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SOME GENERALIZED CONVERGENCE TYPES USING IDEALS IN AMENABLE SEMIGROUPS

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Abstract. The aim of this study is to introduce the concepts of $\mathcal{I}$-statistically convergence, $\mathcal{I}$-statistically pre-Cauchy sequence and $\mathcal{I}$-strongly p-summability for functions defined on discrete countable amenable semigroups and to examine some properties of these concepts.

1. Introduction and Background

The concept of statistical convergence was introduced by Fast [7] and this concept has been studied by Šalát [16], Fridy [8], Connor [1] and many others. The idea of $\mathcal{I}$-convergence which is a generalization of statistical convergence was introduced by Kostyrko et al. [9] which is based on the structure of the ideal $\mathcal{I}$ of subset of the set natural numbers $\mathbb{N}$. Then, Das et al. [2] introduced a new notion, namely $\mathcal{I}$-statistical convergence by using ideal. Recently, Das and Savaş [3] introduced the notion of $\mathcal{I}$-statistically pre-Cauchy sequence.

The concepts of summability in amenable semigroups were studied in [5, 6, 10, 11]. In [13], Nuray and Rhoades introduced the notions of convergence and statistical convergence in amenable semigroups. Also, the notion of almost statistical convergence in amenable semigroups studied by Nuray and Rhoades [14]. Furthermore, the concepts of asymptotically statistical equivalent functions defined on amenable semigroups investigated by Nuray and Rhoades [14].

The aim of this study is to introduce the concepts of $\mathcal{I}$-statistically convergence, $\mathcal{I}$-statistically pre-Cauchy sequence and $\mathcal{I}$-strongly p-summability for functions defined on discrete countable amenable semigroups and to examine some properties of these concepts. For the particular case; when the amenable semigroup is the additive positive integers, our definitions and theorems yield the results of [2, 3].

Let G be a discrete countable amenable semigroup with identity in which both right and left cancelation laws hold, and $w(G)$ and $m(G)$ denote the spaces of all real valued functions and all bounded real functions on G, respectively. $m(G)$ is a Banach space with the supremum norm $\|f\|_\infty = \sup\{|f(g)| : g \in G\}$. Namioka [12] showed that, if G is a countable amenable group, there exists a sequence $\{S_n\}$ of finite subsets of G such that

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i) $G = \bigcup_{n=1}^{\infty} S_n$,
ii) $S_n \subset S_{n+1} \quad (n = 1, 2, ...)$,
iii) $\lim\limits_{n\to\infty} \frac{|S_n g \cap S_n|}{|S_n|} = 1, \quad \lim\limits_{n\to\infty} \frac{|g S_n \cap S_n|}{|S_n|} = 1,$

for all $g \in G$, where $|A|$ denotes the number of elements the finite set A.

Any sequence of finite subsets of G satisfying $(i), (ii)$ and (iii) is called a Folner sequence for G.

The sequence $S_n = \{0, 1, 2, ..., n-1\}$ is a familiar Folner sequence giving rise to the classical Cesàro method of summability.

Now, we recall the basic definitions and concepts (See, [2, 8, 9, 13]).

A sequence $x = (x_k)$ is statistically convergent to L if for every $\varepsilon > 0$

$$\lim_{n\to\infty} \frac{1}{n} \Big| \big\{ k \le n : |x_k - L| \ge \varepsilon \big\} \Big| = 0.$$

A family of sets $\mathcal{I} \subseteq 2^{\mathbb{N}}$ is called an ideal if and only if

(i) $\emptyset \in \mathcal{I}$, (ii) For each $A, B \in \mathcal{I}$ we have $A \cup B \in \mathcal{I}$, (iii) For each $A \in \mathcal{I}$ and each $B \subseteq A$ we have $B \in \mathcal{I}$.

An ideal is called non-trivial if $\mathbb{N} \notin \mathcal{I}$ and non-trivial ideal is called admissible if $\{n\} \in \mathcal{I}$ for each $n \in \mathbb{N}$. All ideals in this paper are assumed to be admissible.

