terminology of [10], [5] and [6] and the following conventions: (X,τ), (Y,σ) and (Z,γ) (or simply X, Y and Z) will always denote topological spaces on which no separation axioms are assumed, unless explicitly stated.

2 Preliminaries

A subset A of a topological space (X,τ) is said to be *semi-open* [9] if for some open set O, $O \subseteq A \subseteq Cl(O)$, where $Cl(O)$ denotes the closure of O in (X,τ). The complement A^c or $X - A$ of a semi-open set A is called *semi-closed* [3]. The family of all semi-open (resp. semi-closed) sets in (X,τ) is denoted by $SO(X,\tau)$ (resp. $SC(X,\tau)$). The intersection of all semi-closed sets containing A is called the semi-closure of A [3] and is denoted by $sCl(A)$. A map $f : (X,\tau) \to (Y,\sigma)$ is said to be *semi-continuous* [9] (resp. *irresolute* [7]) if for every $A\epsilon\sigma$ (resp. $A\epsilon SO(Y,\sigma)$), $f^{-1}(A)\epsilon SO(X,\tau)$; equivalently, f is semi-continuous (resp. irresolute) if and only if, for every closed set A (resp. semi-closed set A) of (Y,σ), $f^{-1}(A)\epsilon SC(X,\tau)$. f is *pre-semi-closed* [7] (resp. *pre-semi-open* [1], resp. *semi-open* [11]) if $f(A)\epsilon\ SC(Y,\sigma)$ (resp. $f(A)\epsilon\ SO(Y,\sigma)$) for every $A\epsilon SC(X,\tau)$ (resp. $A\epsilon SO(X,\tau)$, resp. $A\epsilon\tau$). f is a *semi-homeomorphism (B)* [2] if f is bijective, continuous and semi-open. f is a *semi-homeomorphism (C.H)* [7] if f is bijective, irresolute and pre-semi-open.

Before entering into our work we recall the following definitions and propositions, due to Caldas and Dontchev [5].

Definition 1. Let B be a subset of a topological space (X,τ). B is called a Λ_s-*set* (resp. V_s-*set*) [5], if $B = B^{\Lambda_s}$ (resp. $B = B^{V_s}$), where $B^{\Lambda_s} = \bigcap\{O : O \supseteq B,\ O\epsilon SO(X,\tau)\}$ and $B^{V_s} = \bigcup\{F : F \subseteq B,\ F^c\epsilon SO(X,\tau)\}$.

Definition 2. In a topological space (X,τ), a subset B is called

(i) *generalized* Λ_s*-set* (written as $g.\Lambda_s$-set) of (X, τ) [5], if $B^{\Lambda_s} \subseteq F$ whenever $B \subseteq F$ and $F \epsilon SC(X, \tau)$.

(ii) *generalized* V_s*-set* (written as $g.V_s$-set) of (X, τ) [5], if B^c is a $g.\Lambda_s$-set of (X, τ).

Remark 2.1. From Definitions 1, 2 and [5] (Propositions 2.1, 2.2), we have the following implications, none of which is reversible:

Open sets →Semi-open sets→ Λ_s-sets→ $g.\Lambda_s$-sets , and
Closed sets→ Semi-closed sets→ V_s-sets→ $g.V_s$-sets

Definition 3. (i) A map $f : (X, \tau) \to (Y, \sigma)$ is called *generalized* Λ_s*-continuous* (written as $g.\Lambda_s$-continuous) [6] if $f^{-1}(A)$ is a $g.\Lambda_s$-set in (X, τ) for every open set A of (Y, σ).

(ii) A map $f : (X, \tau) \to (Y, \sigma)$ is called *generalized* Λ_s*-irresolute* (written as $g.\Lambda_s$-irresolute) [6] if $f^{-1}(A)$ is a $g.\Lambda_s$-set in (X, τ) for every $g.\Lambda_s$-set of (Y, σ).

(iii) A map $f : (X, \tau) \to (Y, \sigma)$ is called *generalized* V_s*-closed* (written as $g.V_s$-closed) [6] if for each closed set F of X, $f(F)$ is a $g.V_s$-set.

3 G.Λ_s-open maps and g.Λ_s-homeomorphisms

In this section we introduce the concepts of generalized Λ_s-open maps, generalized Λ_s^c-homeomorphisms and generalized Λ_s^I-homeomorphisms and we study some of their properties.

