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SOME KOROVKIN TYPE APPROXIMATION THEOREMS FOR MULTIVARIATE BERNSTEIN TYPE RATIONAL OPERATORS VIA SUMMABILITY METHODS

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Abstract. In this paper, we derive approximation theorems in the multivariate case for rational operators using power series convergence and A-statistical convergence. By selecting a special case of power series convergence and considering A-statistical convergence, we study approximation properties of a non-tensor product BBH type operator, which doesn't converge in the classical sense. Finally, we demonstrate that our new approximation results are stronger than some previously established results.

1. Introduction

Approximation theory and summability theory have important applications in functional analysis, harmonic analysis, partial differential equations, measure theory and probability theory (see; [44]). Korovkin-type theorems play a central role in approximation theory (see; [9]). The use of summability methods in approximation theory contributes studies by providing a more general limit approach for non-convergent sequences or series. After, Gadjiev and Orhan [29] proved Korovkin-type theorems for sequences of positive linear operators via statistical convergence, many researchers have investigated Korovkin-type theorems with various motivations, considering summability methods such as A-statistical convergence, ideal convergence, power series convergence etc. (see; [1]- [6]).

A Korovkin-type theorem was proved by Gadjiev and Çakar in [30] on a subclass of $C[0,\infty)$, which represents the space of continuous and bounded functions on $[0,\infty)$. With the aid of this theorem the uniform approximation of Bleimann Butzer and Hahn (BBH) operator which is a Bernstein type rational operator was obtained, by considering the test functions $\left(\frac{s}{1+s}\right)^{\tau}$ for $\tau = 0,1,2$. Thus, the problem of examining approximation properties of BBH operators with the test functions $\left(\frac{s}{1+s}\right)^{\tau}$ for $\tau = 0,1,2$ was solved, since neither the Korovkin theorem nor the weighted Korovkin theorem proved in [27], [28] could be applied to BBH operators and their

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generalizations (see; [30]). When the classical Korovkin-type theorem didn't work, some extensions of this theorem were proved via A-statistical convergence, ideal convergence, power series statistical convergence (see; [7], [24], [40]).

The main objective of this paper is to give Korovkin type theorems in multivariate case using power series convergence, Abel convergence, Borel convergence and A-statistical convergence. Additionally, we investigate the approximation properties of multivariate BBH-type operators which do not converge in the ordinary sense.

Next, we start to mention about power series convergence and regularity of it [12]:
Let (ρ_j) be a real sequence with $\rho_0 > 0$ and $\rho_1, \rho_2, ... \geq 0$, such that the corresponding power series $\rho_y = \sum_{j=0}^{\infty} \rho_j y^j$ has radius of convergence R with $0 < R \leq \infty$. If for all $y \in (0, R)$,

$$\lim_{y \to R^-} \frac{1}{\rho(y)} \sum_{j=0}^{\infty} x_j \rho_j y^j = L,$$

then we say that $x = (x_j)$ is convergent in the sense of power series method P.

Theorem 1.1. [12]*A power series method P is considered regular if and only if for any $j \in \mathbb{N}_0$*

$$\lim_{0<y \to R^-} \frac{\rho_j y^j}{\rho(y)} = 0.$$

Korovkin-type theorems can be found via power series convergence in [41], [42]. Power series convergence includes the Abel and Borel summability methods. For Abel summability method, Korovkin-type theorems can be found in [43], [45]. Further results in this direction on different spaces can be found [11], [33], [35].
As special cases of power series convergence, we recall the Abel and Borel convergences; respectively.
Assume that $\rho_j = 1$, in this case $R = 1$ and $\rho(y) = \frac{1}{1-y}$. Thus power series convergence reduce to the Abel convergence. Let $x = (x_j)$ be a real sequence. If the series

$$\sum_{j=0}^{\infty} x_j y^j \tag{1.1}$$

is convergent for any $y \in (0, 1)$ and

$$\lim_{y \to 1^-} (1 - y) \sum_{j=0}^{\infty} x_j y^j = \alpha$$

then x is said to be Abel convergent to real number α ([12], [34]).
Assume that $\rho_j = \frac{1}{j!}$, in this case $R = \infty$ and $\rho(y) = e^y$. Thus the power series convergence reduces to the Borel convergence. Let $x = (x_j)$ be a real sequence. If the series

$$\sum_{j=0}^{\infty} x_j y^j \tag{1.2}$$

is convergent for any $y > 0$ and

$$\lim_{y \to \infty} e^{-y} \sum_{j=0}^{\infty} \frac{x_j}{j!} y^j = \alpha$$

then x is said to be Borel convergent to real number α. Borel summability method is regular if and only if it is non-polynomial. ([12], [34])

In [37], Söylemez and Ünver studied an application of Korovkin-type theorems via Abel convergence, considering Cheney Sharma operators. This study allows us the use of weaker conditions than the classic ones, leading to more general results. Further applications of power series convergence for different operators can be found in ([13]-[22])

The remainder of the paper is organized as follows : In Section 2, after recalling some definitions and notations in the multivariate setting, we present the multivariate BBH-type operators (L_n) which do not converge in the classical sense. In Section 3, we demonstrate some Korovkin-type theorems in the multivariate case, taking into account power series convergence and its particular cases; Abel and Borel convergence. In Section 4, as a special case, using the Korovkin type theorem proved in section 3, we establish the Abel convergence of the operators $\{L_n(\phi;x)\}_{n\in\mathbb{N}}$ to $\phi(x)$ on $\acute{S}$ for ϕ belonging to a suitable subspace of continuous functions that denoted by $\hat{H}^d_\omega\left(\acute{S}\right)$. In Section 5, we give a Korovkin-type theorem via A-statistical convergence, and using this theorem, examine the approximation properties of the operators (L_n).

