Bulletin T.CXLIV de l'Académie serbe des sciences et des arts – 2012
Classe des Sciences mathématiques et naturelles
Sciences mathématiques, № 37

RELATIONS BETWEEN KIRCHHOFF INDEX AND LAPLACIAN–ENERGY–LIKE INVARIANT

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(Presented at the 3rd Meeting, held on March 30, 2012)

A b s t r a c t. *The Kirchhoff index Kf and the Laplacian–energy–like invariant LEL are two graph invariants defined in terms of the Laplacian eigenvalues. If $\mu_1 \geq \mu_2 \geq \cdots \geq \mu_{n-1} > \mu_n = 0$ are the Laplacian eigenvalues of a connected n-vertex graph, then $Kf = n \sum_{i=1}^{n-1} 1/\mu_i$ and $LEL = \sum_{i=1}^{n-1} \sqrt{\mu_i}$. We examine the conditions under which $Kf > LEL$. Among other results we show that $Kf > LEL$ holds for all trees, unicyclic, bicyclic, tricyclic, and tetracyclic connected graphs, except for a finite number of graphs. These exceptional graphs are determined.*

AMS Mathematics Subject Classification (2000): 05C50

Key Words: Laplacian spectrum (of graph), Laplacian eigenvalue, Kirchhoff index, Laplacian–energy–like invariant, LEL

1. *Introduction*

In this paper we are concerned with simple graph, that is a graph possessing no directed, weighted, or multiple edges, and no self–loops. In addition, we assume that the graphs considered are connected. Let G be such a graph and let n and m be the number of its vertices and edges, respectively. Let $\mu_1, \mu_2, \ldots, \mu_n$ be the Laplacian eigenvalues of G, forming its Laplacian spectrum. For details of Laplacian spectral graph theory see [2, 10, 9]. It is important for us that if the graph G is connected, then $n-1$ of its Laplacian eigenvalues are real positive numbers, whereas one eigenvalue is equal to zero. In what follows the Laplacian spectrum of the graph G will be denoted by $Spec(G) = \{\mu_1, \mu_2, \ldots, \mu_n\}$, assuming that $\mu_1 \geq \mu_2 \geq \cdots \geq \mu_{n-1} > \mu_n = 0$.

Two graph invariants based on Laplacian eigenvalues have been much studied in last few years. These are the *Kirchhoff index*,

$$Kf = Kf(G) = n \sum_{i=1}^{n-1} \frac{1}{\mu_i} \tag{0.1}$$

and the *Laplacian–energy–like invariant*,

$$LEL = LEL(G) = \sum_{i=1}^{n-1} \sqrt{\mu_i}\ . \tag{0.2}$$

Recall that the ordinary distance between two vertices v_i and v_j in a connected graph G is defined as the length (= number of edges) of a shortest path that connects v_i and v_j. Klein and Randić [15] conceived the *resistance distance*, defined in terms of electric resistance in a network corresponding to the considered graph, in which the resistance between any two adjacent nodes is 1 Ohm. The sum of resistance distances between all pairs of vertices of a graph was conceived as a novel graph invariant [15, 1] and – in view of the fact that electric resistances are calculated by means of the Kirchhoff laws – named the "*Kirchhoff index*". The fact that the Kirchhoff index satisfies the relation (0.1) was independently established in [12] and [23]. Of the numerous investigations on the Kirchhoff index we mention here only a few most recent [4, 6, 8, 20, 21].

Another Laplacian–spectrum–based graph invariant was put conceived by Liu and Liu [17], and defined via Eq. (0.2). Details of the theory of LEL and an exhaustive list of references can be found in the recent surveys [11, 16]; for some most recent works on this topic see [22, 13, 14, 19].

