

## A Note on Bi-contra-continuous Maps

*Una Nota sobre Aplicaciones Bi-contra-continuas*

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### Abstract

The aim of this paper is to define and develop stronger forms of contra-continuous mappings and contra- $\alpha$ -continuous mappings that are known as bi-contra-continuous and bi-contra- $\alpha$ -continuous mappings.

**Key words and phrases:** Bi-Contra-Continuous, Bi-contra- $\alpha$ -Continuous, Bi-Contra-Strongly- $\alpha$ -Continuous.

### Resumen

El propósito de este artículo es definir y desarrollar formas más fuertes de aplicaciones contra-continuas y contra- $\alpha$ -continuas que son conocidas como aplicaciones bi-contra-continuas y bi-contra- $\alpha$ -continuas.

**Palabras y frases clave:** Bi-Contra-Continua, Bi-contra- $\alpha$ -Continua, Bi-Contra-Fuertemente- $\alpha$ -Continua.

## 1 Introduction

In the year 1963, Levine [11] introduced the concepts of semi-open sets and semi-continuity in a topological space and investigated some of their properties. Njastad [14] defined and studied the properties of  $\alpha$ -sets. The notion of continuity was generalized in 1958 by Ptak [16]. Since then, a number of weak, strong and almost continuous functions have been defined and studied. In the year 1996, Dontchev [4] introduced a concept known as contra-continuous. He defined  $f : (X, \tau) \rightarrow (Y, \sigma)$  to be a contra continuous map if  $f^{-1}(U)$  is closed in  $(X, \tau)$  for each open set  $U$  in  $(Y, \sigma)$ . The aim of this paper is to introduce the concept of bi- contra-continuity in terms of open sets and  $\alpha$ -open sets in a topological space and to generalize various properties in this context.

We hope these types of mappings (strongly contra or two way contra continuous) will be of much use in Biotechnology, where they need strong or two way contra effects. This mapping is obtained by fixing a contra mapping between a set of viruses to a set of antiviruses and another contra mapping between the negative viruses of the viruses set to the the positive viruses of the anti viruses set.

## 2 Preliminaries

Throughout this paper  $(X, \tau)$ ,  $(Y, \sigma)$  and  $(Z, \gamma)$  simply referred as  $X$ ,  $Y$  and  $Z$  denote topological spaces on which no separation axioms are assumed unless explicitly stated. Let  $A$  be a subset of a topological space  $X$ . We denote the closure of  $A$  and the interior of  $A$  with respect to  $\tau$  as  $cl(A)$  and  $int(A)$  respectively.

**Definition 2.1.** A subset  $A$  of  $X$  is called an  $\alpha$ -set [14] (resp. semi-open set [11] and pre-open set [13]) if  $A \subset int(cl(int(A)))$  (resp.  $A \subset cl(int(A))$  and  $A \subset int(cl(A))$ ).

We denote the family of  $\alpha$ -sets (resp. semi-open sets and pre-open sets) in  $X$  by  $\alpha O(X)$  (resp.  $SO(X)$  and  $PO(X)$ ).

**Definition 2.2.** A map  $f : X \rightarrow Y$  is called

- i)  $\alpha$ -continuous [15] (resp. semi-continuous [11] and pre-continuous [13]) if the inverse image of each open set in  $Y$  is an  $\alpha$ -open set (resp. semi-open set and pre-open set) in  $X$ .
- ii) contra-continuous [4] if the inverse image of an open set in  $Y$  is closed in  $X$ .
- iii)  $\alpha$ -open (resp. semi-open and pre-open) if the image of each open in  $X$  is

an open (resp. semi-open and pre-open) set in  $Y$ .

iv) strongly  $\alpha$ -open map [10] if the image of every  $\alpha$ -open set in  $X$  is an  $\alpha$ -open set in  $Y$ .

**Theorem 2.3.** *i) A subset  $A$  of a topological space  $(X, \tau)$  is an  $\alpha$ -open set if and only if  $A$  is semi-open and pre-open [17].*

*ii) A map  $f : X \rightarrow Y$  is  $\alpha$ -continuous if and only if it is both semi-continuous and pre-continuous.*

**Definition 2.4.** A map  $f : X \rightarrow Y$  is  $\alpha$ -irresolute [12], (resp. semi-irresolute [3], and pre-irresolute [17]) if the inverse image of every  $\alpha$ -open set (resp. semi-open set and pre-open set) in  $Y$  is an  $\alpha$ -open set (resp. semi-open set and pre-open set) in  $X$ .