2. Prelimineries

In this section, we mention about the multi index notations and some definitions. Let us consider the set $\acute{S}\subset\mathbb{R}^d,\ d\in\mathbb{N}$, given by

$$\acute{S}=\left\{\mathbf{x}=(x_1,\cdots,x_d)\in\mathbb{R}^d:0\le x_i<\infty,\ 1\le i\le d\right\}.$$

The l_1 norm of $x=(x_1,\cdots,x_d)\in\acute{S}$ is denoted by $|x|=\sum_{i=1}^d x_i$ and the Euclidean norm is given by $\sum\limits_{i=1}^d x_i^2=\|\mathbf{x}\|^2$. Also, $\mathbf{k}=(k_1,\cdots,k_d)\in\mathbb{N}^d\cup\{\mathbf{0}\}$ and $n\in\mathbb{N}$ we have the following representations:

$$|\mathbf{k}|:=k_1+k_2+\cdots+k_d,\quad \mathbf{k}!:=k_1!k_2!\cdots k_d!,$$
$$\mathbf{x}^{\mathbf{k}}:=(x_1^{k_1}x_2^{k_2}\cdots x_d^{k_d}),\ \mathbf{x}\in\mathbb{R}^d,\qquad \alpha\mathbf{x}:=(\alpha x_1,\cdots,\alpha x_d),\ \text{for }\alpha\in\mathbb{R},$$
$$\binom{n}{\mathbf{k}}:=\frac{n!}{\mathbf{k}!(n-|\mathbf{k}|)!}\qquad \sum_{0\le|\mathbf{k}|\le n}:=\sum_{k_1=0}^{n}\sum_{k_2=0}^{n-k_1}\cdots\sum_{k_d=0}^{n-k_1-\cdots-k_{d-1}}.$$

On the other hand, we simply write $\phi(\mathbf{x})=\phi(x_1,\cdots,x_d)$ rather $\phi(x_1,\cdots,x_d)$ for $x=(x_1,\cdots,x_d)\in\acute{S}$, we also write $\mathbf{a}_n x$ rather $(a_{1,n}x_1,a_{2,n}x_2,...,a_{d,n}x_d)$. $\mathbf{1}$ denotes a function such that $\phi(x_1,\cdots,x_d)=1$ and, for any $x=(x_1,\cdots,x_d)$, $y=(y_1,\cdots,y_d)\in\acute{S}$, $x\le y$ means that $x_i\le y_i$ for each $i=1,2,\cdots,d$.

Let $C_B\left(\acute{S}\right)$ denote the space of all real valued continuous and bounded functions defined on $\acute{S}$, equipped with norm

$$\|\phi\|_{C_B}=\sup_{\mathbf{x}\in\acute{S}}|\phi(\mathbf{x})|.$$

Recall that the well-known *total modulus of continuity of a function* $\phi\in C_B\left(\acute{S}\right)$ is defined as

$$\Omega(\phi;\delta):=\sup\left\{|\phi(\mathbf{x})-\phi(\mathbf{t})|:|x_i-t_i|\le\delta_i,\ \mathbf{x},\mathbf{t}\in\acute{S},\ \delta=(\delta_1,\delta_2,\ldots,\delta_d)\in\acute{S}\right\}$$

(see, e.g. [9], [23]).

Below, we give the definition of the general function of modulus of continuity.

Definition 2.1. [10]*A non-negative function* $\omega(\mathbf{u})$ *defined in* $\acute{S}\subset\mathbb{R}^d$ *is called a function of modulus of continuity, if it satisfies the following conditions for any* $\delta=(\delta_1,\cdots,\delta_d)$, $\mu=(\mu_1,\cdots,\mu_d)\in\acute{S}$

(1) $\omega(\delta)$ is continuous for all δ_i, $i=1,\cdots,d$,
(2) $\omega(\mathbf{0})=0$, *where* $\mathbf{0}=(0,0,\cdots,0)$,
(3) $\omega(\delta)=\omega(\delta_1,\cdots,\delta_d)$ *is non-decreasing, i.e.;* $\omega(\delta)\geq\omega(\mu)$ *for* $\delta\geq\mu$,
(4) $\omega(\delta)$ *is sub-additive, i.e.;* $\omega(\delta+\mu)\leq\omega(\delta)+\omega(\mu)$.

Let $\hat{H}^d_\omega\left(\acute{S}\right)$ denote the space of all real valued functions defined on $\acute{S}$ satisfying

$$|\phi(\mathbf{x})-\phi(\mathbf{t})|\leq\omega\left(\left|\frac{x_1}{1+|\mathbf{x}|}-\frac{t_1}{1+|\mathbf{t}|}\right|,\cdots,\left|\frac{x_d}{1+|\mathbf{x}|}-\frac{t_d}{1+|\mathbf{t}|}\right|\right), \tag{2.1}$$

for all $x=(x_1,...,x_d)$, $t=(t_1,...,t_d)\in\acute{S}$. It is easy to see that $\hat{H}^d_\omega\left(\acute{S}\right)\subset C_B\left(\acute{S}\right)$. By letting univariate modulus of continuity in (1.1), we obtain the subclass H_ω defined in [30].

In [39] Söylemez et al. under the above definitions of multiindex notations, constructed non-tensor multivariate BBH operators and show a uniform approximation of these operators in the class $\hat{H}^d_\omega\left(\acute{S}\right)$. In [32], Özarslan et al. introduced a Balázs-type generalization of non tensor bivariate BBH operators and gave a Korovkin-type theorem in the multivariable case for Balázs-type BBH operators, considering a class which was produced by the univariate modulus of continuity function.