A sequence $x = (x_k)$ is said to be $\mathcal{I}$-convergent to L if for every $\varepsilon > 0$

$$A(\varepsilon) = \big\{ k \in \mathbb{N} : |x_k - L| \ge \varepsilon \big\} \in \mathcal{I}.$$

A sequence $x = (x_k)$ is said to be $\mathcal{I}$-statistically convergent to L if for every $\varepsilon > 0$ and $\delta > 0$

$$\left\{ n \in \mathbb{N} : \frac{1}{n} \Big| \big\{ k \le n : |x_k - L| \ge \varepsilon \big\} \Big| \ge \delta \right\} \in \mathcal{I}.$$

A sequence $x = (x_k)$ is said to be $\mathcal{I}$-statistical pre-Cauchy if for any $\varepsilon > 0$ and $\delta > 0$

$$\left\{ n \in \mathbb{N} : \frac{1}{n^2} \big| \big\{ (j,k) : |x_j - x_k| \ge \varepsilon, \ j, k \le n \big\} \big| \ge \delta \right\} \in \mathcal{I}.$$

Let G be a discrete countable amenable semigroup with identity in which both right and left cancelation laws hold. A function $f \in w(G)$ is said to be convergent to s for any Folner sequence $\{S_n\}$ for G if, for every $\varepsilon > 0$ there exists a $k_0 \in \mathbb{N}$ such that $|f(g) - s| < \varepsilon$, for all $m > k_0$ and $g \in G \setminus S_m$.

Let G be a discrete countable amenable semigroup with identity in which both right and left cancelation laws hold. A function $f \in w(G)$ is said to be a Cauchy sequence for any Folner sequence $\{S_n\}$ for G if, for every $\varepsilon > 0$ there exists a $k_0 \in \mathbb{N}$ such that $|f(g) - f(h)| < \varepsilon$, for all $m > k_0$ and $g, h \in G \setminus S_m$.

Let G be a discrete countable amenable semigroup with identity in which both right and left cancelation laws hold. A function $f \in w(G)$ is said to be strongly summable to s for any Folner sequence $\{S_n\}$ for G if

$$\lim_{n\to\infty} \frac{1}{|S_n|} \sum_{g \in S_n} |f(g) - s| = 0.$$

Let G be a discrete countable amenable semigroup with identity in which both right and left cancelation laws hold and $0 < p < \infty$. A function $f \in w(G)$ is said to be strongly p-summable to s for any Folner sequence $\{S_n\}$ for G if

$$\lim_{n\to\infty} \frac{1}{|S_n|} \sum_{g \in S_n} |f(g) - s|^p = 0.$$

The upper and lower Folner densities of a set $S \subset G$ are defined by

$$\overline{\delta}(S) = \limsup_{n\to\infty} \frac{1}{|S_n|}\big|\{g \in S_n : g \in S\}\big|$$

and

$$\underline{\delta}(S) = \liminf_{n\to\infty} \frac{1}{|S_n|}\big|\{g \in S_n : g \in S\}\big|,$$

respectively. If $\overline{\delta}(S) = \underline{\delta}(S)$, then

$$\delta(S) = \lim_{n\to\infty} \frac{1}{|S_n|}\big|\{g \in S_n : g \in S\}\big|,$$

is called Folner density of S. Take $G = \mathbb{N}$, $S_n = \{0, 1, 2, ..., n-1\}$ and S be the set of positive integers with leading digit 1 in the decimal expansion. The set S has no Folner density with respect to the Folner sequence $\{S_n\}$, since $\underline{\delta}(S) = \frac{1}{9}$ and $\overline{\delta}(S) = \frac{5}{9}$.

Let G be a discrete countable amenable semigroup with identity in which both right and left cancelation laws hold. A function $f \in w(G)$ is said to be statistically convergent to s for any Folner sequence $\{S_n\}$ for G if, for every $\varepsilon > 0$

$$\lim_{n\to\infty} \frac{1}{|S_n|}\Big|\big\{g \in S_n : |f(g) - s| \geq \varepsilon\big\}\Big| = 0.$$

Let G be a discrete countable amenable semigroup with identity in which both right and left cancelation laws hold. A function $f \in w(G)$ is said to be a statistically Cauchy sequence for any Folner sequence $\{S_n\}$ for G if, for every $\varepsilon > 0$ and $m \geq 0$, there exists an $h \in G \setminus S_m$ such that

$$\lim_{n\to\infty} \frac{1}{|S_n|}\Big|\big\{g \in S_n : |f(g) - f(h)| \geq \varepsilon\big\}\Big| = 0.$$

2. Main Results

In this section, we introduced the concepts of $\mathcal{I}$-statistically convergence, $\mathcal{I}$-statistically pre-Cauchy sequence and $\mathcal{I}$-strongly p-summability for functions defined on discrete countable amenable semigroups and examined some properties of these concepts.