### 3 Bi-Contra-Continuous maps

**Definition 3.1.** Let  $f : X \rightarrow Y$  be a surjective map. Then  $f$  is called a bi-contra-Continuous map if  $f$  is contra-continuous and  $f^{-1}(V)$  is open in  $X$  implies  $V$  is closed in  $Y$ .

*Example 3.2.* Let  $X = \{a, b, c, d\}$ ,  $\tau = \{\emptyset, \{a, b\}, \{a, b, c\}, \{a, b, d\}, X\}$ ,  $Y = \{p, q, r\}$  and  $\sigma = \{\emptyset, \{r\}, \{q\}, \{q, r\}, Y\}$ . Define a map  $f : (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = f(b) = p$ ,  $f(c) = q$  and  $f(d) = r$ . Then  $f$  is a bi-contra-Continuous map.

Regarding the restriction  $f_A$  of a map  $f : (X, \tau) \rightarrow (Y, \sigma)$  to a subset  $A$  of  $X$ , we have the following:

**Theorem 3.3.** *Let  $f : X \rightarrow Y$  is a bi-contra-continuous and surjective mapping. If  $A$  is a clopen subset of  $X$ , then its restriction  $f_A : (A, \tau_A) \rightarrow (Y, \sigma)$  is a bi-contra-continuous map.*

*Proof.* Let  $f$  be bi-contra-continuous. Since  $f$  is contra-continuous, if  $V$  is open in  $Y$ , then it implies  $f^{-1}(V)$  is closed in  $X$ . Given  $A$  is clopen. So  $f^{-1}(V) \cap A$  is closed in  $A$ . Also  $(f_A)^{-1}(V) = f^{-1}(V) \cap A$  is closed in  $A$ . Hence  $(f_A)^{-1}$  is contra-continuous.

Now let  $(f_A)^{-1}(V)$  be open in  $A$ .  $(f_A)^{-1}(V) = f^{-1}(V) \cap A$ . As  $A$  is clopen,  $f^{-1}(V)$  is open in  $X$ . Since  $f$  is bi-contra-continuous,  $V$  is closed in  $Y$ .

*Remark 3.4.* In the Theorem 3.3, the assumption of clopen of  $A$  can not be given up as seen from example 3.2.

**Theorem 3.5.** (*Pasting lemma for bi-contra-continuous map*) Let  $X = A \cup B$  be a topological space with topology  $\tau$  and  $Y$  be a topological space with topology  $\sigma$ . Let  $f : (A, \tau_A) \rightarrow (Y, \sigma)$  and  $g : (B, \tau_B) \rightarrow (Y, \sigma)$  be bi-contra-continuous maps such that  $f(x) = g(x)$  for every  $x \in A \cap B$ . Suppose  $A$  and  $B$  are clopen in  $X$ . Then the combination  $h : (X, \tau) \rightarrow (Y, \sigma)$  is bi-contra-continuous.

*Proof.* Let  $V$  be open in  $Y$ . Now  $h^{-1}(V) = f^{-1}(V) \cup g^{-1}(V)$  by elementary set theory. Since  $f$  and  $g$  are contra-continuous,  $f^{-1}(V)$  and  $g^{-1}(V)$  are closed in  $A$  and  $B$  respectively where  $A$  and  $B$  are closed in  $X$ . So  $h^{-1}(V)$  is closed in  $X$ .

Let  $h^{-1}(V)$  is open in  $X$ .  $h^{-1}(V) = f^{-1}(V) \cup g^{-1}(V)$ . So  $f^{-1}(V)$  and  $g^{-1}(V)$  are open in  $A$  and  $B$  respectively. Since  $f$  and  $g$  are bi-contra-continuous,  $V$  is closed in both  $A$  and  $B$ . Hence  $V$  is closed in  $X$ .

We may recall that a space  $(X, \tau)$  is almost compact if every open cover has a finite proximate sub cover.

**Theorem 3.6.** *The image of a compact space under a bi-contra-continuous and continuous map is almost compact.*

*Proof.* Let  $(V_\alpha)_{\alpha \in I}$  be an open cover of  $Y$ . Since  $f$  is bi-contra-continuous, it is contra-continuous. So for each  $\alpha$  in the index set, the inverse image of  $V_\alpha$  under  $f$  is closed. Again as  $f$  is continuous the inverse image of each  $V_\alpha$  is open under  $f$ . Hence the inverse image of  $V_\alpha$  under  $f$  is clopen for each  $\alpha$  and these clopen sets cover  $X$ . As  $X$  is compact a finite collection of  $f^{-1}(V_\alpha)$ , where  $\alpha \in J$  is a finite subcover of  $X$ . Since  $f$  is bi-contra-continuous,  $(V_\alpha)_{\alpha \in J}$  is closed. Also  $f(X) = Y = \bigcup_{\alpha \in J} cl(V_\alpha)$ . So  $Y$  is almost compact.

