Screen bi-slant lightlike submanifolds of indefinite Kaehler manifolds

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Abstract. The purpose of present paper is to introduce the study of screen bi-slant lightlike submanifolds of indefinite Kaehler manifolds. We obtain a characterization result for the existence of screen bi-slant lightlike submanifolds of indefinite Kaehler manifolds. Then, we derive a necessary and sufficient condition for the induced connection on a screen bi-slant lightlike submanifold of an indefinite Kaehler manifold to be a metric connection. Further, we derive the integrability conditions for the various distributions associated with such lightlike submanifolds. Finally, we emphasize on the study of totally umbilical screen bi-slant lightlike submanifolds of indefinite Kaehler manifolds.

1 Introduction

The two well known classes of submanifolds namely, holomorphic and totally real submanifolds of an almost Hermitian manifold, arise due to the action of almost complex structure $\bar{J}$ when for every vector field $Y(\neq 0)$ tangent to any point $p \in N$, the angle becomes 0 or $\pi/2$, respectively, between the tangent space T_pN and $\bar{J}Y$. Then, Chen [4, 5] as a generalization of holomorphic and totally real submanifolds introduced the notion of slant submanifold in 1990. In this continuation, Lotta [11, 12] investigated the concept of slant submanifolds in contact geometry. After that, Carriazo [2] introduced the geometry of bi-slant submanifolds of almost Hermitian manifolds as well as almost contact metric manifolds as a generalization of slant submanifolds. On the similar note, several new generalized classes of slant submanifolds namely, pseudo-slant submanifolds, semi-slant submanifolds, hemi-slant submanifolds came into existence and the subject matter was dealt in detail by Carriazo [3], Papaghiuc [13] and Sahin [15].

On the other hand, the study of lightlike submanifolds due to its interesting geometric properties attracted many geometers since last two decades. One may note

that, the geometry of lightlike submanifolds has broad application area and has been successfully employed in the theory of black holes, asymptotically flat spacetimes, Killing horizon and electromagnatic as well as radiation fields (see, [6] and [9]). Then, Sahin [14, 17], initiated the study of slant lightlike submanifolds in indefinite almost Hermitian manifolds and indefinite Sasakian manifolds. Afterwards, some more generalizations of slant lightlike submanifolds viz. screen slant lightlike submanifolds, screen pseudo-slant lightlike submanifolds, semi-slant lightlike submanifolds of indefinite Hermitian manifolds were considered and developed by others (for details, see [10, 16, 18, 19]). But the theory of bi-slant submanifolds is yet to be explored in lightlike geometry.

Therefore in this paper, we introduce screen bi-slant lightlike submanifolds in indefinite Kaehler manifolds and justify the existence of this type of submanifolds by giving a characterization result. Then, we derive a necessary and sufficient condition for the induced connection on a screen bi-slant lightlike submanifold of an indefinite Kaehler manifold to be a metric connection. Further, we establish some integrability conditions for the distributions associated with such lightlike submanifolds. Finally, we emphasize on the study of totally umbilical screen bi-slant lightlike submanifolds of indefinite Kaehler manifolds.

2   Preliminaries

In the present section, we will review essential formulas and notations of lightlike submanifolds following [6].

Assume a submanifold (N_n, g) of semi-Riemannian manifold $(\bar{N}_{m+n}, \bar{g})$ such that $\bar{g}$ is metric with index q satisfying $m, n \geq 1$ and $1 \leq q \leq m + n - 1$. If the metric $\bar{g}$ is degenerate on TN, then T_pN and $T_pN^\perp$ both becomes degenerate and there exists a radical (null) subspace $Rad(T_pN)$ such that $Rad(T_pN) = T_pN \cap T_pN^\perp$. If $Rad(TN) : p \in N \to Rad(T_pN)$ is a smooth distribution on N with rank $r > 0$, $1 \leq r \leq n$, then N is called an r-lightlike submanifold of $\bar{N}$. While the radical distribution $Rad(TN)$ of TN is defined as:

$$Rad(TN) = \cup_{p \in N}\{\xi \in T_pN | g(u, \xi) = 0, \ \forall \ u \in T_pN, \xi \neq 0\}.$$

Further, $S(TN)$ be the screen distribution in TN such that $TN = Rad(TN) \perp S(TN)$ and similarly $S(TN^\perp)$ is the screen transversal vector bundle in $TN^\perp$ such that $TN^\perp = Rad(TN) \perp S(TN^\perp)$.