2. *Relations between* **Kf** *and* **LEL** *for graphs with given cyclomatic number*

In spite of the intense research done on both Kf and LEL, the relation between these two closely related Laplacian–spectrum–based graph invariants has not been investigated until quite recently [5]. In [5] the following two results have been established:

Theorem 2.1 *Let G be a connected graph of order n with m edges. If $2m \leq (n-1)\,n^{2/3}$, then $LEL(G) < Kf(G)$.*

Theorem 2.2 *Let G be a connected graph of order n with m edges. Let δ be the smallest degree of a vertex of G. If $2m \leq (n-2)\,n^{2/3} + \delta$, then $LEL(G) < Kf(G)$.*

Theorems 2.1 and 2.2 immediately imply:

Corollary 2.3 *Let G be a connected graph of order n. If $Kf(G) < LEL(G)$, then G must have more than $\frac{1}{2}(n-1)\,n^{2/3}$ edges.*

Corollary 2.4 *Let G be a connected graph of order n. Let δ be the smallest degree of a vertex of G. If $Kf(G) < LEL(G)$, then G must have more than $\frac{1}{2}[(n-2)\,n^{2/3} + \delta]$ edges.*

In case when the value of δ cannot be specified, we have the following weakened variant of Corollary 2.4:

Corollary 2.5 *Let G be a connected graph of order n. If $Kf(G) < LEL(G)$, then G must have more than $\frac{1}{2}[(n-2)\,n^{2/3} + 1]$ edges.*

Combining Corollaries 2.3 and 2.5, it is evident that in order that the relation $Kf(G) < LEL(G)$ be obeyed, the graph G must possess more than

$$\frac{1}{2}\,\min\left\{(n-1)\,n^{2/3}\;,\;(n-2)\,n^{2/3} + 1\right\}$$

edges. It is easy to show that the inequality

$$(n-2)\,n^{2/3} + 1 < (n-1)\,n^{2/3}$$

holds for all values of n , $n \geq 3$.

Theorem 2.6 *Let $\mathcal{G}(\rfloor)$ be the set of connected graphs with cyclomatic number c. For any fixed value of c, the number of elements of $\mathcal{G}(\rfloor)$ for which $Kf < LEL$ holds is finite.*

P r o o f. An n-vertex graph with cyclomatic number c has $n + c - 1$ edges. No matter how large c is, there always will exist some (finite) positive integer $n_0 = n_0(c)$, such that the inequality

$$n + c - 1 < \frac{1}{2}\left[(n-2)\, n^{2/3} + 1\right]$$

be satisfied for all values of $n \geq n_0$. Therefore graphs for which $Kf < LEL$ must possess less than n_0 vertices and, consequently, their number is finite. □

Remark 2.7 *By direct numerical testing we can verify that* n_0 *in Theorem 2.6 is equal to 4, 6, 6, 7, 8 for cyclomatic number 0, 1, 2, 3, and 4. This means that for* $c = 0, 1, 2, 3, 4$, *connected graphs for which the Kirchhoff index is smaller than the Laplacian–energy–like invariant can possess at most 3, 5, 5, 6, and 7 vertices, respectively.*

Remark 2.8 *For the complete graph* K_n *we have* [10, 9] $Spec(K_n) = \{n, n, \ldots, n, 0\}$. *Therefore,* $Kf(K_n) = n - 1$ *and* $LEL(K_n) = (n-1)\sqrt{n}$. *Therefore,* $Kf(K_n) < LEL(K_n)$ *holds for all* $n > 1$.

Corollary 2.9 [5] *The only tree (i. e., a connected graph with* $c = 0$*) for which* $Kf < LEL$ *holds is* K_2.

Corollary 2.10 [5] *The only connected unicyclic graph (i. e., graph with* $c = 1$*) for which* $Kf < LEL$ *holds is* K_3.