Definition 4. A map $f : (X, \tau) \to (Y, \sigma)$ is called *generalized* Λ_s*-open* (written as $g.\Lambda_s$-open) if for each open set A of X, $f(A)$ is a $g.\Lambda_s$-set.

Obviously every semi-open map is $g.\Lambda_s$-open. The converse is not always true, as the following example shows.

Example 3.1. Let $X = \{a, b, c\}$, $Y = \{a, b, c, d\}$, $\tau = \{\emptyset, \{b, c\}, X\}$ and $\sigma = \{\emptyset, \{c, d\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be a map defined by $f(a) = a$, $f(b) = b$ and $f(c) = d$. Then, for X which is open in (X, τ), $f(X) = \{a, b, d\}$ is not a semi-open set of Y. Hence f is not a semi-open map. However, f is a $g.\Lambda_s$-open map.

We consider now some composition properties in terms of $g.\Lambda_s$-sets.

Theorem 3.2. *Let* $f : (X, \tau) \to (Y, \sigma)$, $g : (Y, \sigma) \to (Z, \gamma)$ *be two maps such that* $g \circ f : (X, \tau) \to (Z, \gamma)$ *is a* $g.\Lambda_s$*-open map. Then* *(i) g is $g.\Lambda_s$-open, if f is continuous and surjective.*

(ii) f is $g.\Lambda_s$-open, if g is irresolute, pre-semi-closed and bijective.

Proof. (i) Let A be an open set in Y. Since $f^{-1}(A)$ is open in X, $(g\circ f)(f^{-1}(A))$ is a $g.\Lambda_s$-set in Z and hence $g(A)$ is $g.\Lambda_s$-set in Z. This implies that g is a $g.\Lambda_s$-open map.
(ii) Let A be an open set in X. Then $(g\circ f)(A)$ is a $g.\Lambda_s$-set in (Z,γ). Since g is irresolute, pre-semi-closed and bijective, $g^{-1}(g\circ f)(A)$ is a $g.\Lambda_s$-set in (Y,σ). Really, suppose that $(g\circ f)(A)=B$ and $g^{-1}(B)\subseteq F$ where F is semi-closed in (Y,σ). Therefore $B\subseteq g(F)$ holds and $g(F)$ is semi-closed, because g is pre-semi-closed. Since B is $(g\circ f)(A)$, $B^{\Lambda_s}\subseteq g(F)$ and $g^{-1}(B^{\Lambda_s})\subseteq F$. Hence, since g is irresolute, we have $(g^{-1}(B))^{\Lambda_s}\subseteq g^{-1}(B^{\Lambda_s})\subseteq F$. Thus $g^{-1}(B)=g^{-1}(g\circ f)(A)$ is a $g.\Lambda_s$-set in (Y,σ). Since g is injective, $f(A)=g^{-1}(g\circ f)(A)$ is $g.\Lambda_s$-set in Y. Therefore f is $g.\Lambda_s$-open. □

Remark 3.3. A bijection $f:(X,\tau)\to(Y,\sigma)$ is pre-semi-open if and only if f is pre-semi-closed.

Theorem 3.4. *(i) If $f:(X,\tau)\to(Y,\sigma)$ is a $g.\Lambda_s$-open map and $g:(Y,\sigma)\to(Z,\gamma)$ is bijective, irresolute and pre-semi-closed, then $g\circ f:(X,\tau)\to(Z,\gamma)$ is a $g.\Lambda_s$-open map.*
(ii) If $f:(X,\tau)\to(Y,\sigma)$ is an open map and $g:(Y,\sigma)\to(Z,\gamma)$ is a $g.\Lambda_s$-open map, then $g\circ f:(X,\tau)\to(Z,\gamma)$ is a $g.\Lambda_s$-open map.

Proof. (i) Let A be an arbitrary open set in (X,τ). Then $f(A)$ is a $g.\Lambda_s$-set in (Y,σ) because f is $g.\Lambda_s$-open. Since g is bijective, irresolute and pre-semi-closed $(g\circ f)(A)=g(f(A))$ is $g.\Lambda_s$-open. Really. Let $g(f(A)\subseteq F$ where F is any semi-closed set in (Z,γ). Then $f(A)\subseteq g^{-1}(F)$ holds and $g^{-1}(F)$ is semi-closed because g is irresolute. Since g is pre-semi-open (Remark 3.3) $(g(f(A)))^{\Lambda_s}\subseteq g((f(A))^{\Lambda_s})\subseteq F$. Hence $g(f(A))$ is $g.\Lambda_s$-set in (Z,γ). Thus $g\circ f$ is $g.\Lambda_s$-open.