Moreover, there exists a local null frame $\{N_i'\}$ of null sections with values in the orthogonal complement of $S(TN^\perp)$ in $S(TN^\perp)^\perp$ such that

(2.1) $\bar{g}(N_i', \xi_j) = \delta_{ij}, \quad \bar{g}(N_i', N_j') = 0, \text{ for } i, j \in \{1, 2, .., r\},$

where $\{\xi_i\}$ is any local basis of $\Gamma(Rad(TN))$. It implies that $tr(TN)$ and $ltr(TN)$, respectively, be the vector bundles in $T\bar{N}|_N$ and $S(TN^\perp)^\perp$ with the property

$$tr(TN) = ltr(TN) \perp S(TN^\perp),$$

and

(2.2) $T\bar{N}|_N = TN \oplus tr(TN) = S(TN) \perp (Rad(TN) \oplus ltr(TN)) \perp S(TN^\perp).$

Further, the Gauss and Weingarten formulas, for $Y_1, Y_2 \in \Gamma(TN)$ and $V \in \Gamma(tr(TN))$, are given by

$$(2.3) \qquad \bar{\nabla}_{Y_1} Y_2 = \nabla_{Y_1} Y_2 + h(Y_1, Y_2), \quad \bar{\nabla}_{Y_1} V = -A_V Y_1 + \nabla^t_{Y_1} V,$$

where $\{h(Y_1, Y_2), \nabla^t_{Y_1} V\}$ and $\{\nabla_{Y_1} Y_2, A_V Y_1\}$ belong to $\Gamma(tr(TN))$ and $\Gamma(TN)$, respectively and $\bar{\nabla}$ represents the Levi-Civita connection on $\bar{N}$. Here, h is a symmetric bilinear second fundamental form on $\Gamma(TN)$ and A_V is linear shape operator on N. In view of decomposition given by (2.2), the Gauss and Weingarten formulas become

$$(2.4) \qquad \bar{\nabla}_{Y_1} Y_2 = \nabla_{Y_1} Y_2 + h^l(Y_1, Y_2) + h^s(Y_1, Y_2),$$

$$(2.5) \qquad \bar{\nabla}_{Y_1} N' = -A_{N'} Y_1 + \nabla^l_{Y_1} N' + D^s(Y_1, N'),$$

$$(2.6) \qquad \bar{\nabla}_{Y_1} W = -A_W Y_1 + D^l(Y_1, W) + \nabla^s_{Y_1} W,$$

where $Y_1, Y_2 \in \Gamma(TN), N' \in \Gamma(ltr(TN))$ and $W \in \Gamma(S(TN^\perp))$. Furthermore, employing Eqs. (2.4)-(2.6), we derive

$$(2.7) \qquad g(A_W Y_1, Y_2) = \bar{g}(h^s(Y_1, Y_2), W) + \bar{g}(Y_2, D^l(Y_1, W)).$$

$$(2.8) \qquad \bar{g}(A_W Y_1, N') = \bar{g}(D^s(Y_1, N'), W).$$

Suppose that P denotes the projection morphism of TN on its screen distribution $S(TN)$, then some new geometric objects of S(TN) on N are given as

$$(2.9) \qquad \nabla_{Y_1} P Y_2 = \nabla^*_{Y_1} P Y_2 + h^*(Y_1, PY_2), \quad \nabla_{Y_1} \xi = -A^*_\xi Y_1 + \nabla^{*t}_{Y_1} \xi.$$