Definition 2.1. *Let G be a discrete countable amenable semigroup with identity in which both right and left cancelation laws hold. $f \in w(G)$ is said to be $\mathcal{I}$-statistical convergent to s, for any Folner sequence $\{S_n\}$ for G, if for every $\varepsilon, \delta > 0$*

$$\left\{n \in \mathbb{N} : \frac{1}{|S_n|}\Big|\big\{g \in S_n : |f(g) - s| \geq \varepsilon\big\}\Big| \geq \delta\right\} \in \mathcal{I}.$$

In this case we write $f \xrightarrow{S_{\mathcal{I}}} s$.

The set of all $\mathcal{I}$-statistical convergent functions will be denoted $S_{\mathcal{I}}(G)$.

Theorem 2.2. *The $\mathcal{I}$-statistical convergence of $f \in w(G)$ depends on the particular choice of the Folner sequence.*

By assuming $\mathcal{I} = \mathcal{I}_d$, let us show this by an example.

Example 2.3. *Let $G = \mathbb{Z}^2$ and take two Folner sequences as follows:*

$$\{S_n^1\} = \left\{(i,j) \in \mathbb{Z}^2 : |i| \le n, |j| \le n\right\} \quad \textit{and} \quad \{S_n^2\} = \left\{(i,j) \in \mathbb{Z}^2 : |i| \le n, |j| \le n^2\right\}$$

and define $f(g) \in w(G)$ by

$$f = \begin{cases} 1 \quad , & \textit{if } \ (i,j) \in A, \\ 0 \quad , & \textit{if } \ (i,j) \notin A. \end{cases}$$

where

$$A = \left\{(i,j) \in \mathbb{Z}^2 : i \le j \le n, \ i = 0,1,2,...,n; \ n = 1,2,...\right\}.$$

Since for the Folner sequence $\{S_n^2\}$

$$\lim_{n\to\infty} \frac{1}{|S_n^2|}\left|\left\{g \in S_n^2 : |f(g) - 0| \ge \varepsilon\right\}\right| = \lim_{n\to\infty} \frac{\frac{(n+1)(n+2)}{2}}{(2n+1)(2n^2+1)} = 0,$$

i.e.,

$$\left\{n \in \mathbb{N} : \frac{1}{|S_n^2|}\left|\left\{g \in S_n^2 : |f(g) - 0| \ge \varepsilon\right\}\right| \ge \delta\right\} \in \mathcal{I}_d,$$

then $f(g)$ is $\mathcal{I}$-statistically convergent to 0. But, since for the Folner sequence $\{S_n^1\}$

$$\lim_{n\to\infty} \frac{1}{|S_n^1|}\left|\left\{g \in S_n^1 : |f(g) - 0| \ge \varepsilon\right\}\right| = \lim_{n\to\infty} \frac{\frac{(n+1)(n+2)}{2}}{(2n+1)^2} \ne 0,$$

i.e.,

$$\left\{n \in \mathbb{N} : \frac{1}{|S_n^1|}\left|\left\{g \in S_n^2 : |f(g) - 0| \ge \varepsilon\right\}\right| \ge \delta\right\} \notin \mathcal{I}_d,$$

then $f(g)$ is not $\mathcal{I}$-statistically convergent to 0.

Definition 2.4. *Let G be a discrete countable amenable semigroup with identity in which both right and left cancelation laws hold. $f \in w(G)$ is said to be $\mathcal{I}$-statistical pre-Cauchy, for any Folner sequence $\{S_n\}$ for G, if for every $\varepsilon, \delta > 0$ such that*

$$\left\{n \in \mathbb{N} : \frac{1}{|S_n|^2}\left|\left\{(g,h) \in S_n : |f(g) - f(h)| \ge \varepsilon\right\}\right| \ge \delta\right\} \in \mathcal{I}.$$

Theorem 2.5. *If $f \in w(G)$ is $\mathcal{I}$-statistically convergent for Folner sequence $\{S_n\}$ for G, then it is $\mathcal{I}$-statistically pre-Cauchy for same sequence.*