**Theorem 3.7.** *If  $f : X \rightarrow Y$  is open and bi-contra-continuous, and  $g : Y \rightarrow Z$  is continuous and bi-contra-continuous, then the composition  $(g \circ f) : X \rightarrow Z$  is bi-contra-continuous.*

*Proof.* Let  $V$  be open in  $Z$ . Since  $g$  is continuous  $g^{-1}(V)$  is open in  $Y$ . Since  $f$  is bi-contra-Continuous,  $f^{-1}(g^{-1}(V))$  is closed in  $X$ .  $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ . So  $(g \circ f)^{-1}(V)$  is closed in  $X$ .  $f^{-1}(g^{-1}(V))$  is open in  $X$ . Since  $f$  is open,  $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$  is open in  $Y$ . Again  $g$  is contra-continuous. So  $V$  is closed in  $Z$ . Hence  $(g \circ f)^{-1}(V)$  is bi-contra-continuous.

**Theorem 3.8.** *Let  $p : X \rightarrow Y$  be a bi-contra-Continuous map. Let  $Z$  be topological space and let  $g : X \rightarrow Z$  be a continuous map that is constant on each set  $p^{-1}(\{y\})$ , for  $y \in Y$ . Then  $g$  induces a contra-continuous map  $f : Y \rightarrow Z$  such that  $f \circ p = g$ .*

*Proof.* For each  $y \in Y$ , the set  $g(p^{-1}(\{y\}))$  is an one-point set in  $Z$  (since  $g$  is constant on  $p^{-1}(\{y\})$ ). If we let  $f(y)$  denote this point, then we can define a map  $f : Y \rightarrow Z$  such that for each  $x \in X$ ,  $f(p(x)) = g(x)$ .

To show  $f$  is contra-continuous, let  $V$  be an open set in  $Z$ . Since  $g$  is continuous,  $g^{-1}(V)$  is open in  $X$ . Now  $g^{-1}(V) = p^{-1}(f^{-1}(V))$  and  $p^{-1}(f^{-1}(V))$  is open. Since  $p$  is bi-contra-continuous  $f^{-1}(V)$  is closed in  $Y$ . Hence  $f$  is contra-continuous.

## 4 Bi-Contra- $\alpha$ -Continuous maps

**Definition 4.1.** A mapping  $f : X \rightarrow Y$  is called a contra- $\alpha$ -continuous [6] (resp. contra-semi-continuous [5] and contra-pre-continuous [7]) if the inverse image of an open set in  $Y$  is an  $\alpha$ -closed (resp. semi-closed and pre-closed) set in  $X$ .

**Definition 4.2.** A surjective mapping  $f : X \rightarrow Y$  is called a bi- contra- $\alpha$ -continuous (resp. bi- contra-semi-continuous and bi-contra-pre-continuous) if  $f$  is contra- $\alpha$ -continuous (resp. contra-semi-continuous and contra-pre-continuous) and  $f^{-1}(V)$  is open in  $X$  implies  $V$  is  $\alpha$ -closed (resp. semi-closed and pre-closed).

*Remark 4.3.* Every contra-continuous map is contra- $\alpha$ -continuous but not the converse as is justified by the following example.

*Example 4.4.* Let  $X = \{a, b, c, d\}$ ,  $\tau = \{\emptyset, \{a\}, X\}$ ,  $Y = \{p, q, r\}$ ,  $\sigma = \{\emptyset, \{p\}, \{q\}, \{p, q\}, Y\}$ . Define,  $f : (X, \tau) \rightarrow (Y, \sigma)$  by as  $f(a) = f(d) = r$ ,  $f(b) = p$  and  $f(c) = q$ . Here  $f$  is contra- $\alpha$ -continuous but not contra-continuous. Since  $f^{-1}(p) = b$  where  $\{b\}$  is  $\alpha$ -closed but not closed.

**Theorem 4.5.** *If  $f : (X, \alpha O(X)) \rightarrow (Y, \alpha O(Y))$  is a bi-contra-continuous map, then  $f : (X, \tau) \rightarrow (Y, \sigma)$  is a bi- contra- $\alpha$ -continuous map.*

*Proof.* clear.