where $\{h^*(Y_1, PY_2), \nabla^{*t}_{Y_1} \xi\} \in \Gamma(Rad(TN))$ and $\{\nabla^*_{Y_1} PY_2, A^*_\xi Y_1\} \in \Gamma(S(TN))$. Further, employing Eqs. (2.5), (2.6) and (2.9), we attain

$$(2.10) \qquad \bar{g}(h^l(Y_1, PY_2), \xi) = g(A^*_\xi Y_1, PY_2).$$

Let $\bar{\nabla}$ is a metric connection on $\bar{N}$, therefore for $Y_1, Y_2, Y_3 \in \Gamma(TN)$, one has

$$(2.11) \qquad (\nabla_{Y_1} g)(Y_2, Y_3) = \bar{g}(h^l(Y_1, Y_2), Y_3) + \bar{g}(h^l(Y_1, Y_3), Y_2),$$

which implies that the induced connection ∇ on N is not a metric connection. For curvature tensors $\bar{R}$ of $\bar{\nabla}$, the equation of Codazzi is given by

$$
\begin{aligned}
(\bar{R}(Y_1, Y_2) Y_3)^\perp =& (\nabla_{Y_1} h^l)(Y_2, Y_3) - (\nabla_{Y_2} h^l)(Y_1, Y_3) + D^l(Y_1, h^s(Y_2, Y_3)) \\
& - D^l(Y_2, h^s(Y_1, Y_3)) + (\nabla_{Y_1} h^s)(Y_2, Y_3) - (\nabla_{Y_2} h^s)(Y_1, Y_3) \\
& + D^s(Y_1, h^l(Y_2, Y_3)) - D^s(Y_2, h^l(Y_1, Y_3)),
\end{aligned}
$$
$$(2.12)$$

where

$$(2.13) \qquad (\nabla_{Y_1} h^s)(Y_2, Y_3) = \nabla^s_{Y_1} h^s(Y_2, Y_3) - h^s(\nabla_{Y_1} Y_2, Y_3) - h^s(Y_2, \nabla_{Y_1} Y_3),$$

$$(2.14) \qquad (\nabla_{Y_1} h^l)(Y_2, Y_3) = \nabla^l_{Y_1} h^l(Y_2, Y_3) - h^l(\nabla_{Y_1} Y_2, Y_3) - h^l(Y_2, \nabla_{Y_1} Y_3),$$

for any $Y_1, Y_2, Y_3 \in \Gamma(TN)$.

Definition 2.1. [1]. An indefinite almost Hermitian manifold $(\bar{N}, \bar{g}, \bar{J})$ is said to be an indefinite Kaehler manifold if

$$(2.15) \qquad \bar{J}^2 = -I, \ \bar{g}(\bar{J}Y_1, \bar{J}Y_2) = \bar{g}(Y_1, Y_2), \ (\bar{\nabla}_{Y_1}\bar{J})Y_2 = 0, \quad \forall \ Y_1, Y_2 \in \Gamma(T\bar{N}),$$

where $\bar{\nabla}$ be the Levi-Civita connection defined on $\bar{N}$.

An indefinite Kaehler manifold along with constant holomorphic sectional curvature c is known as indefinite complex space form and it is denoted by $\bar{N}(c)$. Then, its curvature tensor is given by

$$\bar{R}(Y_1, Y_2)Y_3 = \frac{c}{4}\{\bar{g}(Y_2, Y_3)Y_1 - \bar{g}(Y_1, Y_3)Y_2 + \bar{g}(JY_2, Y_3)JY_1$$
$$(2.16) \qquad\qquad - \bar{g}(JY_1, Y_3)JY_2 + 2\bar{g}(Y_1, JY_2)JY_3\},$$

for $Y_1, Y_2, Y_3 \in \Gamma(T\bar{N})$.

3 Screen bi-slant lightlike submanifolds

Firstly, we state the following lemma given by Sahin [14], which is very useful for the coming part of this article.