Proof. Let $f \in w(G)$ be $\mathcal{I}$-statistical convergent to s for Folner sequence $\{S_n\}$ for G. Then for every $\varepsilon, \delta > 0$, we have

$$A_{\varepsilon,\delta} = \left\{n \in \mathbb{N} : \frac{1}{|S_n|}\left|\left\{g \in S_n : |f(g) - s| \ge \frac{\varepsilon}{2}\right\}\right| \ge \delta\right\} \in \mathcal{I}.$$

Hence, for all $n \in A^c_{\varepsilon,\delta}$ where c stands for the complement, we get

$$\frac{1}{|S_n|}\left|\left\{g \in S_n : |f(g) - s| \ge \frac{\varepsilon}{2}\right\}\right| < \delta,$$

i.e.,

$$\frac{1}{|S_n|}\left|\left\{g \in S_n : |f(g) - s| \ge \frac{\varepsilon}{2}\right\}\right| > 1 - \delta,$$

Now, taking

$$B_\varepsilon = \left\{g \in S_n : |f(g) - s| < \frac{\varepsilon}{2}\right\},$$

we observe that for $g,h \in B_\varepsilon$

$$|f(g)-f(h)| \le |f(g)-s| + |f(h)-s| < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon.$$

Then

$$B_\varepsilon \times B_\varepsilon \subset \big\{(g,h) \in S_n : |f(g)-f(h)| < \varepsilon\big\}$$

which implies that

$$\left(\frac{|B_\varepsilon|}{|S_n|}\right)^2 \le \frac{1}{|S_n|^2}\big|\big\{(g,h) \in S_n : |f(g)-f(h)| < \varepsilon\big\}\big|.$$

Thus, for all $n \in A^c_{\varepsilon,\delta}$

$$\frac{1}{|S_n|^2}\big|\big\{(g,h) \in S_n : |f(g)-f(h)| < \varepsilon\big\}\big| \ge \left(\frac{|B_\varepsilon|}{|S_n|}\right)^2 > (1-\delta)^2,$$

i.e.,

$$\frac{1}{|S_n|^2}\big|\big\{(g,h) \in S_n : |f(g)-f(h)| \ge \varepsilon\big\}\big| < 1-(1-\delta)^2.$$

Let $\mu > 0$ be given. Choosing $\delta > 0$ so that $1-(1-\delta)^2 < \mu$, we see that for all $n \in A^c_{\varepsilon,\delta}$

$$\frac{1}{|S_n|^2}\big|\big\{(g,h) \in S_n : |f(g)-f(h)| \ge \varepsilon\big\}\big| < \mu$$

and so

$$\left\{n \in \mathbb{N} : \frac{1}{|S_n|^2}\big|\big\{(g,h) \in S_n : |f(g)-f(h)| \ge \varepsilon\big\}\big| \ge \mu\right\} \subset A_{\varepsilon,\delta}.$$

Since $A_{\varepsilon,\delta} \in \mathcal{I}$, so

$$\left\{n \in \mathbb{N} : \frac{1}{|S_n|^2}\big|\big\{(g,h) \in S_n : |f(g)-f(h)| \ge \varepsilon\big\}\big| \ge \mu\right\} \in \mathcal{I}.$$

Hence, f is $\mathcal{I}$-statistical pre-Cauchy sequence. □

Theorem 2.6. *$f \in m(G)$ is $\mathcal{I}$-statistically pre-Cauchy for Folner sequence $\{S_n\}$ for G if and only if for every $\varepsilon > 0$,*

$$\left\{n \in \mathbb{N} : \frac{1}{|S_n|^2}\sum_{g,h\in S_n}|f(g)-f(h)| \ge \varepsilon\right\} \in \mathcal{I}.$$

Proof. Firstly suppose that for every $\varepsilon > 0$,

$$\left\{n \in \mathbb{N} : \frac{1}{|S_n|^2}\sum_{g,h\in S_n}|f(g)-f(h)| \ge \varepsilon\right\} \in \mathcal{I}.$$

Note that for any $\varepsilon > 0$ and $n \in \mathbb{N}$, we have

$$\frac{1}{|S_n|^2}\sum_{g,h\in S_n}|f(g)-f(h)| \ge \varepsilon\left(\frac{1}{|S_n|^2}\big|\big\{(g,h) \in S_n : |f(g)-f(h)| \ge \varepsilon\big\}\big|\right)$$