**Theorem 4.6.** *Every bi-contra-continuous map is a bi-contra- $\alpha$ -continuous map.*

*Proof.* We know that contra-continuous implies contra- $\alpha$ -continuous. Let  $f^{-1}(V)$  be open in  $X$ . Since  $f$  is bi-contra-continuous,  $V$  is closed in  $Y$ . Hence  $V$  is  $\alpha$ -closed in  $Y$ .

*Remark 4.7.* The converse of the Theorem 4.6, need not be true as is seen from the following example.

*Example 4.8.* Let  $X = \{a, b, c, d\}$ ,  $\tau = \{\emptyset, \{a\}, \{a, d\}, \{a, b, c\}, X\}$ ,  $Y = \{p, q, r, t\}$  and  $\sigma = \{\emptyset, \{p\}, \{t\}, \{r\}, \{p, t\}, \{t, r\}, \{p, r\}, \{p, t, r\}, Y\}$ . Define a map  $f : (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = q$ ,  $f(c) = r$ ,  $f(b) = p$ , and  $f(d) = t$ . Here  $f$  is bi- contra- $\alpha$ -continuous but not bi- contra-continuous. Since  $f^{-1}(p) = b$  where  $\{b\}$  is not a closed set in  $X$ .

**Definition 4.9.** A function  $f : X \rightarrow Y$  is called a :

- (i) contra- $\alpha$ -open (resp. contra- $\alpha$ -closed) map [2] if the image of every open set (resp. closed set) in  $X$  is  $\alpha$ -closed set (resp.  $\alpha$ -open) in  $Y$ .
- (ii) contra- strongly  $\alpha$ -open map, if the image of every  $\alpha$ -open set in  $X$  is  $\alpha$ -closed in  $Y$ .

**Theorem 4.10.** *If  $f : (X, \alpha O(X)) \rightarrow (Y, \alpha O(Y))$  is contra- $\alpha$ -open, then  $f : (X, \tau) \rightarrow (Y, \sigma)$  is contra-strongly-  $\alpha$ -open.*

*Proof.* Is obvious.

**Theorem 4.11.** *If  $f : X \rightarrow Y$  is surjective, contra- $\alpha$ -continuous and contra- $\alpha$ -open, then  $f$  is bi-contra- $\alpha$ -continuous.*

*Proof.* Let  $f$  be a contra- $\alpha$ -continuous map and let  $f^{-1}(V)$  be open in  $X$ . Since  $f$  is contra- $\alpha$ -open  $f(f^{-1}(V)) = V$  is  $\alpha$ -closed. Hence  $f$  is bi-contra- $\alpha$ -continuous.

**Theorem 4.12.** *Let  $f : (X, \tau) \rightarrow (Y, \sigma)$  be an open surjective,  $\alpha$ - irresolute map and  $g : (Y, \sigma) \rightarrow (Z, \gamma)$  be a bi-contra- $\alpha$ -continuous map. Then  $(g \circ f)$  is a bi-contra- $\alpha$ -continuous map.*

*Proof.* Let  $V$  be an open set in  $Z$ . Since  $g$  is bi-contra- $\alpha$ -continuous,  $g^{-1}(V)$  is  $\alpha$ -closed in  $Y$ . As  $f$  is  $\alpha$ -irresolute,  $f^{-1}(g^{-1}(V))$  is  $\alpha$ -closed in  $X$ . Hence  $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$  is  $\alpha$ -closed in  $X$ . Hence  $(g \circ f)^{-1}$  is contra- $\alpha$ -continuous.

Let  $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$  be open in  $X$ . Since  $f$  is open and surjective,  $g^{-1}(V)$  is open in  $Y$ . Since  $g$  is bi- contra- $\alpha$ -continuous,  $V$  is  $\alpha$ -closed in  $Z$ . Hence  $(g \circ f)$  is bi-contra- $\alpha$ -continuous.

**Theorem 4.13.** *If  $p : X \rightarrow Y$  is a bi-contra- $\alpha$ -continuous map and  $g : X \rightarrow Z$  is a continuous map, where  $Z$  is a space that is constant on each set  $p^{-1}(\{y\})$ , for  $y \in Y$ , then  $g$  induces a contra- $\alpha$ -continuous map  $f : Y \rightarrow Z$  such that  $f \circ p = g$ .*

*Proof.* Similar to that for theorem 3.8.

