

**SOME STABILITY THEOREMS FOR SOME ITERATION
PROCESSES USING CONTRACTIVE CONDITION OF
INTEGRAL TYPE**

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ABSTRACT. In this paper we establish some stability results for Picard and Mann iteration processes in metric space and normed linear space by employing contractive condition of integral type. Our results are generalizations and extensions of some of the existing ones in literature especially Olatinwo [8].

1. INTRODUCTION

Let (E, d) be a complete metric space, $T : E \rightarrow E$ a selfmap of E . Suppose that $F_T = \{p \in E : T(p) = p\}$ is the set of fixed points of T in E .

Let $\{x_n\}_{n=0}^{\infty} \subset E$ be the sequence generated by an iteration procedure involving the operator T , that is,

$$x_{n+1} = f(T, x_n), n = 0, 1, \dots, \quad (1.1)$$

where $x_0 \in E$ is the initial approximation and f is some function.

Let $\{y_n\}_{n=0}^{\infty} \subset E$ be an arbitrary sequence in E , and set

$$\varepsilon_n = d(y_{n+1}, f(T, y_n)), n = 0, 1, \dots,$$

then, the iteration procedure (1.1) is said to be T -stable or stable with respect to T if and only if

$$\lim_{n \rightarrow \infty} \varepsilon_n = 0 \Rightarrow \lim_{n \rightarrow \infty} y_n = p.$$

If in (1.1),

$$x_{n+1} = f(T, x_n) = Tx_n, n = 0, 1, \dots, \quad (1.2)$$

then, we have the Picard iteration process, which has been employed to approximate the fixed points of mappings satisfying the inequality relation

$$d(Tx, Ty) \leq \alpha d(x, y), \forall x, y \in E \text{ and } \alpha \in [0, 1[. \quad (1.3)$$

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Condition (1.3) is called the Banach's contraction condition. Any operator satisfying (1.3) is called strict contraction. Also, condition (1.3) is significant in the celebrated Banach's fixed point theorem [1].

In the Banach space setting, we shall state some of the iteration processes generalizing (1.2) as follows:

For $x_0 \in E$, the sequence $\{x_n\}_{n=0}^{\infty}$ defined by

$$x_{n+1} = (1 - \alpha_n)x_n + \alpha_n T(x_n), n = 0, 1, \dots, \quad (1.4)$$

where $\{\alpha_n\}_{n=0}^{\infty} \subset [0, 1]$ is called the Mann iteration process (see Mann [7]).

Several stability results have been obtained by various authors using different contractive definitions. Harder and Hicks [4] obtained interesting stability results for some iteration procedures using various contractive definitions. Rhoades [11, 12] generalized the results of Harder and Hicks [4] to a more general contractive mapping. In Osilike [9], a generalization of some of the results of Harder and Hicks [4] and Rhoades [12] was obtained by employing the following contractive definition: there exist a constant $L \geq 0$ and $\alpha \in [0, 1[$ such that

$$d(Tx, Ty) \leq Ld(x, Tx) + \alpha d(x, y), \forall x, y \in E. \quad (1.5)$$

Imoru and Olatinwo [5] obtained some stability results for Picard and Mann iteration procedures by using a more general contractive condition than those of Harder and Hicks [4], Rhoades [12], Osilike [9], Osilike and Udomene [10] and Berinde [2]. In the paper [5], the following contractive definition was employed: there exist $\alpha \in [0, 1[$ and a monotone increasing function $\varphi : \mathbb{R}_+ \rightarrow \mathbb{R}_+$, with $\varphi(0) = 0$, such that,

$$d(Tx, Ty) \leq \varphi(d(x, Tx)) + \alpha d(x, y), \forall x, y \in E. \quad (1.6)$$

A function $h : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is called a comparison function if:

- (i) h is monotone increasing;
- (ii) $\lim_{n \rightarrow \infty} h^n(t) = 0, \forall t \geq 0$ (where h^n denotes the n -times repeated composition of h with itself).

We remark here that every comparison function satisfies the condition $h(0) = 0$.