Lemma 3.1. *Let N be an r-lightlike submanifold of an indefinite Kaehler manifold $\bar{N}$ of index $2q$ (provided $2q < dim(N)$). Then, screen distribution $S(TN)$ of lightlike submanifold N is Riemannian.*

Definition 3.1. Assume that N be a q-lightlike submanifold of an indefinite Kaehler manifold $\bar{N}$ with index $2q$. Then, N is said to be a screen bi-slant lightlike submanifold of $\bar{N}$, if the following conditions are satisfied:

(i) $Rad(TN)$ is invariant with respect to $\bar{J}$, that is, $\bar{J}Rad(TN) = Rad(TN)$.

(ii) There exists non-degenerate orthogonal distributions D_1 and D_2 on N such that $S(TN) = D_1 \perp D_2$.

(iii) For each non-zero vector field tangent to D_i for $i = 1, 2$, at $y \in U \subset N$, the angle $\theta_i(Y)$ between $\bar{J}Y$ and the vector space D_{i_y} is constant, that is, it is independent of the choice of $y \in U \subset N$ and $Y \in D_{i_y}$.

This constant angle $\theta_i(Y)$ for $i = 1, 2$, is known as slant angle of the distribution D_i, respectively. Moreover, a screen bi-slant lightlike submanifold is said to be proper if $D_i \neq \{0\}$ and $\theta_i \neq 0, \pi/2$, for $i = 1, 2$.
In view of Definition 3.1, the tangent bundle TN of N has the following decomposition

$$(3.1) \qquad\qquad\qquad TN = Rad(TN) \perp D_1 \perp D_2.$$

For any $Y \in \Gamma(TN)$, we write

$$(3.2) \qquad\qquad\qquad \bar{J}Y = fY + \omega Y,$$

where fY and ωY are the tangential and transversal components of $\bar{J}Y$, respectively. Similarly, for any $V \in \Gamma(tr(TN))$,

$$(3.3) \qquad \bar{J}V = tV + nV,$$

where tV and nV are the tangential and transversal components $\bar{J}V$, respectively. Consider ϕ_1, ϕ_2 and ϕ_3 be the projections of TM on D_1, D_2 and $Rad(TN)$, respectively. Similarly, Consider η_1, η_2 and η_3 be the projections of $tr(TN)$ on $\bar{J}D_1, \bar{J}D_2$ and $ltr(TN)$, respectively, where $\bar{J}D_1$ and $\bar{J}D_2$ are non-degenerate subbundles in $S(TN^\perp)$. Then, for $Y \in \Gamma(TN)$, we have

$$(3.4) \qquad Y = \phi_1 Y + \phi_2 Y + \phi_3 Y.$$

On applying $\bar{J}$ to (3.4), we obtain

$$\bar{J}Y = \bar{J}\phi_1 Y + \bar{J}\phi_2 Y + \bar{J}\phi_3 Y,$$

which yields

$$(3.5) \qquad \bar{J}Y = f\phi_1 Y + \omega\phi_1 Y + f\phi_2 Y + \omega\phi_2 Y + \bar{J}\phi_3 Y.$$

Further, (3.5) can be written as

$$(3.6) \qquad \bar{J}Y = fY + \omega\phi_1 Y + \omega\phi_2 Y,$$

where $fY = f\phi_1 Y + f\phi_2 Y + \bar{J}\phi_3 Y$. Also, for $W \in \Gamma(tr(TN))$, we have

$$(3.7) \qquad W = \eta_1 W + \eta_2 W + \eta_3 W,$$

then applying $\bar{J}$ to (3.7), we obtain

$$\bar{J}W = \bar{J}\eta_1 W + \bar{J}\eta_2 W + \bar{J}\eta_3 W,$$

which yields

$$(3.8) \qquad \bar{J}W = t\eta_1 W + n\eta_1 W + t\eta_2 W + n\eta_2 W + \bar{J}\eta_3 W.$$

Differentiating (3.5) and then by equating the tangent components and transversal components, we obtain

$$(3.9) \qquad (\nabla_X f)Y = A_{\omega Y} X + th^s(X, Y),$$

$$(3.10) \qquad \bar{J}h^l(X, Y) = h^l(X, fY) + D^l(X, \omega Y)$$

and

$$(3.11) \qquad (\nabla_X \omega)Y = -h^s(X, fY) + nh^s(X, Y),$$

where $(\nabla_X f)Y = \nabla_X fY - f\nabla_X Y$ and $(\nabla_X \omega)Y = \nabla^s \omega Y - \omega\nabla_X Y$, for $X, Y \in \Gamma(TN)$.