**Theorem 4.14.** *The function  $f : X \rightarrow Y$  is bi-contra- $\alpha$ -continuous if and only if it is both bi-contra-semi-continuous and bi-contra-pre-continuous.*

*Proof.* Let  $f$  be a bi-contra- $\alpha$ -continuous map. If  $V$  is an open set in  $Y$ , then  $f^{-1}(V)$  is  $\alpha$ -closed. Hence it is semi-closed and pre-closed. Let  $f^{-1}(V)$  be open in  $X$ . Since  $f$  is bi-contra- $\alpha$ -continuous,  $V$  is  $\alpha$ -closed in  $Y$ . Hence  $V$  is semi-closed and pre-closed. So,  $f$  is both bi-contra-semi-continuous and bi-contra-pre-continuous.

Conversely, let  $f$  be both bi-contra-semi-continuous and bi-contra-pre-continuous. So, if  $V$  is open in  $Y$  then it implies  $f^{-1}(V)$  is both semi-closed and pre-closed. Hence  $f^{-1}(V)$  is  $\alpha$ -closed. Let  $f^{-1}(V)$  be open in  $X$ . Then  $V$  is both semi-closed and pre-closed in  $Y$ . Hence  $V$  is  $\alpha$ -closed in  $Y$ .

## 5 Bi-Contra-Strongly- $\alpha$ -Continuous maps and bi-Contra- $\alpha^*$ -Continuous maps

**Definition 5.1.** Let  $f : X \rightarrow Y$  be a surjective map. Then  $f$  is called bi-contra-strongly- $\alpha$ -continuous (resp. bi-contra-strongly-semi-continuous and bi-contra-strongly-pre-continuous) provided  $f$  is contra- $\alpha$ -continuous (resp. contra-semi-continuous and contra-pre-continuous) and  $f^{-1}(V)$  is open in  $X$  if and only if  $V$  is  $\alpha$ -closed (resp. semi-closed and pre-closed) in  $Y$ .

*Example 5.2.* The Sierpinski space  $X = Y = \{a, b\}$ , by setting  $\tau = \{\emptyset, \{a\}, X\}$  and  $\sigma = \{\emptyset, \{b\}, Y\}$  with  $f : (X, \tau) \rightarrow (Y, \sigma)$  as an identity map is an example of a bi-contra-strongly- $\alpha$ -Continuous map.

*Remark 5.3.* Every bi-contra-strongly- $\alpha$ -continuous map is bi-contra- $\alpha$ -continuous, but the converse need not always be true as justified by example 4.6, where  $\{q, r\}$  is an  $\alpha$ -closed set in  $(Y, \sigma)$  but  $f^{-1}(\{q, r\}) = \{a, c\}$  is not an open set in  $(X, \tau)$ .

**Theorem 5.4.** *If  $f : X \rightarrow Y$  is bi-contra-strongly-semi-continuous and bi-contra -strongly-pre-continuous, then  $f$  is bi-contra- strongly- $\alpha$ -continuous.*

*Remark 5.5.* The converse of the Theorem 5.4, is not always true as is seen from the following example.

*Example 5.6.* Let  $X = \{a, b, c, d\}$ ,  $\tau = \{\emptyset, \{a, b\}, \{c, d\}, X\}$ ,  $Y = \{p, q, r\}$  and  $\sigma = \{\emptyset, \{p\}, \{p, r\}, Y\}$ . Define a map  $f : (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = f(b) = p$ ,  $f(c) = q$  and  $f(d) = r$ . Here  $f$  is bi-contra strongly- $\alpha$ -continuous but not contra strongly-pre-continuous, Since  $\{q\}$  is pre-closed in  $Y$  but  $f^{-1}(q) = \{c\}$  is not an open set of  $X$ .

**Definition 5.7.** A function  $f : X \rightarrow Y$  is called a contra- $\alpha$ -irresolute map if the inverse image of an  $\alpha$ -open set in  $Y$  is an  $\alpha$ -closed set in  $X$ .

*Remark 5.8.* Every contra- $\alpha$ -irresolute map is a contra- $\alpha$ -continuous map but the converse need not be true as is seen from the following example.

*Example 5.9.* Let  $X = \{a, b, c, d\}$ ,  $\tau = \{\emptyset, \{a\}, \{b, c, d\}, X\}$ ,  $Y = \{p, q, r\}$  and  $\sigma = \{\emptyset, \{p\}, Y\}$ . Define a map  $f : (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = p$ ,  $f(b) = q$  and  $f(c) = f(d) = r$ . Here  $f$  is contra- $\alpha$ -continuous but not contra- $\alpha$ -irresolute.