Now, we consider the following generalization of BBH-type operators for $\phi\in C_B\left(\acute{S}\right)$ in several variables which is not a tensor product setting:

$$L_n(\phi;\mathbf{x})=\frac{b_n}{(1+|a_n\mathbf{x}|)^n}\sum_{0\leq|\mathbf{k}|\leq n}\binom{n}{\mathbf{k}}(\mathbf{a}_n\mathbf{x})^{\mathbf{k}}\phi\left(\frac{\mathbf{k}}{n+1-|\mathbf{k}|}\right), \tag{2.2}$$

where $x=(x_1,x_2,...,x_d)\in\acute{S}$, $b_n\geq0$, $a_{i,n}\geq0$ for all $i=1,2,...d$, $n\in\mathbb{N}$. By using power series statistical convergence defined in [6] the approximation properties of the operators (2.2) were studied in [40]. Throughout the paper, we use the following test functions:

$$\dot{e}_0(\mathbf{x})=1$$

$$\dot{e}_i(\mathbf{x})=\frac{x_i}{1+|\mathbf{x}|},$$

$$\dot{e}_d(\mathbf{x})=\sum_{i=1}^{d}\left(\frac{x_i}{1+|\mathbf{x}|}\right)^2.$$

3. Main Results

In this section, we prove some Korovkin-type theorems via power series convergence. Firstly, we recall the Korovkin-type theorem given in [30].

Theorem 3.1. [30]*Let* (A_n) *be a sequence of positive linear operators from* $H_\omega \to C_B[0,\infty)$. *Then we have*

$$\lim_{n\to\infty}\|A_n(\phi)-\phi\|_{C_B}=0,$$

if and only if

$$\lim_{n\to\infty}\|A_n(e_i)-e_i\|_{C_B}=0,\ i=0,1,2,$$

where $e_i(x)=\left(\frac{x}{1+x}\right)^i$.

The following three theorems are more useful whenever Theorem 3.1 and multivariate version of it (see; Theorem 2.1 in [39]) can't be used.

Theorem 3.2. *Let* $\{T_n(\phi)\}_{n\in\mathbb{N}}$ *be a sequence of linear positive operators from* $\hat{H}_\omega^d\left(\acute{S}\right)$ *to* $C_B\left(\acute{S}\right)$ *such that* $\sup_{0<y-R}\frac{1}{p(y)}\sum_{n=0}^{\infty}\|T_n(\dot{e}_0)\|p_ny^n<\infty$ *for any* $y\in(0,R)$. *If*

$$\lim_{y\to R^-}\frac{1}{p(y)}\left\|\sum_{n=0}^{\infty}\{T_n(\dot{e}_0)-\dot{e}_0\}p_ny^n\right\|_{C_B}=0, \tag{3.1}$$

$$\lim_{y\to R^-}\frac{1}{p(y)}\left\|\sum_{n=0}^{\infty}\{T_n(\dot{e}_i)-\dot{e}_i\}p_ny^n\right\|_{C_B}=0,\ \textit{for all}\ \ i=1,\cdots,d,d+1$$

are satisfied, then for $\phi\in\hat{H}_\omega^d\left(\acute{S}\right)$ *we have*

$$\lim_{y\to R^-}\frac{1}{p(y)}\left\|\sum_{n=0}^{\infty}(T_n(\phi)-\phi)p_ny^n\right\|=0. \tag{3.2}$$

Proof. Suppose that $\phi\in\hat{H}_\omega^d\left(\acute{S}\right)$ and $x=(x_1,...,x_d)$, $\mathbf{t}=(t_1,...,t_d)$ are any two elements of $\acute{S}$. Then, from the properties of the general function of modulus of continuity, for any given $\epsilon>0$ there exists a $\eta_i>0$ $i=1,2,...,d$ and taking $\eta=\min\{\eta_1,\eta_2,...,\eta_d\}$, we may write

$$|\phi(\mathbf{t})-\phi(\mathbf{x})|<\epsilon \text{ whenever } \left|\frac{t_i}{1+|\mathbf{t}|}-\frac{x_i}{1+|\mathbf{x}|}\right|<\eta\ (i=1,2,...,d). \tag{3.3}$$

Otherwise, if $\left|\frac{t_{i_0}}{1+|\mathbf{t}|}-\frac{x_{i_0}}{1+|\mathbf{x}|}\right|\geq\eta$ for some $i_0\in\{1,2,....,d\}$, then we have

$$\left\|\frac{\mathbf{t}}{1+|\mathbf{t}|}-\frac{\mathbf{x}}{1+|\mathbf{x}|}\right\|=\sqrt{\sum_{i=1}^{d}\left(\frac{t_i}{1+|\mathbf{t}|}-\frac{x_i}{1+|\mathbf{x}|}\right)^2}\geq\left|\frac{t_{i_0}}{1+|\mathbf{t}|}-\frac{x_{i_0}}{1+|\mathbf{x}|}\right|\geq\eta.$$

From the boundedness of ϕ on $\acute{S}$, one has

$$|\phi(\mathbf{t})-\phi(\mathbf{x})|\leq\frac{2\|\phi\|_{C_B}}{\eta^2}\left\|\frac{\mathbf{t}}{1+|\mathbf{t}|}-\frac{\mathbf{x}}{1+|\mathbf{x}|}\right\|^2,$$

when $\left\| \frac{\mathbf{t}}{1+|\mathbf{t}|} - \frac{\mathbf{x}}{1+|\mathbf{x}|} \right\| = \sqrt{\sum_{i=1}^{d} \left(\frac{t_i}{1+|\mathbf{t}|} - \frac{x_i}{1+|\mathbf{x}|} \right)^2} \geq \eta,$ for some $i_0 \in \{1, 2, ..., d\}$.
Hence we obtain for all $\mathbf{x}, \mathbf{t} \in \acute{S}$ that

$$|\phi(\mathbf{t}) - \phi(\mathbf{x})| \leq \epsilon + \frac{2\|\phi\|_{C_B}}{\eta^2} \sum_{i=1}^{d} \left(\frac{t_i}{1+|\mathbf{t}|} - \frac{x_i}{1+|\mathbf{x}|} \right)^2 \tag{3.4}$$
$$= \epsilon + \frac{2\|\phi\|_{C_B}}{\eta^2} \left\| \frac{\mathbf{t}}{1+|\mathbf{t}|} - \frac{\mathbf{x}}{1+|\mathbf{x}|} \right\|^2.$$