(ii) The proof follows immediately from the definitions. □

Definition 5. A bijection $f:(X,\tau)\to(Y,\sigma)$ is called a *generalized Λ_s^c-homeomorphism* (written $g.\Lambda_s^c$-homeomorphism) if f is both $g.\Lambda_s$-continuous and $g.\Lambda_s$-open.

In order to obtain an alternative description of the $g.\Lambda_s^c$-homeomorphisms, we first prove the following three theorems which are in [6].

Theorem 3.5. *Let $f:(X,\tau)\to(Y,\sigma)$ be $g.\Lambda_s$-irresolute. Then f is $g.\Lambda_s$-continuous, but not conversely.*

Proof. Since every open set is semi-open and every semi-open set is $g.\Lambda_s$-set (Remark 2.1) it is proved that f is $g.\Lambda_s$-continuous. □

The converse needs not be true, as seen from the following example.

Example 3.6. Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$ and $\sigma = \{\emptyset, \{a, b\}, Y\}$. The identity map $f : (X, \tau) \to (Y, \sigma)$ is $g.\Lambda_s$-continuous but it is not $g.\Lambda_s$-irresolute, since for the $g.\Lambda_s$-set $\{b, c\}$ of (Y, σ) the inverse image $f^{-1}(\{b, c\}) = \{b, c\}$ is not a $g.\Lambda_s$-set of (X, τ).

Theorem 3.7. *A map $f : (X, \tau) \to (Y, \sigma)$ is $g.\Lambda_s$-irresolute (resp. $g.\Lambda_s$-continuous) if and only if, for every $g.V_s$-set A (resp. closed set A) of (Y, σ) the inverse image $f^{-1}(A)$ is a $g.V_s$-set of (X, τ).*

Proof. Necessity: If $f : (X, \tau) \to (Y, \sigma)$ is $g.\Lambda_s$-irresolute, then every $g.\Lambda_s$-set B of (Y, σ), $f^{-1}(B)$ is $g.\Lambda_s$-set in (X, τ). If A is any $g.V_s$-set of (Y, σ), then A^c is a $g.\Lambda_s$-set (Definition 2(ii)). Thus $f^{-1}(A^c)$ is a $g.\Lambda_s$-set, but $f^{-1}(A^c) = (f^{-1}(A))^c$ so that $f^{-1}(A)$ is a $g.V_s$-set.
Sufficiency: If, for all $g.V_s$-set A of $(Y, \sigma) f^{-1}(A)$ is a $g.V_s$-set in (X, τ), then if B is any $g.\Lambda_s$-set of (Y, σ) then B^c is a $g.V_s$-set. Also $f^{-1}(B^c) = (f^{-1}(B))^c$ is a $g.V_s$-set. Thus $f^{-1}(B)$ is a $g.\Lambda_s$-set.

In a similar way we prove the case $g.\Lambda_s$-continuous. □

Theorem 3.8. *If a map $f : (X, \tau) \to (Y, \sigma)$ is bijective irresolute and pre-semi-closed, then*

(i) for every $g.\Lambda_s$-set B of (Y, σ), $f^{-1}(B)$ is a $g.\Lambda_s$-set of (X, τ) (i.e., f is $g.\Lambda_s$-irresolute).

(ii) for every $g.\Lambda_s$-set A of (X, τ), $f(A)$ is a $g.\Lambda_s$-set of (Y, σ) (i.e., f is $g.\Lambda_s$-preopen).