P r o o f. In Fig. 1 are depicted all unicyclic graphs with 3, 4, and 5 vertices. Numerical calculation shows that $Kf < LEL$ holds only for the graph H_1. □

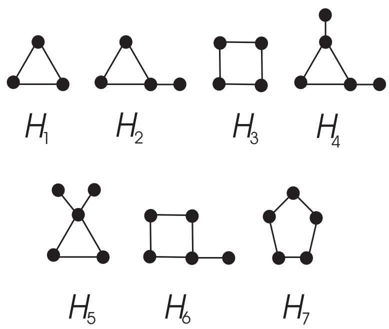

Fig. 1. The connected unicyclic graphs with 3, 4, and 5 vertices.

Corollary 2.11 [5] *The only connected bicyclic graph (i. e., graph with $c = 2$) for which $Kf < LEL$ holds is $K_4 - e$ i. e., the graph H_8 in Fig. 2.*

P r o o f. In Fig. 2 are depicted all bicyclic graphs with 4 and 5 vertices. Numerical calculation shows that $Kf < LEL$ holds only for the graph H_8. □

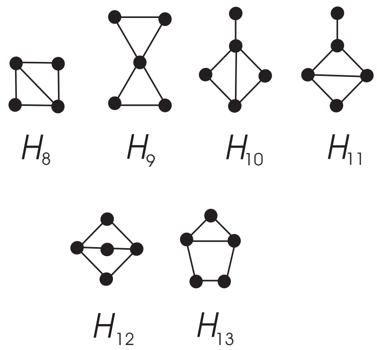

Fig. 2. The connected bicyclic graphs with 4 and 5 vertices.

Corollary 2.12 *The only connected tricyclic graphs (i. e., graphs with $c = 3$) for which $Kf < LEL$ holds are $H_{14} \cong K_4$, H_{16} , H_{17} , and H_{18} , depicted in Fig. 3.*

P r o o f. In Fig. 3 are shown all tricyclic graphs with 4, 5, and 6 vertices. By numerical calculation we obtained the following results:

graph	Kf	LEL	graph	Kf	LEL
H_{14}	3.00	6.00	H_{28}	13.88	8.61
H_{15}	8.50	7.24	H_{29}	13.83	8.61
H_{16}	7.00	7.30	H_{30}	14.52	8.55
H_{17}	6.95	7.33	H_{31}	15.24	8.57
H_{18}	6.42	7.38	H_{32}	14.50	8.63
H_{19}	11.50	8.69	H_{33}	12.70	8.70
H_{20}	14.20	8.51	H_{34}	12.55	8.68
H_{21}	15.20	8.54	H_{35}	11.25	8.75
H_{22}	12.43	8.65	H_{36}	11.34	8.74
H_{23}	11.75	8.70	H_{37}	12.67	8.68
H_{24}	16.50	8.46	H_{38}	12.00	8.72
H_{25}	16.00	8.45	H_{39}	13.50	8.60
H_{26}	19.00	8.51	H_{40}	14.50	8.63
H_{27}	14.14	8.54			

□

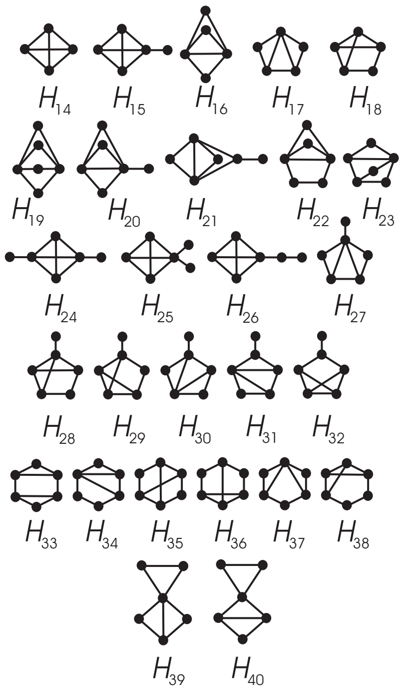

Fig. 3. The connected tricyclic graphs with 4, 5, and 6 vertices.