If we apply the operators (T_n) to (3.4), then we have for all $y \in (0, R)$ that

$$\begin{aligned} \frac{1}{p(y)} \left| \sum_{n=0}^{\infty} \left(T_n(\phi(\mathbf{t}); \mathbf{x}) - \phi(\mathbf{x}) \right) p_n y^n \right| &\leq \frac{1}{p(y)} \sum_{n=0}^{\infty} \left(T_n(|\phi(\mathbf{t}) - \phi(\mathbf{x})|; \mathbf{x}) \right) p_n y^n \\ &\quad + \|\phi\|_{C_B} \frac{1}{p(y)} \left| \sum_{n=0}^{\infty} \left(T_n(\mathbf{1}; \mathbf{x}) - 1 \right) p_n y^n \right| \\ &=: I_y^1 + I_y^2. \end{aligned}$$

From (3.1) and using the fact that $\|\phi\|_{C_B}$ is finite when $\phi \in \hat{H}_\omega^d \left(\acute{S} \right)$, we obtain

$$\lim_{y \to R^-} I_y^2 = 0$$

$$\lim_{y \to R^-} \|\phi\|_{C_B} \frac{1}{p(y)} \left| \sum_{n=0}^{\infty} \left(T_{n,d}(\mathbf{1}; \mathbf{x}) - 1 \right) p_n y^n \right| = 0.$$

Since the operator (T_n) is linear and positive and from (3.1), we obtain that

$$\begin{aligned} I_y^1 &\leq \epsilon \frac{1}{p(y)} \left| \sum_{n=0}^{\infty} \left(T_n(\mathbf{1}; \mathbf{x}) - 1 \right) p_n y^n \right| + \epsilon \\ &+ \frac{2\|\phi\|_{C_B}}{\eta^2} \frac{1}{p(y)} \sum_{n=0}^{\infty} \left\{ \left| T_n \left(\sum_{i=1}^{d} \left(\frac{t_i}{1+|\mathbf{t}|} \right)^2; \mathbf{x} \right) - \sum_{i=1}^{d} \left(\frac{x_i}{1+|\mathbf{x}|} \right)^2 \right. \right. \\ &- 2 \sum_{i=1}^{d} \frac{x_i}{1+|\mathbf{x}|} \left[T_n \left(\frac{t_i}{1+|\mathbf{t}|}; \mathbf{x} \right) - \frac{x_i}{1+|\mathbf{x}|} \right] \\ &+ \left. \left. \sum_{i=1}^{d} \left(\frac{x_i}{1+|\mathbf{x}|} \right)^2 \left(T_n(\mathbf{1}; \mathbf{x}) - 1 \right) \right\} p_n y^n \right| \\ &\leq \left(\epsilon + \frac{2d\|\phi\|_{C_B}}{\eta^2} \right) \frac{1}{p(y)} \left| \sum_{n=0}^{\infty} \left\{ T_n(\mathbf{1}; \mathbf{x}) - 1 \right\} p_n y^n \right| \\ &+ \frac{4\|\phi\|_{C_B}}{\eta^2} \frac{1}{p(y)} \sum_{i=1}^{d} \left| \sum_{n=0}^{\infty} \left\{ T_n \left(\left(\frac{t_i}{1+|\mathbf{t}|} \right); \mathbf{x} \right) - \left(\frac{x_i}{1+|\mathbf{x}|} \right) \right\} p_n y^n \right| \\ &+ \frac{2\|\phi\|_{C_B}}{\eta^2} \frac{1}{p(y)} \left| \left\{ \sum_{n=0}^{\infty} T_n \left(\left(\sum_{i=1}^{d} \frac{t_i}{1+|\mathbf{t}|} \right)^2; \mathbf{x} \right) - \sum_{i=1}^{d} \left(\frac{x_i}{1+|\mathbf{x}|} \right)^2 \right\} p_n y^n \right|, \end{aligned}$$

for all $y \in (0, R)$. Then, one can have

$$\begin{aligned}
&\frac{1}{p(y)} \left\| \sum_{n=0}^{\infty} \left(T_n(\phi) - \phi\right) p_n y^n \right\|_{C_B} \\
&\leq K \left\{ \frac{1}{p(y)} \left\| \sum_{n=0}^{\infty} \left(T_n(\dot{e}_d) - \dot{e}_d\right) p_n y^n \right\|_{C_B} \right. \\
&+ \frac{1}{p(y)} \left\| \sum_{n=0}^{\infty} \left(T_n(\dot{e}_i) - \dot{e}_i\right) p_n y^n \right\|_{C_B} \\
&\left. + \frac{1}{p(y)} \left\| \sum_{n=0}^{\infty} \left(T_n(\dot{e}_0) - \dot{e}_0\right) p_n y^n \right\|_{C_B} \right\},
\end{aligned}$$

where $K = \max\left\{\epsilon + \frac{2d\|\phi\|_{C_B}}{\eta^2}, \frac{4\|\phi\|_{C_B}}{\eta^2}, \right\}$. Hence, we deduce

$$\lim_{y \to R^-} I_y^1 = 0,$$

which ends the proof. □

We can express Theorem 3.2 with Abel convergence and Borel convergence for particular cases as follows:

Theorem 3.3. *Let $\{T_n(\phi)\}_{n\in\mathbb{N}}$ be a sequence of linear positive operators from $\hat{H}_\omega^d\left(\acute{S}\right)$ to $C_B\left(\acute{S}\right)$ such that $\sum_{n=0}^{\infty} \|T_n(\dot{e}_0)\| y^n < \infty$ for any $y \in (0, 1)$. If*

$$\begin{aligned}
&\lim_{y\to 1^-} (1-y) \left\| \sum_{n=0}^{\infty} \left\{T_n(\dot{e}_0) - \dot{e}_0\right\} y^n \right\|_{C_B} = 0, \\
&\lim_{y\to 1^-} (1-y) \left\| \sum_{n=0}^{\infty} \left\{T_n(\dot{e}_i) - \dot{e}_i\right\} y^n \right\|_{C_B} = 0, \ \textit{for all } i = 1, \cdots, d, d+1
\end{aligned} \tag{3.5}$$

are satisfied, then for $\phi \in \hat{H}_\omega^d\left(\acute{S}\right)$ we have

$$\lim_{y\to 1^-} (1-y) \left\| \sum_{n=0}^{\infty} \left(T_n(\phi) - \phi\right) y^n \right\|_{C_B} = 0.$$

Theorem 3.4. *Let $\{T_n(\phi)\}_{n\in\mathbb{N}}$ be a sequence of linear positive operators from $\hat{H}_\omega^d\left(\acute{S}\right)$ to $C_B\left(\acute{S}\right)$ such that $\sum_{n=0}^{\infty} \|T_n(\dot{e}_0)\| y^n < \infty$ for any $y > 0$. If*

$$\begin{aligned}
&\lim_{y\to \infty} e^{-y} \left\| \sum_{n=0}^{\infty} \left\{T_n(\dot{e}_0) - \dot{e}_0\right\} \frac{y^n}{n!} \right\|_{C_B} = 0, \\
&\lim_{y\to \infty} e^{-y} \left\| \sum_{n=0}^{\infty} \left\{T_n(\dot{e}_i) - \dot{e}_i\right\} \frac{y^n}{n!} \right\|_{C_B} = 0, \ \textit{for all } \ i = 1, \cdots, d, d+1
\end{aligned}$$

are satisfied, then for $\phi \in \hat{H}_\omega^d\left(\acute{S}\right)$ we have

$$\lim_{y\to \infty} e^{-y} \left\| \sum_{n=0}^{\infty} \left(T_n(\phi) - \phi\right) \frac{y^n}{n!} \right\|_{C_B} = 0.$$

The following example shows that power series convergence of the operators holds, but ordinary convergence does not hold. Let $p_j = 1$, in this case $R = 1$ and $p(y) = \frac{1}{1-y}$, and we obtain Abel convergence.

Example 3.5. *We can give the following sequence as an example for the sequences* (b_n) *and* $(a_{i,n})$:

$$a_{i,n} := \begin{cases} 0 & , \quad n \text{ is a prime,} \\ 1 & , \quad \text{otherwise,} \end{cases} \quad \text{for each } i = 1, 2, ...d$$

$$b_n := \begin{cases} 0 & , \quad n \text{ is a perfect square,} \\ 1 & , \quad \text{otherwise.} \end{cases}$$

Observe that the sequences (b_n) *and* $(a_{i,n})$ *are not convergent, but they are Abel convergent (they are bounded and statistically convergent) (*[34], [36]*).*

4. A Particular case of Power Series Convergence

In this section, we study a particular case of power series convergence to give more detailed examples. By selecting the Abel convergence, we investigate the approximation properties of the operators (L_n) under the following conditions:

$$\begin{aligned} \lim_{y \to 1^-} (1-y) \sum_{n=0}^{\infty} \left|1 - a_{i,n}\right| y^n &= 0, \text{ for all } i = 1, 2, ...d \\ \lim_{y \to 1^-} (1-y) \sum_{n=0}^{\infty} \left|1 - b_n\right| y^n &= 0, \\ \lim_{y \to 1^-} (1-y) \sum_{n=0}^{\infty} \left|1 - (a_{i,n})^2 \frac{n(n-1)}{(n+1)^2} b_n\right| y^n &= 0, \text{ for all } i = 1, 2, ...d \\ \lim_{y \to 1^-} (1-y) \sum_{n=0}^{\infty} \left|1 - b_n \frac{n}{n+1} (a_{i,n})\right| y^n &= 0, \text{ for all } i = 1, 2, ...d. \end{aligned} \tag{4.1}$$

Before studying the promised approximation properties of these operators, we give the following lemma which can be proved as in [39]:

Lemma 4.1.

$$L_n(\mathbf{1}; \mathbf{x}) = b_n \tag{4.2}$$

$$L_n\left(\frac{t_i}{1+|\mathbf{t}|}; \mathbf{x}\right) = b_n \frac{n}{n+1} \left(\frac{a_{i,n} x_i}{1 + |\mathbf{a}_n \mathbf{x}|}\right), \quad \textit{for} \quad i = 1, \cdots, d \tag{4.3}$$

$$L_n\left(\sum_{i=1}^{d} \left(\frac{t_i}{1+|\mathbf{t}|}\right)^2; \mathbf{x}\right) = b_n \sum_{i=1}^{d} \left\{ \frac{n(n-1)}{(n+1)^2} \left(\frac{a_{i,n} x_i}{1 + |\mathbf{a}_n \mathbf{x}|}\right)^2 + \frac{n}{(n+1)^2} \frac{a_{i,n} x_i}{1 + |\mathbf{a}_n \mathbf{x}|} \right\}. \tag{4.4}$$