Proof. (i) Let B be a $g.\Lambda_s$-set of (Y, σ). Suppose that $f^{-1}(B) \subseteq F$ where F is semi-closed in (X, τ). Therefore $B \subseteq f(F)$ holds and $f(F)$ is semi-closed, because f is pre-semi-closed. Since B is a $g.\Lambda_s$-set, $B^{\Lambda_s} \subseteq f(F)$, and hence $f^{-1}(B^{\Lambda_s}) \subseteq F$. Therefore we have $(f^{-1}(B))^{\Lambda_s} \subseteq f^{-1}(B^{\Lambda_s}) \subseteq F$. Hence $f^{-1}(B)$ is a $g.\Lambda_s$-set in (X, τ).
(ii) Let A be a $g.\Lambda_s$-set of (X, τ). Let $f(A) \subseteq F$ where F is any semi-closed set in (Y, σ). Then $A \subseteq f^{-1}(F)$ holds and $f^{-1}(F)$ is semi-closed because f is irresolute. Since f is pre-semi-open , $(f(A))^{\Lambda_s} \subseteq f(A^{\Lambda_s}) \subseteq F$. Hence $f(A)$ is a $g.\Lambda_s$-set in (Y, σ). □

Corollary 3.9. *If a map $f : (X,\tau) \to (Y,\sigma)$ is bijective, irresolute and pre-semi-closed, then:*
(i) for every $g.V_s$-set B of (Y,σ), $f^{-1}(B)$ is a $g.V_s$-set of (Y,σ), and
(ii) for every $g.V_s$-set A of (X,τ), $f(A)$ is a $g.V_s$-set of (Y,σ).

Proposition 3.10. *Every semi-homeomorphism (B) and semi-homeomorphism (C.H) is a $g.\Lambda_s^c$-homeomorphism.*

Proof. It is proved from the definitions and Theorem 3.8. □

The converse of Proposition 3.10 is not true as seen from the following examples.

Example 3.11.
$g.\Lambda_s^c$-homeomorphisms need not be semi-homeomorphisms (B).
Let $X = Y = \{a,b,c\}$, $\tau = \{\emptyset, \{a\}, X\}$ and $\sigma = \{\emptyset, \{b\}, \{a,b\}, Y\}$. Then the $g.\Lambda_s$-sets of (X,τ) are $\emptyset$, X, $\{a\}$, $\{a,b\}$, $\{a,c\}$ and the $g.\Lambda_s$-sets of (Y,σ) are $\emptyset$, Y, $\{b\}$, $\{a,b\}$, $\{b,c\}$. Let f be a map from (X,τ) to (Y,σ) defined by $f(a) = b$, $f(b) = a$ and $f(c) = c$. Here f is a $g.\Lambda_s^c$-homeomorphism from (X,τ) to (Y,σ). However f is not a semi-homeomorphism (B), since f is not continuous.

Example 3.12.
$g.\Lambda_s^c$-homeomorphisms need not be semi-homeomorphisms (C.H).
Let $X = \{a,b,c\}$ and $\tau = \{\emptyset, \{a\}, \{b,c\}, X\}$. Let $f : (X,\tau) \to (X,\tau)$ be a bijection defined by $f(a) = b$, $f(b) = a$ and $f(c) = c$. Since f is not irresolute f is not a semi-homeomorphism (C.H). However, f is a $g.\Lambda_s^c$-homeomorphism.

We characterize $g.\Lambda_s^c$-homeomorphism and $g.\Lambda_s$-open maps. The proofs are obvious and hence omitted.

Proposition 3.13. *For any bijection $f : (X,\tau) \to (Y,\sigma)$ the following statements are equivalent.*
(i) Its inverse map $f^{-1} : (Y,\sigma) \to (X,\tau)$ is $g.\Lambda_s$-continuous.
(ii) f is $g.\Lambda_s$-open.
(iii) f is $g.V_s$-closed.

Proposition 3.14. *Let $f : (X,\tau) \to (Y,\sigma)$ be a bijective and $g.\Lambda_s$-continuous map. Then the following statements are equivalent.*
(i) f is a $g.\Lambda_s$-open map.
(ii) f is a $g.\Lambda_s^c$-homeomorphism.
(iii) f is a $g.V_s$-closed map.

Now we introduce a class of maps which are included in the class of $g.\Lambda_s^c$-homeomorphisms and includes the class de homeomorphisms. Moreover, this class of maps is closed under the composition of maps.

Definition 6. A bijection $f : (X, \tau) \to (Y, \sigma)$ is said to be a *generalized* Λ_s^I-*homeomorphism* (written $g.\Lambda_s^I$-homeomorphism) if both f and f^{-1} preserve $g.\Lambda_s$-sets, i.e., if both f and f^{-1} are $g.\Lambda_s$-irresolute. We say that two spaces (X, τ) and (Y, σ) are $g.\Lambda_s^I$-*homeomorphic* if there exists a Λ_s^I-homeomorphism from (X, τ) in (Y, σ).