Hence, for any $\delta > 0$

$$\left\{n \in \mathbb{N} : \frac{1}{|S_n|^2}\big|\big\{(g,h) \in S_n : |f(g)-f(h)| \ge \varepsilon\big\}\big| \ge \delta\right\}$$
$$\subset \left\{n \in \mathbb{N} : \frac{1}{|S_n|^2}\sum_{g,h\in S_n}|f(g)-f(h)| \ge \delta\varepsilon\right\}.$$

Due to our acceptance, the set on the right hand side belongs to $\mathcal{I}$ which implies that

$$\left\{n \in \mathbb{N} : \frac{1}{|S_n|^2}\big|\{(g,h) \in S_n : |f(g) - f(h)| \geq \varepsilon\}\big| \geq \delta\right\} \in \mathcal{I}.$$

This shows that f is $\mathcal{I}$-statistically pre-Cauchy.

Conversely, suppose that $f \in m(G)$ is $\mathcal{I}$-statistically pre-Cauchy. Since $f \in m(G)$, set $\|f\|_\infty = M$. Let $\delta > 0$ be given. Then, for every $n \in \mathbb{N}$ we can write

$$\frac{1}{|S_n|^2}\sum_{g,h\in S_n}|f(g) - f(h)| \leq \frac{\varepsilon}{2} + 2M\left(\frac{1}{|S_n|^2}\big|\{(g,h) \in S_n : |f(g) - f(h)| \geq \frac{\varepsilon}{2}\}\big|\right).$$

Since f is $\mathcal{I}$-statistically pre-Cauchy, for every $\delta > 0$

$$A_{\varepsilon,\delta} = \left\{n \in \mathbb{N} : \frac{1}{|S_n|^2}\big|\{(g,h) \in S_n : |f(g) - f(h)| \geq \frac{\varepsilon}{2}\}\big| \geq \delta\right\} \in \mathcal{I}.$$

Hence, for all $n \in A^c_{\varepsilon,\delta}$, we get

$$\frac{1}{|S_n|^2}\big|\{(g,h) \in S_n : |f(g) - f(h)| \geq \frac{\varepsilon}{2}\}\big| < \delta$$

and so

$$\frac{1}{|S_n|^2}\sum_{g,h\in S_n}|f(g) - f(h)| \leq \frac{\varepsilon}{2} + 2M\delta.$$

Let $\mu > 0$ be given. Then, choosing $\varepsilon, \delta > 0$ so that $\frac{\varepsilon}{2} + 2M\delta < \mu$ we observe that for all $n \in A^c_{\varepsilon,\delta}$,

$$\frac{1}{|S_n|^2}\sum_{g,h\in S_n}|f(g) - f(h)| < \mu,$$

i.e.,

$$\left\{n \in \mathbb{N} : \frac{1}{|S_n|^2}\sum_{g,h\in S_n}|f(g) - f(h)| \geq \mu\right\} \subset A_{\varepsilon,\delta} \in \mathcal{I}.$$

This completed the proof. □

Definition 2.7. *Let G be a discrete countable amenable semigroup with identity in which both right and left cancelation laws hold. $f \in w(G)$ is said to be $\mathcal{I}$-summable to s, for any Folner sequence $\{S_n\}$ for G, if for every $\varepsilon > 0$*

$$\left\{n \in \mathbb{N} : \left|\frac{1}{|S_n|}\sum_{g\in S_n} f(g) - s\right| \geq \varepsilon\right\} \in \mathcal{I}.$$

In this case we write $f \xrightarrow{C_{\mathcal{I}}} s$.

The set of all $\mathcal{I}$-summable functions will be denoted $C_{\mathcal{I}}(G)$.

Definition 2.8. *Let G be a discrete countable amenable semigroup with identity in which both right and left cancelation laws hold. $f \in w(G)$ is said to be $\mathcal{I}$-strongly summable to s, for any Folner sequence $\{S_n\}$ for G, if for every $\varepsilon > 0$*

$$\left\{n \in \mathbb{N} : \frac{1}{|S_n|}\sum_{g\in S_n}|f(g) - s| \geq \varepsilon\right\} \in \mathcal{I}.$$

In this case we write $f \xrightarrow{[C]_{\mathcal{I}}} s$.

The set of all $\mathcal{I}$-strongly summable functions will be denoted $C_{\mathcal{I}}[G]$.