Lemma 4.2. *The following inequalities hold for the operators (2.2)*
i)

$$\left\| \sum_{n=0}^{\infty} \left(L_n(\dot{e}_0) - \dot{e}_0\right) y^n \right\|_{C_B} = \sum_{n=0}^{\infty} \left|1 - b_n\right| y^n.$$

ii)

$$\left\|\sum_{n=0}^{\infty}\left(L_n\left(\dot{e}_i\right)-\dot{e}_i\right)y^n\right\|_{C_B}$$
$$\leq\sum_{n=0}^{\infty}b_n\left(\sum_{i=1}^{d}\left|1-a_{i,n}\right|\right)y^n+\sum_{n=0}^{\infty}\left|b_n\frac{n}{n+1}\left(a_{i,n}\right)-1\right|y^n.$$

ii)

$$\left\|\sum_{n=0}^{\infty}\left(L_n\left(\dot{e}_d\right)-\dot{e}_d\right)y^n\right\|_{C_B}$$
$$\leq\sum_{n=0}^{\infty}db_n\left(\sum_{i=1}^{d}\left|1-a_{i,n}\right|\right)\left(d+\left|\mathbf{a}_n\right|\right)y^n+2d\sum_{n=0}^{\infty}b_n\left(\sum_{i=1}^{d}\left|1-a_{i,n}\right|\right)y^n$$
$$+d\sum_{n=0}^{\infty}\left(\sum_{i=1}^{d}\left|b_n\frac{n\left(n-1\right)}{\left(n+1\right)^2}\left(a_{i,n}\right)^2-1\right|\right)y^n+d\sum_{n=0}^{\infty}b_n\frac{n}{\left(n+1\right)^2}y^n.$$

Proof of Lemma 4.2 can be obtained Theorem 3.1 in [40]. In the following theorem, we show the Abel convergence of the sequence of the multivariate Bleimann, Butzer and Hahn-type operators $L_n(\phi)$ to the function $\phi\in\hat{H}_{\omega}^{d}\left(\acute{S}\right)$ on $\acute{S}$.

Theorem 4.3. *Let (L_n) be the operator defined by (2.2) and suppose that $\left\{b_n\frac{n}{(n+1)^2}\right\}_0^{\infty}$ is Abel null, (b_n) and $(a_{i,n})$ hold the condition* (4.1)*, for each $i=1,2,3,...,d$. Then for any $\phi\in\hat{H}_{\omega}^{d}\left(\acute{S}\right)$ we have*

$$\lim_{y\to1^-}\left(1-y\right)\left\|\sum_{n=0}^{\infty}\left(L_n\left(\phi\right)-\phi\right)y^n\right\|_{C_B}=0.$$

Proof. By using Theorem 3.3, we prove (3.5) holds for the operator (L_n). Indeed, from Lemma 4.1 and Lemma 4.2 (i) by the hypothesis, we have

$$\lim_{y\to1^-}\left(1-y\right)\left\|\sum_{n=0}^{\infty}\left(L_n\left(\mathbf{1};\mathbf{x}\right)-1\right)y^n\right\|_{C_B}=0.$$

Moreover, from Lemma 4.2 (ii), we have for all $i=1,2,3,...,d$

$$\left\|\sum_{n=0}^{\infty}\left(L_n\left(\dot{e}_i\right)-\dot{e}_i\right)y^n\right\|_{C_B}$$
$$\leq\sum_{n=0}^{\infty}b_n\left(\sum_{i=1}^{d}\left|1-a_{i,n}\right|\right)y^n+\sum_{n=0}^{\infty}\left|b_n\frac{n}{n+1}\left(a_{i,n}\right)-1\right|y^n.$$

By using (4.1) and the hypothesis, we reach to

$$\lim_{y\to1^-}\left(1-y\right)\left\|\sum_{n=0}^{\infty}\left(L_n\left(\dot{e}_i\right)-\dot{e}_i\right)y^n\right\|_{C_B}=0,\ i=1,\cdots,d.$$

Finally, from Lemma 4.2 (iii), we obtain that

$$\begin{aligned}
&\left\|\sum_{n=0}^{\infty}\left(L_n\left(\dot{e}_d\right)-\dot{e}_d\right)y^n\right\|_{C_B}\\
&\leq \sum_{n=0}^{\infty} db_n\left(\sum_{i=1}^{d}\left|1-a_{i,n}\right|\right)\left(d+\left|\mathbf{a}_n\right|\right)y^n+2d\sum_{n=0}^{\infty}b_n\left(\sum_{i=1}^{d}\left|1-a_{i,n}\right|\right)y^n\\
&+d\sum_{n=0}^{\infty}\left(\sum_{i=1}^{d}\left|b_n\frac{n\left(n-1\right)}{\left(n+1\right)^2}\left(a_{i,n}\right)^2-1\right|\right)y^n+d\sum_{n=0}^{\infty}b_n\frac{n}{\left(n+1\right)^2}y^n.
\end{aligned}$$

From (4.1), we get

$$\lim_{y\to 1^-}\left(1-y\right)\left\|\sum_{n=0}^{\infty}\left(L_n\left(\dot{e}_d\right)-\dot{e}_d\right)y^n\right\|_{C_B}=0.$$

Thus, we reach to the desired result. □

5. A -Statistical convergence

In this section, taking into account A-statistical convergence, we give a Korovkin type theorem.

Now, we recall that for an infinite non-negative regular summability matrix $A = (a_{jn})$ a real sequence $x := (x_n)$ is called A-statistically convergent to a number L if, for every $\varepsilon > 0$, $\lim_{j\to\infty}\sum_{n:|x_n-L|\geq\varepsilon}a_{jn} = 0$ holds. Then we say that a real sequence $x = (x_n)$ is said to be *A-statistically convergent* (see, [25], [26]) to a real number L. If A is the identity matrix I, then I-statistical convergence reduces to the classical convergence, and, if $A = C_1$, the Cesáro matrix of order one, then it coincides with statistical convergence. Some Korovkin type approximation results obtained using A-statistical convergence may be found [8], [31], [35], [38]. In [24], a Korovkin-type theorem was obtained to prove A-statistical convergence of tensor product multivariate BBH operators which have a different construction from the operators (2.2).