In a fully analogous manner, by examining all the 154 connected tetracyclic graphs with seven or fewer vertices, we arrive at:

Corollary 2.13 *The only connected tetracyclic graphs (i. e., graphs with* $c = 4$*) for which* $Kf < LEL$ *holds are* H_{41} , H_{42} , H_{43} , *and* H_{44} , *depicted in Fig. 4.*

Remark 2.14 *There are 2, 20, and 132 connected tetracyclic graphs with 5, 6, and 7 vertices, respectively. Among the 7-vertex species no one satisfies the inequality* $Kf < LEL$.

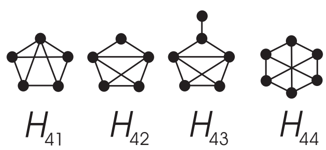

Fig. 4. The only connected tetracyclic graphs for which LEL is greater than the Kirchhoff index.

3. *More relations between* **Kf** *and* **LEL**

Theorem 3.15 *Let G be a connected graph and e its edge, such that $G-e$ is also connected. If $Kf(G) > LEL(G)$, then $Kf(G-e) > LEL(G-e)$.*

P r o o f. Let $Spec(G) = \{\mu_1, \mu_2, \dots, \mu_{n-1}, 0\}$ and $Spec(G-e) = \{\mu'_1, \mu'_2, \dots, \mu'_{n-1}, 0\}$. As well known [10, 9], the Laplacian eigenvalues of $G-e$ interlace the Laplacian eigenvalues of G, i. e.,

$$\mu_1 \geq \mu'_1 \geq \mu_2 \geq \mu'_2 \geq \cdots \geq \mu_{n-1} \geq \mu'_{n-1} > \mu_n = \mu'_n = 0 \ .$$

These inequalities immediately imply

$$\sum_{i=1}^{n-1} \sqrt{\mu_i} \geq \sum_{i=1}^{n-1} \sqrt{\mu'_i} \qquad \text{i. e.,} \qquad LEL(G) \geq LEL(G-e)$$

and

$$\sum_{i=1}^{n-1} \frac{1}{\mu_i} \leq \sum_{i=1}^{n-1} \frac{1}{\mu'_i} \qquad \text{i. e.,} \qquad Kf(G) \leq Kf(G-e) \ .$$

□

Corollary 3.16 *If $Kf(G) > LEL(G)$ and if $e_1, e_2, \dots, e_t$ are edges of G, such that $G - e_1 - e_2 - \cdots - e_t$ is connected, then*

$$Kf(G - e_1 - e_2 - \cdots - e_t) > LEL(G - e_1 - e_2 - \cdots - e_t) \ .$$

In a fully analogous manner as Theorem 3.15, we can prove also:

Theorem 3.17 *Let $G+e$ be the graph obtained by adding a new edge to the connected graph G. If $Kf(G) < LEL(G)$, then $Kf(G+e) < LEL(G+e)$.*

Corollary 3.18 *If G is a connected graph of order n with cyclomatic number $c \geq 0$, such that $Kf(G) < LEL(G)$, then we can construct a connected graph $G^\dagger$ of order n, with cyclomatic number $c^\dagger$, $c < c^\dagger \leq (n-1)(n-2)/2$, such that $Kf(G^\dagger) < LEL(G^\dagger)$.*

Corollary 3.19 *If $n \geq 4$, then $Kf(K_n - e) < LEL(K_n - e)$ holds.*

Lemma 3.20 [3] *Let G be a connected graph of order n with Laplacian spectrum $Spec(G) = \{\mu_1, \mu_2, \dots, \mu_{n-1}, 0\}$. If G^* is the graph obtained by connecting a new vertex to all vertices of G, then $Spec(G^*) = \{n+1, \mu_1 + 1, \mu_2 + 1, \dots, \mu_{n-1} + 1, 0\}$.*

The product of G_1 and G_2 is the graph $G_1 \times G_2$ whose vertex set is the Cartesian product $V(G_1) \times V(G_2)$. Suppose $v_1,\ v_2 \in V(G_1)$ and $u_1,\ u_2 \in V(G_2)$. Then $(v_1,\ u_1)$ and $(v_2,\ u_2)$ are adjacent in $G_1 \times G_2$ if and only if one of the following conditions is satisfied: (i) $v_1 = v_2$ and $\{u_1,\ u_2\} \in E(G_2)$, or (ii) $\{v_1,\ v_2\} \in E(G_1)$ and $u_1 = u_2$ [2].