Remark 3.15. Every semi-homeomorphism (C.H) is a $g.\Lambda_s^I$-homeomorphism by (Theorem 3.8). Every $g.\Lambda_s^I$-homeomorphism is a $g.\Lambda_s^c$-homeomorphism. The converses are not true from the following examples.

Example 3.16.
$g.\Lambda_s^I$-homeomorphisms need not be semi-homeomorphisms (C.H).
Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a\}, \{b, c\}, X\}$. Then the $g.\Lambda_s$-sets of (X, τ) are $\emptyset$, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{a, c\}$, $\{b, c\}$ and X. Let $f : (X, \tau) \to (X, \tau)$ be a map defined by $f(a) = b$, $f(b) = a$, $f(c) = c$. Here f is a $g.\Lambda_s^I$-homeomorphism. However f is not a semi-homeomorphism (CH), since it is not irresolute.

Example 3.17.
$g.\Lambda_s^c$-homeomorphisms need not be $g.\Lambda_s^I$-homeomorphisms.
Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$ and $\sigma = \{\emptyset, \{a, b\}, Y\}$. The identity map $f : (X, \tau) \to (Y, \sigma)$ is not a $g.\Lambda_s^I$-homeomorphism since for the $g.\Lambda_s$-set $\{b, c\}$ of (Y, σ), the inverse image $f^{-1}(\{b, c\}) = \{b, c\}$ is not a $g.\Lambda_s$-set of (X, τ), i.e., f is not $g.\Lambda_s$-irresolute (and so it is not a semi-homeomorphism (C.H)). However f is a $g.\Lambda_s^c$-homeomorphism.

Remark 3.18. From the propositions, examples and remarks above, we have the following diagram of implications.

		semi-homeom. (B)	$\overset{\rightarrow}{\not\leftarrow}$	$g.\Lambda_s^c$-homeom.
homeomorphism	$\nearrow\not\swarrow$	$\not\uparrow\ \not\downarrow$	$\nearrow\not\swarrow$	$\not\downarrow\ \uparrow$
	$\searrow\not\nwarrow$	semi-homeom. (C.H)	$\overset{\not\leftarrow}{\rightarrow}$	$g.\Lambda_s^I$-homeom.

4 Additional Properties.

Definition 7. A subset B of a topological space (X, τ) is said to be $g.\Lambda_s$-*compact relative to* X, if for every cover $\{A_i : i \in \Omega\}$ of B by $g.\Lambda_s$-subsets of (X, τ), i.e., $B \subset \bigcup\{A_i : i \in \Omega\}$ where A_i $(i \in \Omega)$ are $g.\Lambda_s$-sets in (X, τ), there exists a finite subset Ω_o of Ω such that $B \subset \bigcup\{A_i : i \in \Omega_o\}$. If X is $g.\Lambda_s$-compact relative to X, (X,τ) is said to be a $g.\Lambda_s$-compact space.

Proposition 4.1. *Every $g.V_s$-set of a $g.\Lambda_s$-compact space (X,τ) is $g.\Lambda_s$-compact relative to X.*

Since the proof is similar to the sg-compactnees (see [4],Theorem 4.1), it is omitted.

Proposition 4.2. *Let $f : (X,\tau) \to (Y,\sigma)$ be a map and let B be a $g.\Lambda_s$-compact set relative to (X,τ). Then,*
(i) If f is $g.\Lambda_s$-continuous, then $f(B)$ is compact in (Y,σ).
(ii) If f is $g.\Lambda_s$-irresolute, then $f(B)$ is $g.\Lambda_s$-compact relative to Y.

Proof. (i) Let $\{U_i : i \in \Omega\}$ be any collection of open subsets of (Y,σ) such that $f(B) \subset \bigcup\{U_i : i \in \Omega\}$. Then $B \subset \bigcup\{f^{-1}(U_i) : i \in \Omega\}$ holds and there exists a finite subset Ω_o of Ω such that $B \subset \bigcup\{f^{-1}(U_i) : i \in \Omega_o\}$ which shows that $f(B)$ is compact in (Y,σ).
(ii) Analogous to (i). □

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