For the sequence of the operators (2.2) we can not use Theorem 3.1 and multivariate version of it proved in [39], therefore it may be benefical to consider the following theorem.

Theorem 5.1. *Let $A = (a_{jn})$ be a nonnegative regular summability matrix and $\{T_n(\phi)\}_{n\in\mathbb{N}}$ be a sequence of linear positive operators from $\hat{H}_\omega^d\left(\acute{S}\right)$ to $C_B\left(\acute{S}\right)$, $st_A-\lim_{n\to\infty}b_n = 1$, $st_A-\lim_{n\to\infty}a_{i,n} = 1$ for all $i = 1, 2, ..., d$. If*

$$\begin{aligned}
&st_A-\lim\nolimits_{n\to\infty}\left\|T_n\left(\dot{e}_0\right)-\dot{e}_0\right\|_{C_B}=0,\\
&st_A-\lim\nolimits_{n\to\infty}\left\|T_n\left(\dot{e}_i\right)-\dot{e}_i\right\|_{C_B}=0,\ \textit{for all}\ \ i=1,\cdots,d,d+1
\end{aligned}\tag{5.1}$$

are satisfied, then for $\phi\in\hat{H}_\omega^d\left(\acute{S}\right)$ we have

$$st_A-\lim_{n\to\infty}\left\|T_n\left(\phi\right)-\phi\right\|_{C_B}=0.$$

Proof. From (3.4), we deduce

$$\begin{aligned}
&\left|T_n\left(\phi\left(\mathbf{t}\right);\mathbf{x}\right)-\phi\left(\mathbf{x}\right)\right| \\
&\leq \varepsilon+\left(\varepsilon+\left\|\phi\right\|_{C_B}+\frac{2d\left\|\phi\right\|_{C_B}}{\eta^2}\right)\left|T_n\left(\dot{e}_0\right)-\dot{e}_0\right| \\
&+\frac{4\left\|\phi\right\|_{C_B}}{\eta^2}\sum_{i=1}^{d}\left|T_n\left(\dot{e}_i\right)-\dot{e}_i\right|+\frac{2\left\|\phi\right\|_{C_B}}{\eta^2}\left|T_n\left(\dot{e}_d\right)-\dot{e}_d\right|,
\end{aligned}$$

which implies

$$\left\|T_n\left(\phi\right)-\phi\right\|_{C_B}\leq\varepsilon+K\left\{\sum_{i=0}^{d+1}\left\|T_n\left(\dot{e}_i\right)-\dot{e}_i\right\|_{C_B}\right\},$$

where $K=\max\left\{\varepsilon+\left\|\phi\right\|_{C_B}+\frac{2d\|\phi\|_{C_B}}{\eta^2},\frac{4\|\phi\|_{C_B}}{\eta^2}\right\}$.
For a given $s>0$ we select $\varepsilon>0$ such that $\varepsilon<s$. Next, we establish the following sets

$$\begin{aligned}
\grave{U}&:=\left\{n\in\mathbb{N}:\left\|T_n\left(\phi\right)-\phi\right\|_{C_B}\geq s\right\}, \\
\acute{U}_i&:=\left\{n\in\mathbb{N}:\left\|T_n\left(\dot{e}_i\right)-\dot{e}_i\right\|_{C_B}\geq\frac{s-\varepsilon}{K(d+2)}\right\}, i=0,1,2,...,d+1.
\end{aligned}$$

Then, by (5.1), we have $\acute{U}\subset\left(\bigcup_{i=0}^{d+1}\acute{U}_i\right)$. Hence, for all $n\in\mathbb{N}$,

$$\sum_{n\in\acute{U}}a_{jn}\leq\left(\sum_{n\in\acute{U}_1}a_{jn}+\sum_{n\in\acute{U}_2}a_{jn}+\sum_{n\in\acute{U}_3}a_{jn}+...+\sum_{n\in\acute{U}_{d+1}}a_{jn}\right)$$

letting $j\to\infty$ and we get from (5.1) that

$$\lim_{j\to\infty}\sum_{n\in\acute{U}}a_{jn}=0,$$

which ends the proof. □

In the following, we study aproximation properties of the operators (2.2) via A-statistical convergence.

Theorem 5.2. *Let $A=(a_{jn})$ be a nonnegative regular summability matrix and $\{L_n(\phi)\}_{n\in\mathbb{N}}$ be a sequence of linear positive operators defined in (2.2) from $\hat{H}_\omega^d\left(\acute{S}\right)$ to $C_B\left(\acute{S}\right)$, $st_A-\lim_{n\to\infty}b_n=1$, $st_A-\lim_{n\to\infty}a_{i,n}=1$ for all $i=1,2,...,d$. If*

$$\begin{aligned}
&st_A-\lim\nolimits_{n\to\infty}\left\|L_n\left(\dot{e}_0\right)-\dot{e}_0\right\|_{C_B}=0, \\
&st_A-\lim\nolimits_{n\to\infty}\left\|L_n\left(\dot{e}_i\right)-\dot{e}_i\right\|_{C_B}=0,\ \textit{for all}\ \ i=1,\cdots,d,d+1
\end{aligned}\tag{5.2}$$

are satisfied, then for $\phi\in\hat{H}_\omega^d\left(\acute{S}\right)$ we have

$$st_A-\lim_{n\to\infty}\left\|L_n\left(\phi\right)-\phi\right\|_{C_B}=0.$$

Proof. By using the Theorem 5.1, it is enough to show that (5.2) holds for (L_n). From Lemma 4.2 (ii), we have

$$\begin{aligned}&\left\|L_n\left(\dot{e}_i\right)-\dot{e}_i\right\|_{C_B}\\&\leq b_n\left(\sum_{i=1}^{d}\left|1-a_{i,n}\right|\right)+\left|b_n\frac{n}{n+1}\left(a_{i,n}\right)-1\right|.\end{aligned}$$