Lemma 3.21 [7, 18] *Let G_1 and G_2 be graphs on n_1 and n_2 vertices, respectively. Then $Spec(G_1 \times G_2)$ consists of all possible sums $\mu_i(G_1) + \mu_j(G_2)$, $1 \le i \le n_1$ and $1 \le j \le n_2$.*

Let $H_n = K_p \times K_2$. Then $n = 2p$. In particular, $H_4 = C_4$ for $n = 4$. We have $LEL(C_4) = \sqrt{4} + 2\sqrt{2} < 1 + 2 + 2 = Kf(C_4)$ and $LEL(H_6) = 2\sqrt{5} + 2\sqrt{3} + \sqrt{2} \approx 9.35 < 9.4 = 2.4 + 4 + 3 = Kf(H_6)$. But we have the following:

Theorem 3.22 *Let G be a graph of order $n \ge 8$ (n is even) and let H_n be a subgraph of G. Then $LEL(G) > Kf(G)$.*

P r o o f. We have $n = 2p$. Since $Spec(K_p) = \{\underbrace{p, p, \ldots, p}_{p-1}, 0\}$, from Lemma 3.21 it follows that

$$Spec(H_n) = \{\underbrace{p+2,\ p+2, \ldots,\ p+2}_{p-1},\ \underbrace{p,\ p, \ldots,\ p}_{p-1},\ 2,\ 0\}$$

Since H is a subgraph of G, we have $\mu_i(G) \ge \mu_i(H)$, that is,

$$\mu_i(G) \ge p+2 \quad \text{for } i = 1,\ 2, \ldots,\ p-1;$$

$$\mu_i(G) \ge p \quad \text{for } i = p,\ p+1, \ldots,\ 2p-2;$$

$$\mu_{2p-1}(G) \ge 2 \quad \text{and} \quad \mu_{2p}(G) = 0\ .$$

Thus we have

$$\begin{aligned} LEL(G) &= \sum_{i=1}^{n-1} \sqrt{\mu_i(G)} \ge (p-1)\sqrt{p+2} + (p-1)\sqrt{p} + \sqrt{2} \\ &\ge 2(p-1)\sqrt{p} + \sqrt{2} \end{aligned} \tag{3.3}$$

and

$$Kf(G) = n \sum_{i=1}^{n-1} \frac{1}{\mu_i(G)} \le (p-1)\frac{2p}{p+2} + (p-1)\frac{2p}{p} + \frac{2p}{2} \le 5p-4\ . \tag{3.4}$$

From (3.3) and (3.4),

$$\begin{aligned}
LEL(G) &\geq 3\sqrt{6}+3\sqrt{4}+\sqrt{2}\approx 14.763 > 14\\
&= 4+6+4 \geq Kf(G) \quad \text{for } p=4\\
LEL(G) &\geq 4\sqrt{7}+4\sqrt{5}+\sqrt{2}\approx 20.941 > 18.714\\
&\approx \frac{40}{7}+8+5 \geq Kf(G) \quad \text{for } p=5\\
LEL(G) &\geq 5\sqrt{8}+5\sqrt{6}+\sqrt{2}\approx 27.804 > 23.5\\
&= \frac{15}{2}+10+6 \geq Kf(G) \quad \text{for } p=6\ .
\end{aligned}$$

For $p \geq 7$, one can see easily that

$$LEL(G) \geq 2(p-1)\sqrt{p}+\sqrt{2} > 5p-4 \geq Kf(G)\ .$$

This completes the proof. □

Let H'_n be the graph of order n ($n = 2p+1$) obtained from H_n in such a way that $H'_n = \overline{\overline{H_n} \cup K_1}$, where $H_n = K_p \times K_2$.