**Definition 5.10.** A map  $f : X \rightarrow Y$  is called as a bi-contra- $\alpha^*$ -continuous (resp. bi-contra-*semi* $^*$ -continuous and bi-contra-*pre* $^*$ -continuous) map if  $f$  is contra- $\alpha$ -irresolute (resp. contra-*semi*-irresolute and contra-*pre*-irresolute) and  $f^{-1}(V)$  is  $\alpha$ -open (resp. *semi*-open and *pre*-open) in  $X$  if and only if  $V$  is closed in  $Y$ .

It is worthwhile to note that all the contra- $\alpha$ -irresolute functions need not be a bi-contra- $\alpha^*$ -continuous map. In the following example, we define a contra- $\alpha$ -irresolute function which is not a bi-contra- $\alpha^*$ -continuous.

*Example 5.11.* Let  $X = \{a, b, c, d\}$ ,  $\tau = \{\emptyset, \{a\}, \{a, b\}, \{c, d\}, \{a, c, d\}, X\}$ ,  $Y = \{p, q, r\}$  and  $\sigma = \{\emptyset, \{p\}, Y\}$ . Define a map  $f : (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = q$ ,  $f(b) = p$  and  $f(c) = f(d) = r$ . Here  $f^{-1}(p, q) = \{a, b\}$  is  $\alpha$ -open but  $\{p, q\}$  is not closed. So  $f$  is not bi-contra- $\alpha^*$ -continuous.

**Theorem 5.12.** *Every bi-contra- $\alpha^*$ -continuous map is a bi-contra- strongly- $\alpha$ -continuous map.*

*Proof.* obvious.

The converse of the above theorem need not be true as is seen from the following example.

*Example 5.13.* Let  $X = Y = \{a, b, c, d\}$ ,  $\tau = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}, X\}$ ,  $\sigma = \{\emptyset, \{b\}, \{d\}, \{b, d\}, \{c, d\}, \{b, c, d\}, Y\}$ . Define  $f : (X, \tau) \rightarrow (Y, \sigma)$  be the identity map. The map  $f$  is bi-contra-strongly- $\alpha$ -continuous. Since  $\{a, d\}$  is  $\alpha$ -open in  $X$  but  $\{a, d\}$  is not closed in  $Y$ . So  $f$  is not bi-contra- $\alpha^*$ -continuous.

**Theorem 5.14.** *Let  $f : X \rightarrow Y$  be a surjective, strongly- $\alpha$ - open and  $\alpha$ -irresolute map and  $g : Y \rightarrow Z$  be a bi-contra- $\alpha^*$ -continuous map. Then  $(g \circ f)$  is an bi-contra-  $\alpha^*$ -continuous map.*

*Proof.* Let  $V$  be an  $\alpha$ -open set in  $Z$ . Since  $g$  is bi- contra- $\alpha^*$ -continuous,  $g^{-1}(V)$  is  $\alpha$ -closed in  $Y$ . Again  $f$  is  $\alpha$ -irresolute and so  $f^{-1}(g^{-1}(V))$  is  $\alpha$ -closed in  $X$ . Hence  $(g \circ f)$  is contra- $\alpha$ -irresolute.

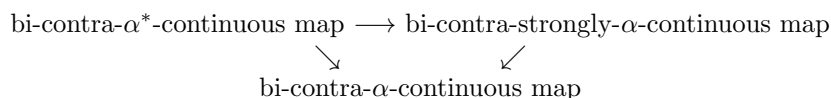
Let  $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$  be  $\alpha$ -open in  $X$ . Since  $f$  is strongly- $\alpha$ -open and surjective,  $g^{-1}(V)$  is  $\alpha$ -open in  $Y$ . Again  $g$  is bi-contra- $\alpha^*$ -continuous and so  $g^{-1}(V)$  is  $\alpha$ -open if and only if  $V$  is closed in  $Z$ . Thus  $(g \circ f)$  is bi-contra- $\alpha^*$ -continuous.



**Theorem 5.15.** *If  $f : X \rightarrow Y$  is bi-contra-semi\*-continuous and bi-contra-pre\*-continuous, then  $f$  is bi-contra- $\alpha^*$ -continuous.*

The converse of the above theorem is not always true as is seen from example 5.11.

*Remark 5.16.* From the above theorems and examples we get the following diagram, and none of its implications is reversible.



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