for all $i=1,2,...,d$. Next, we establish the following sets for any $\varepsilon>0$,

$$N_i:=\left\{n\in\mathbb{N}:\left\|L_n\left(\dot{e}_i\right)-\dot{e}_i\right\|_{C_B}\geq\varepsilon\right\},$$

$$N_i^1:=\left\{n\in\mathbb{N}:\left|b_n\left(\sum_{i=1}^{d}\left|1-a_{i,n}\right|\right)\right|\geq\frac{\varepsilon}{2}\right\},$$

$$N_i^2:=\left\{n\in\mathbb{N}:\left|b_n\frac{n}{n+1}\left(a_{i,n}\right)-1\right|\geq\frac{\varepsilon}{2}\right\},$$

for $i=1,\cdots,d$, it is clear that $N_i\subset N_i^1\cup N_i^2$. Therefore, we can write

$$\sum_{n\in N_i}a_{jn}\leq\left(\sum_{n\in N_i^1}a_{jn}+\sum_{n\in N_i^2}a_{jn}\right)$$

letting $j\to\infty$ and we get from the hypothesis that

$$st_A-\lim_{n\to\infty}\left\|L_n\left(\dot{e}_i\right)-\dot{e}_i\right\|_{C_B}=0,$$

for $i=1,\cdots,d$. Moreover, by using Lemma 4.2 (iii), we reach to

$$\begin{aligned}&\left\|L_n\left(\dot{e}_d\right)-\dot{e}_d\right\|_{C_B}\\&=\left|L_n\left(\sum_{i=1}^{d}\left(\frac{t_i}{1+|\mathbf{t}|}\right)^2;\mathbf{x}\right)-\sum_{i=1}^{d}\left(\frac{x_i}{1+|\mathbf{x}|}\right)^2\right|\\&\leq\sum_{i=1}^{d}\left\{db_n\left|1-a_{i,n}\right|\left(d+\left|\mathbf{a}_n\right|\right)+2db_n\left|1-a_{i,n}\right|\right.\\&\left.+d\left|b_n\frac{n\left(n-1\right)}{\left(n+1\right)^2}\left(a_{i,n}\right)^2-1\right|+db_n\frac{n}{\left(n+1\right)^2}\right\}.\end{aligned}$$

Since $st_A-\lim_{n\to\infty}b_n$, $st_A-\lim_{n\to\infty}\frac{n(n-1)}{(n+1)^2}=1$ and $st_A-\lim\limits_{n\to\infty}a_{i,n}=1$ for all $i=1,2,...,d$, observe that $st_A-\lim\limits_{n\to\infty}b_n\frac{n(n-1)}{(n+1)^2}\left(a_{i,n}\right)^2=1$. Now, we define the following sets for any $\varepsilon>0$ that

$$K:=\left\{n\in\mathbb{N}:\left\|\left(L_n\left(\dot{e}_d\right)-\dot{e}_d\right)\right\|_{C_B}\geq\varepsilon\right\},$$

$$K_i^1:=\left\{n\in\mathbb{N}:\left|db_n\left|1-a_{i,n}\right|\left(d+\left|\mathbf{a}_n\right|\right)\right|\geq\frac{\varepsilon}{4d}\right\},$$

$$K_i^2:=\left\{n\in\mathbb{N}:2db_n\left|1-a_{i,n}\right|\geq\frac{\varepsilon}{4d}\right\},$$

$$K_i^3:=\left\{n\in\mathbb{N}:\left|b_n\frac{n\left(n-1\right)}{\left(n+1\right)^2}\left(a_{i,n}\right)^2-1\right|\geq\frac{\varepsilon}{4d}\right\},$$

$$K_i^4:=\left\{n\in\mathbb{N}:b_n\frac{n}{\left(n+1\right)^2}\geq\frac{\varepsilon}{4d}\right\},$$

we easily see that

$$K \subset \bigcup_{i=1}^{d} \left(K_i^1 \cup K_i^2 \cup K_i^3 \cup K_i^4 \right)$$

which yields

$$\sum_{n \in K} a_{jn} \le \sum_{i=1}^{d} \left(\sum_{n \in K_i^1} a_{jn} + \sum_{n \in K_i^2} a_{jn} + \sum_{n \in K_i^3} a_{jn} + \sum_{n \in K_i^4} a_{jn} \right)$$

letting $j \to \infty$ and we get from the hypothesis that

$$\lim_{j \to \infty} \sum_{n \in K} a_{jn} = 0,$$

it follows that

$$st_A - \lim_{n \to \infty} \| L_n (\dot{e}_d) - \dot{e}_d \|_{C_B} = 0.$$

Thus, the proof is completed. □

The example below demonstrates the existence of a sequence (b_n), $(a_{i,n})$ where A-statistical convergence holds, but classical convergence does not hold.

Example 5.3. *Let (b_n) and $(a_{i,n})$ be the sequences defined by*

$$b_n = \begin{cases} \frac{1}{2} & , \quad \text{if } n \text{ is a perfect square,} \\ 1 + \frac{1}{n} & , \quad \text{otherwise,} \end{cases}$$

$$a_{i,n} = \begin{cases} \frac{1}{2} & , \quad \text{if } n \text{ is a perfect square,} \\ e^{-\frac{i}{n}} & , \quad \text{otherwise.} \end{cases} \quad \text{for each } i = 1, 2, ..., d.$$

It is easy to see that (b_n) and $(a_{i,n})$ are not convergent, but they are statistically convergent, i.e., C_1-statistically convergent.

6. Concluding Remarks

This paper presents several Korovkin-type theorems by utilizing power series convergence and A-statistical convergence, which are stronger than Theorem 3.1. Note that Theorem 3.1 can not be used for the operators (2.2), but these theorems can be used. By selecting, Abel convergence as a special case, we can give more detail example in Example 3.5.

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