Theorem 3.23 *Let G be a graph of order $n \geq 5$ (n is odd) and let H'_n be a subgraph of G. Then $LEL(G) > Kf(G)$.*

P r o o f. We have $n = 2p+1$. Since

$$Spec(H_n) = \{\underbrace{p+2,\ p+2,\dots,\ p+2}_{p-1},\ \underbrace{p,\ p,\dots,\ p}_{p-1},\ 2,\ 0\}$$

by Lemma 3.20,

$$Spec(H'_n) = \{n, \underbrace{p+3,\ p+3,\dots,\ p+3}_{p-1},\ \underbrace{p+1,\ p+1,\dots,\ p+1}_{p-1},\ 3,\ 0\}\ .$$

Since H'_n is a subgraph of G, we have $\mu_i(G) \geq \mu_i(H'_n)$, that is,

$$\mu_1(G) \geq n \quad ; \quad \mu_i(G) \geq p+3 \quad \text{for } i = 2,3,\dots,p\ ;$$

$$\mu_i(G) \geq p+1 \quad \text{for } i = p+1, p+2, \dots, 2p-1\ ;$$

$$\mu_{2p}(G) \geq 3 \quad \text{and} \quad \mu_{2p+1}(G) = 0\ .$$

Thus we have

$$\begin{aligned} LEL(G) &= \sum_{i=1}^{n-1} \sqrt{\mu_i(G)} \\ &\geq \sqrt{n} + (p-1)\sqrt{p+3} + (p-1)\sqrt{p+1} + \sqrt{3} \\ &> \sqrt{2p+1} + 4.88(p-1) + \sqrt{3} \text{ for } p \geq 4 \ . \end{aligned}$$

and

$$\begin{aligned} Kf(G) &= \sum_{i=1}^{n-1} \frac{n}{\mu_i(G)} \\ &\leq \frac{n}{n} + (p-1)\frac{2p+1}{p+3} + (p-1)\frac{2p+1}{p+1} + \frac{2p+1}{3} \\ &\leq \frac{14}{3}p - \frac{26}{3} + \frac{20}{p+3} + \frac{2}{p+1} \ . \end{aligned}$$

Now,

$$LEL(G) \geq 2\sqrt{5} + 2\sqrt{3} \approx 7.936 > 5.333 \approx 2 + \frac{10}{3} \geq Kf(G) \quad \text{for } p = 2$$

and

$$\begin{aligned} LEL(G) &\geq \sqrt{7} + 2\sqrt{6} + 4 + \sqrt{3} \approx 13.277 \\ &> 9.166 \approx 1 + \frac{7}{3} + \frac{7}{2} + \frac{7}{3} \geq Kf(G) \quad \text{for } p = 3 \ . \end{aligned}$$

For $p \geq 4$, one can see easily that

$$LEL(G) \geq \sqrt{2p+1} + 4.88(p-1) + \sqrt{3} > \frac{14}{3}p - \frac{26}{3} + \frac{20}{p+3} + \frac{2}{p+1} \geq Kf(G) \ .$$

This completes the proof. □

Acknowledgement. B. A., I. G., and K. Ch. D. thank, respectively, for support by the Serbian Ministry of Science (Grant No. 174033), and the Sungkyunkwan University BK21 Project, BK21 Math. Modeling HRD Div. Sungkyunkwan University.

REFERENCES

[1] D. Bonchev, A. T. Balaban, X. Liu, D. J. Klein, Molecular cyclicity and centricity of polycyclic graphs. I. Cyclicity based on resistance distances or reciprocal distances, *Int. J. Quantum Chem.* **50** (1994) 1–20.