In 2006, Imoru and Olatinwo [6] proved some stability results for Picard and Mann iteration processes using the following contractive conditions: there exist a constant $\alpha \in [0, 1[$ and a monotone increasing function $\phi : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ with $\phi(0) = 1$, such that,

$$d(Tx, Ty) \leq \alpha d(x, y)\phi(d(x, Tx)), \forall x, y \in E. \quad (1.7)$$

2. PRELIMINARIES

In a recent paper of Branciari [3], a generalization of Banach [1] was established. In that paper, Branciari [3] employed the following contractive integral inequality condition: there exist $\alpha \in [0, 1[$ such that $\forall x, y \in E$, we have

$$\int_0^{d(Tx, Ty)} \varphi(t)d(t) \leq \alpha \int_0^{d(x, y)} \varphi(t)d(t), \quad (2.1)$$

where $\varphi : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is a Lebesgue-integrable mapping which is summable, nonnegative and such that for each $\varepsilon > 0, \int_0^{\varepsilon} \varphi(t)dv(t) > 0$.

In 2010, Olatinwo [8] introduced the following contractive integral inequality condition: there exist a real number $\alpha \in [0, 1[$ and monotone increasing functions $v, \psi : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ such that $\psi(0) = 0$ and $\forall x, y \in E$, we have

$$\int_0^{d(Tx, Ty)} \varphi(t) dv(t) \leq \psi \left(\int_0^{d(x, Tx)} \varphi(t) dv(t) \right) + \alpha \int_0^{d(x, y)} \varphi(t) dv(t), \quad (2.2)$$

where $\varphi : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ in both cases is as defined in (2.1).

Remark 2.1. If in condition (2.2):

- i) we have $\varphi(t) = 1$ and $v(t) = t$ then we get condition (1.6).
- ii) we have $\varphi(t) = 1$ and $v(t) = t$ and $\psi(u) = Lu, L \geq 0, \forall u \in \mathbb{R}_+$, then we obtain condition (1.5).
- iii) we have $\psi(u) = 0, \forall u \in \mathbb{R}_+$, then we obtain condition (2.1).

Following Branciari [3] and Olatinwo [8], we now state the following contractive conditions of integral type which shall be employed in establishing our results.

For a selfmapping $T : E \rightarrow E$, there exist a constant $\alpha \in [0, 1[$ and monotone increasing functions $\phi, v : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ with $\phi(0) = 1$, such that

$$\int_0^{d(Tx, Ty)} \varphi(t) dv(t) \leq \alpha \phi \left(\int_0^{d(x, Tx)} \varphi(t) dv(t) \right) \int_0^{d(x, y)} \varphi(t) dv(t), \forall x, y \in E, \quad (2.3)$$

where $\varphi : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is a Lebesgue-Stieltjes integrable mapping which is summable, nonnegative and such that for each $\varepsilon > 0, \int_0^\varepsilon \varphi(t) dv(t) > 0$.

Remark 2.2. The contractive condition (2.3) is more general than those considered by Olatinwo [8], Imoru and Olatinwo [6] and several others in the following sense:

i) If in (2.3), we have

$$\phi(u) = \left(\frac{\psi(u)}{\rho} + 1 \right), \rho = \int_0^{d(x, y)} \varphi(t) dv(t) \neq 0, \forall x, y \in E, x \neq y, u \in \mathbb{R}_+,$$

where $\psi : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is also a monotone increasing function such that $\psi(0) = 0$, then we obtain condition (2.2) employed in Olatinwo [8].

ii) If $\phi(u) = 1, \forall u \in \mathbb{R}_+$, then we have the condition (2.1) employed in Branciari [3].

iii) If in condition (2.3), we have $\varphi(t) = 1$ and $v(t) = t$, then we get condition (1.7) employed in Imoru and Olatinwo [6].

iv) If in (2.3)

$$\phi(u) = \left(\frac{\psi(u)}{d(x, y)} + 1 \right), d(x, y) \neq 0, \forall x, y \in E, x \neq y, u \in \mathbb{R}_+,$$

where $\psi : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is also a monotone increasing function such that $\psi(0) = 0$, and

$$\varphi(t) = 1, v(t) = t,$$

then we obtain condition (1.6).

v) If in (2.3)

$$\phi(u) = \left(\frac{Lu}{d(x, y)} + 1 \right), L \geq 0, d(x, y) \neq 0, \forall x, y \in E, x \neq y, u \in \mathbb{R}_+,$$

and

$$\varphi(t) = 1, v(t) = t,$$

then we obtain condition (1.5).

We shall require the following lemmas in the sequel.