Definition 2.9. *Let $0<p<\infty$ and G be a discrete countable amenable semigroup with identity in which both right and left cancelation laws hold. $f\in w(G)$ is said to be $\mathcal{I}$-strongly p-summable to s, for any Folner sequence $\{S_n\}$ for G, if for every $\varepsilon>0$*

$$\left\{n\in\mathbb{N}:\frac{1}{|S_n|}\sum_{g\in S_n}|f(g)-s|^p\geq\varepsilon\right\}\in\mathcal{I}.$$

In this case we write $f\overset{[C]_{\mathcal{I}}^p}{\longrightarrow}s$.

The set of all $\mathcal{I}$-strongly p-summable functions will be denoted $C_{\mathcal{I}}^p[G]$.

Theorem 2.10. *Let $0<p<\infty$. If $f\in w(G)$ is $\mathcal{I}$-strongly p-summable to s for Folner sequence $\{S_n\}$ for G, then it is $\mathcal{I}$-statistically convergent to s for same sequence.*

Proof. For any $f\in w(G)$, fix an $\varepsilon>0$. Then, we can write

$$\begin{aligned}\sum_{g\in S_n}|f(g)-s|^p &= \sum_{\substack{g\in S_n\\ |f(g)-s|\geq\varepsilon}}|f(g)-s|^p+\sum_{\substack{g\in S_n\\ |f(g)-s|<\varepsilon}}|f(g)-s|^p\\ &\geq \left|\left\{g\in S_n:|f(g)-s|\geq\varepsilon\right\}\right|\,\varepsilon^p\end{aligned}$$

and so

$$\frac{1}{\varepsilon^p\ |S_n|}\sum_{g\in S_n}|f(g)-s|^p\geq\frac{1}{|S_n|}\left|\left\{g\in S_n:|f(g)-s|\geq\varepsilon\right\}\right|.$$

Hence, for any $\delta>0$

$$\left\{n\in\mathbb{N}:\frac{1}{|S_n|}\left|\left\{g\in S_n:|f(g)-s|\geq\varepsilon\right\}\right|\geq\delta\right\}$$

$$\subseteq\left\{n\in\mathbb{N}:\frac{1}{|S_n|}\sum_{g\in S_n}|f(g)-s|^p\geq\delta\ \varepsilon^p\right\}.$$

Therefore, due to our acceptance, the right set belongs to $\mathcal{I}$, so we get

$$\left\{n\in\mathbb{N}:\frac{1}{|S_n|}\left|\left\{g\in S_n:|f(g)-s|\geq\varepsilon\right\}\right|\geq\delta\right\}\in\mathcal{I}.$$

This proof completed. □

Theorem 2.11. *Let $0<p<\infty$. If $f\in m(G)$ is $\mathcal{I}$-statistically convergent to s for Folner sequence $\{S_n\}$ for G, then it is $\mathcal{I}$-strongly p-summable to s for same sequence.*

Proof. Let $f\in m(G)$ be $\mathcal{I}$-statistically convergent to s for Folner sequence $\{S_n\}$ for G. Since $f\in m(G)$, set $\|f\|_\infty+s=M$. Let $\varepsilon>0$ be given. Then we have

$$\begin{aligned}\frac{1}{|S_n|}\sum_{g\in S_n}|f(g)-s|^p &= \frac{1}{|S_n|}\sum_{\substack{g\in S_n\\ |f(g)-s|\geq\frac{\varepsilon}{2}}}|f(g)-s|^p+\frac{1}{|S_n|}\sum_{\substack{g\in S_n\\ |f(g)-s|<\frac{\varepsilon}{2}}}|f(g)-s|^p\\ &\leq \frac{M^p}{|S_n|}\left|\left\{g\in S_n:|f(g)-s|\geq\frac{\varepsilon}{2}\right\}\right|+\left(\frac{\varepsilon}{2}\right)^p,\end{aligned}$$

and so

$$\left\{n \in \mathbb{N} : \frac{1}{|S_n|} \sum_{g \in S_n} |f(g) - s|^p \geq \varepsilon\right\}$$
$$\subseteq \left\{n \in \mathbb{N} : \frac{1}{|S_n|} \left|\left\{g \in S_n : |f(g) - s| \geq \frac{\varepsilon}{2}\right\}\right| \geq \frac{\varepsilon}{2M^p}\right\}.$$

Therefore, due to our acceptance, the right set belongs to $\mathcal{I}$, so we get

$$\left\{n \in \mathbb{N} : \frac{1}{|S_n|} \sum_{g \in S_n} |f(g) - s|^p \geq \varepsilon\right\} \in \mathcal{I}.$$

This proof completed. □

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