[2] D. Cvetković, P. Rowlinson, S. Simić, *An Introduction to the Theory of Graph Spectra*, Cambridge Univ. Press, Cambridge, 2010.

[3] K. C. Das, The Laplacian spectrum of a graph, *Comput. Math. Appl.* **48** (2004) 715–724.

[4] K. C. Das, A. D. Güngör, A. S. Çevik, On the Kirchhoff index and the resistance–distance energy of a graph, *MATCH Commun. Math. Comput. Chem.* **67** (2012) 541–556.

[5] K. C. Das, K. Xu, I. Gutman, Comparison between Kirchhoff index and the Laplacian–energy–like invariant, *Lin. Algebra Appl.* **436** (2012) 3661–3671.

[6] E. Estrada, N. Hatano, Topological atomic displacements, Kirchhoff and Wiener indices of molecules, *Chem. Phys. Lett.* **486** (2010) 166–170.

[7] M. Fiedler, Algebraic connectivity of graphs, *Czech. Math. J.* **23** (1973) 298–305.

[8] X. Gao, Y. Luo, W. Liu, Resistance distances and the Kirchhoff index in Cayley graphs, *Discr. Appl. Math.* **159** (2011) 2050–2057.

[9] R. Grone, R. Merris, The Laplacian spectrum of a graph II, *SIAM J. Discr. Math.* **7** (1994) 221–229.

[10] R. Grone, R. Merris, V. S. Sunder, The Laplacian spectrum of a graph, *SIAM J. Matrix Anal. Appl.* **11** (1990) 218–238.

[11] I. Gutman, Comparative studies of graph energies, *Bull. Acad. Serbe Sci. Arts (Cl. Sci. Math. Natur.)*, **37** (2012) 1–17. in press.

[12] I. Gutman, B. Mohar, The quasi-Wiener and the Kirchhoff indices coincide, *J. Chem. Inf. Comput. Sci.* **36** (1996) 982–985.

[13] I. Gutman, B. Zhou, B. Furtula, The Laplacian–energy like invariant is an energy like invariant, *MATCH Commun. Math. Comput. Chem.* **64** (2010) 85–96.

[14] A. Ilić, D. Krtinić, M. Ilić, On Laplacian like energy of trees, *MATCH Commun. Math. Comput. Chem.* **64** (2010) 111–122.

[15] D. J. Klein, M. Randić, Resistance distance, *J. Math. Chem.* **12** (1993) 81–95.

[16] B. Liu, Y. Huang, Z. You, A survey on the Laplacian–energy–like invariant, *MATCH Commun. Math. Comput. Chem.* **66** (2011) 713–730.

[17] J. Liu, B. Liu, A Laplacian–energy–like invariant of a graph, *MATCH Commun. Math. Comput. Chem.* **59** (2008) 397–419.

[18] B. Mohar, The Laplacian spectrum of graphs, in: Y. Alavi, G. Chartrand, O. R. Oellermann, A. J. Schwenk (Eds.), *Graph Theory, Combinatorics, and Applications*, Wiley, New York, 1991, pp. 871–898.

[19] S. W. Tan, On the Laplacian coefficients and Laplacian–like energy of bicyclic graphs, *Lin. Multilin. Algebra*, in press.

[20] H. P. Zhang, X. Y. Jiang, Y. J. Yang, Bicyclic graphs with extremal Kirchhoff index, *MATCH Commun. Math. Comput. Chem.* **61** (2009) 697–712.

[21] H. P. Zhang, Y. J. Yang, C. W. Li, Kirchhoff index of composite graphs, *Discr. Appl. Math.* **107** (2009) 2918–2927.

[22] B. X. Zhu, The Laplacian–energy like of graphs, *Appl. Math. Lett.* **24** (2011) 1604–1607.

[23] H. Y. Zhu, D. J. Klein, I. Lukovits, Extensions of the Wiener number, *J. Chem. Inf. Comput. Sci.* **36** (1996) 420–428.

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