Lemma 2.1. (Berinde [2]) If δ is a real number such that $0 < \delta < 1$, and $\{\varepsilon'_n\}_{n=0}^\infty$ is a sequence of positive numbers such that $\lim_{n \rightarrow \infty} \varepsilon'_n = 0$ then for any sequence of positive numbers $\{u_n\}_{n=0}^\infty$ satisfying

$$u_{n+1} \leq \delta u_n + \varepsilon'_n, n = 0, 1, \dots,$$

we have

$$\lim_{n \rightarrow \infty} u_n = 0.$$

Lemma 2.2. (Olatinwo [8]) Let (E, d) be a complete metric space and $\varphi : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ a Lebesgue-Stieltjes integrable mapping which is summable, nonnegative, and such that for each $\varepsilon > 0$, $\int_0^\varepsilon \varphi(t) dv(t) > 0$. Suppose that $\{u_n\}_{n=0}^\infty, \{v_n\}_{n=0}^\infty \subset E$ and $\{a_n\}_{n=0}^\infty \subset]0, 1[$ are sequences such that

$$\left| d(u_n, v_n) - \int_0^{d(u_n, v_n)} \varphi(t) dv(t) \right| \leq a_n,$$

with $\lim_{n \rightarrow \infty} a_n = 0$. Then

$$d(u_n, v_n) - a_n \leq \int_0^{d(u_n, v_n)} \varphi(t) dv(t) \leq d(u_n, v_n) + a_n. \quad (2.4)$$

Remark 2.3. Lemma 2.2 is also applicable in normed linear space setting since metric is induced by norm. That is, we have

$$d(x, y) = \|x - y\|, \forall x, y \in E,$$

whenever we are working in a normed linear space.

3. MAIN RESULTS

We give here our main results.

Theorem 3.1. Let (E, d) be a complete metric space and $T : E \rightarrow E$ a selfmap of E satisfying condition (2.3). Suppose T has a fixed point p . Let $x_0 \in E$ and let

$$x_{n+1} = Tx_n, n = 0, 1, \dots,$$

be the Picard iteration associated to T . Let $v, \phi : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ be monotone increasing functions such that $\phi(0) = 1$ and $\varphi : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ a Lebesgue-Stieltjes integrable mapping which is summable, nonnegative and such that for each $\varepsilon > 0$, $\int_0^\varepsilon \varphi(t) dv(t) > 0$.

Then, the Picard iteration process is T -stable.

Proof. Let $\{y_{n=0}^\infty\} \subset E$ and $\varepsilon_n = d(y_{n+1}, Ty_n)$, and suppose $\lim_{n \rightarrow \infty} \varepsilon_n = 0$. Then, we shall establish that $\lim_{n \rightarrow \infty} y_n = p$. Then, by using condition (2.3), Lemma 2.2 and the triangle inequality as follows. Let $\{a_n\}_{n=0}^\infty \subset]0, 1[$. Then,

$$\int_0^{d(y_{n+1}, p)} \varphi(t) dv(t) \leq d(y_{n+1}, p) + a_n$$

$$\begin{aligned}
&\leq d(y_{n+1}, Ty_n) + d(Ty_n, p) + a_n \\
&\leq \int_0^{d(y_{n+1}, Ty_n)} \varphi(t) dv(t) + \int_0^{d(Ty_n, p)} \varphi(t) dv(t) + 3a_n \\
&\leq \int_0^{\varepsilon_n} \varphi(t) dv(t) + \alpha \phi \left(\int_0^{d(p, Tp)} \varphi(t) dv(t) \right) \left(\int_0^{d(y_n, p)} \varphi(t) dv(t) \right) + 3a_n \\
&\leq \alpha \phi(0) \left(\int_0^{d(y_n, p)} \varphi(t) dv(t) \right) + \int_0^{\varepsilon_n} \varphi(t) dv(t) + 3a_n,
\end{aligned}$$

therefore

$$\int_0^{d(y_{n+1}, p)} \varphi(t) dv(t) \leq \alpha \int_0^{d(y_n, p)} \varphi(t) dv(t) + \int_0^{\varepsilon_n} \varphi(t) dv(t) + 3a_n. \quad (3.1)$$

We can now express (3.1) in the form $u_{n+1} \leq \delta u_n + \varepsilon'_n$, where

$$0 \leq \delta = \alpha < 1, u_n = \int_0^{d(Ty_n, p)} \varphi(t) dv(t),$$

and

$$\varepsilon'_n = \int_0^{\varepsilon_n} \varphi(t) dv(t) + 3a_n,$$

with

$$\lim_{n \rightarrow \infty} \varepsilon'_n = \lim_{n \rightarrow \infty} \left(\int_0^{\varepsilon_n} \varphi(t) dv(t) + 3a_n \right) = 0,$$

so that by Lemma 2.1 and the fact that $\int_0^\varepsilon \varphi(t) dv(t) > 0$, for each $\varepsilon > 0$ we have

that $\lim_{n \rightarrow \infty} \int_0^{d(Ty_n, p)} \varphi(t) dv(t) = 0$ from which it follows that $\lim_{n \rightarrow \infty} d(Ty_n, p) = 0$, that is $\lim_{n \rightarrow \infty} y_n = p$.

Conversely, let $\lim_{n \rightarrow \infty} y_n = p$. Then, by the contractive condition (2.3), Lemma 2.2 and the triangle inequality again, we have

$$\begin{aligned}
\int_0^{\varepsilon_n} \varphi(t) dv(t) &= \int_0^{d(y_{n+1}, Ty_n)} \varphi(t) dv(t) \\
&\leq d(y_{n+1}, Ty_n) + a_n \\
&\leq d(y_{n+1}, p) + d(p, Ty_n) + a_n \\
&\leq \int_0^{d(y_{n+1}, p)} \varphi(t) dv(t) + \int_0^{d(p, Ty_n)} \varphi(t) dv(t) + 3a_n \\
&\leq \int_0^{d(y_{n+1}, p)} \varphi(t) dv(t) + \alpha \phi \left(\int_0^{d(p, Tp)} \varphi(t) dv(t) \right) \left(\int_0^{d(y_n, p)} \varphi(t) dv(t) \right) + 3a_n \\
&\leq \int_0^{d(y_{n+1}, p)} \varphi(t) dv(t) + \alpha \phi(0) \left(\int_0^{d(y_n, p)} \varphi(t) dv(t) \right) + 3a_n \\
&\leq \int_0^{d(y_{n+1}, p)} \varphi(t) dv(t) + \alpha \int_0^{d(y_n, p)} \varphi(t) dv(t) + 3a_n \rightarrow 0 \text{ as } n \rightarrow \infty.
\end{aligned}$$

Again, using the condition on φ yields $\lim_{n \rightarrow \infty} \varepsilon_n = 0$. \square

Remark 3.1. Theorem 3.1 is a generalization and extension of Theorem 3.1 of Olatinwo [8]. Theorem 3.1 is also a generalization of the results obtained in [5, 6, 2, 4, 11, 12].

Theorem 3.2. Let $(E, \|\cdot\|)$ be a normed linear space and $T : E \rightarrow E$ a selfmapping of E satisfying condition (2.3). Suppose T has a fixed point p . Let $x_0 \in E$, and let

$$x_{n+1} = (1 - \alpha_n)x_n + \alpha_n T(x_n), \alpha_n \in]0, 1], n = 0, 1, \dots$$

be the Mann iteration process such that $0 < \gamma \leq \alpha_n, (n = 0, 1, \dots)$. Let $v, \psi : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ be monotone increasing functions such that $\psi(0) = 0$ and $\varphi : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ a Lebesgue-Stieltjes integrable mapping which is summable, nonnegative and such that for each $\varepsilon > 0, \int_0^\varepsilon \varphi(t)dv(t) > 0$. Then, the Mann iteration process is T -stable.

Proof. Let $\{y_{n=0}^\infty\} \subset E$ and $\varepsilon_n = \|y_{n+1} - (1 - \alpha_n)y_n - \alpha_n T(y_n)\|$, and suppose $\lim_{n \rightarrow \infty} \varepsilon_n = 0$. Then, we shall establish that $\lim_{n \rightarrow \infty} y_n = p$. Then, by using condition (2.3), Lemma 2.3 and the triangle inequality as follows. Let $\{a_n\}_{n=0}^\infty \subset]0, 1[$. Then,

$$\begin{aligned} & \int_0^{\|y_{n+1}-p\|} \varphi(t)dv(t) \leq \|y_{n+1} - p\| + a_n \\ & \leq \|y_{n+1} - (1 - \alpha_n)y_n - \alpha_n T(y_n)\| + \|(1 - \alpha_n)y_n + \alpha_n T(y_n) - p\| + a_n \\ & \leq \varepsilon_n + \|(1 - \alpha_n)y_n + \alpha_n T(y_n) - (1 - \alpha_n + \alpha_n)p\| + a_n \\ & \leq (1 - \alpha_n) \|y_n - p\| + \alpha_n \|T(y_n) - Tp\| + a_n + \varepsilon_n \\ & \leq (1 - \alpha_n) \int_0^{\|y_n-p\|} \varphi(t)dv(t) + \alpha_n \int_0^{\|T(y_n)-Tp\|} \varphi(t)dv(t) + 3a_n + \varepsilon_n \\ & \leq (1 - \alpha_n) \int_0^{\|y_n-p\|} \varphi(t)dv(t) + \alpha_n \alpha \phi \left(\int_0^{\|p-Tp\|} \varphi(t)dv(t) \right) \int_0^{\|y_n-p\|} \varphi(t)dv(t) + 3a_n + \varepsilon_n \\ & \leq (1 - \alpha_n) \int_0^{\|y_n-p\|} \varphi(t)dv(t) + \alpha_n \alpha \phi(0) \int_0^{\|y_n-p\|} \varphi(t)dv(t) + 3a_n + \varepsilon_n \\ & \leq (1 - (1 - \alpha)\alpha_n) \int_0^{\|y_n-p\|} \varphi(t)dv(t) + \varepsilon_n + 3a_n, \end{aligned}$$

therefore

$$\int_0^{\|y_{n+1}-p\|} \varphi(t)dv(t) \leq (1 - (1 - \alpha)\gamma) \int_0^{\|y_n-p\|} \varphi(t)dv(t) + \varepsilon_n + 3a_n. \quad (3.2)$$

We can now express (3.2) in the form $u_{n+1} \leq \delta u_n + \varepsilon'_n$, where

$$0 \leq \delta = 1 - (1 - \alpha)\gamma < 1, u_n = \int_0^{\|y_n-p\|} \varphi(t)dv(t),$$

and

$$\varepsilon'_n = \varepsilon_n + 3a_n,$$

with

$$\lim_{n \rightarrow \infty} \varepsilon'_n = \lim_{n \rightarrow \infty} (\varepsilon_n + 3a_n) = 0,$$

applying Lemma 2.1 in (3.2) yields $\lim_{n \rightarrow \infty} y_n = p$.

Conversely, let $\lim_{n \rightarrow \infty} y_n = p$. Then, by the contractive condition (2.3), Lemma 2.2 and the triangle inequality again, we have

$$\begin{aligned} \int_0^{\varepsilon_n} \varphi(t) dv(t) &= \frac{\|y_{n+1} - (1 - \alpha_n)y_n - \alpha_n T(y_n)\|}{\|T(y_n) - Tp\|} \int_0^{\varepsilon_n} \varphi(t) dv(t) \\ &\leq \|y_{n+1} - (1 - \alpha_n)y_n - \alpha_n T(y_n)\| + a_n \\ &\leq \|y_{n+1} - p\| + \|(1 - \alpha_n + \alpha_n)p - (1 - \alpha_n)y_n - \alpha_n T(y_n)\| + a_n \\ &\leq \|y_{n+1} - p\| + (1 - \alpha_n) \|p - y_n\| + \alpha_n \int_0^{\varepsilon_n} \varphi(t) dv(t) + \alpha_n a_n + a_n \\ &\leq \|y_{n+1} - p\| + (1 - \alpha_n) \|p - y_n\| + \alpha_n a_n + a_n \\ &\quad + \alpha_n \alpha \phi \left(\int_0^{\|p - Tp\|} \varphi(t) dv(t) \right) \int_0^{\|y_n - p\|} \varphi(t) dv(t) \\ &\leq \|y_{n+1} - p\| + (1 - \alpha_n) \|p - y_n\| \\ &\quad + \alpha_n \alpha \int_0^{\|y_n - p\|} \varphi(t) dv(t) + \alpha_n a_n + a_n \rightarrow 0 \text{ as } n \rightarrow \infty. \end{aligned}$$

Again, using the condition on φ yields $\lim_{n \rightarrow \infty} \varepsilon_n = 0$. \square

Remark 3.2. Our Theorem 3.2 of this paper is a generalization of Olatinwo [8]. Theorem 3.2 is also a generalization of the results obtained by Imoru and Olatinwo [6] and this is a further improvement to many existing known results in literature.

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