Lemma 3.2. *For a screen bi-slant lightlike submanifold N of $\bar{N}$, ωD_1 and ωD_2 are orthogonal.*

Proof. Let $X \in \Gamma(D_1)$ and $Y \in \Gamma(D_2)$; then $\bar{g}(\omega X, \omega Y) = \bar{g}(\bar{J}X - fX, \omega Y) = \bar{g}(\bar{J}X, \omega Y) = -\bar{g}(X, \bar{J}\omega Y) = -\bar{g}(X, t\omega Y + n\omega Y) = 0$, which completes the proof. □

Thus, it is clear that there exist $\mu \subset S(TN^\perp)$ such that

$$(3.12) \qquad T\bar{N} = S(TN) \perp \{Rad(TN) \oplus ltr(TN)\} \perp \{\omega(D_1) \perp \omega(D_2) \perp \mu\}.$$

Theorem 3.3. *(Existence Theorem) Consider N be a q-lightlike submanifold of an indefinite Kaehler manifold $\bar{N}$. Then N is a screen bi-slant lightlike submanifold, if and only if,*

(i) $ltr(TN)$ is invariant with respect to $\bar{J}$.

(ii) the screen distribution S(TN) can be split as $S(TN) = D_1 \perp D_2$.

(iii) $f^2\phi_i Y = -cos^2\theta_i(\phi_i Y)$, for i=1,2 and $Y \in \Gamma(S(TN))$.

(iv) $t\omega\phi_i Y = -sin^2\theta_i(\phi_i Y)$, for i=1,2 and $Y \in \Gamma(S(TN))$.

Proof. Assume that N be a screen bi-slant lightlike submanifold of an indefinite Kaehler manifold $\bar{N}$. Suppose that $\bar{J}N' \in \Gamma(Rad(TN))$; then we get $\bar{J}\bar{J}N' = -N' \in \Gamma(ltr(TN))$, since $Rad(TN)$ is invariant with respect to $\bar{J}$, therefore we get a contradiction, which implies that $\bar{J}N' \notin \Gamma(Rad(TN))$. Now, employing Eqs. (2.15) and (3.2), for $Y \in \Gamma(S(TN))$ and $N' \in \Gamma(ltr(TN))$, we acquire $\bar{g}(\bar{J}N', Y) = -\bar{g}(N', \bar{J}Y) = 0$. Therefore, $\bar{J}N'$ does not belong to $S(TN)$. In addition, for $W \in \Gamma(S(TN^\perp))$ and $N' \in \Gamma(ltr(TN))$, employing Eqs. (2.15) and (3.2), we obtain, $\bar{g}(\bar{J}N', W) = -\bar{g}(N', \bar{J}W) = 0$, which implies that $\bar{J}N'$ does not belong to $S(TN^\perp)$. Moreover, the angle between $\bar{J}\phi_i Y$ and D_i is constant for $i = 1, 2$. Therefore, for $Y \in \Gamma(S(TN))$, we have

$$(3.13) \qquad cos\theta_i(\phi_i Y) = \frac{\bar{g}(\bar{J}\phi_i Y, f\phi_i Y)}{|\bar{J}\phi_i Y||f\phi_i Y|} = \frac{-\bar{g}(\phi_i Y, \bar{J}f\phi_i Y)}{|\phi_i Y||f\phi_i Y|}$$
$$= \frac{-\bar{g}(\phi_i Y, f^2\phi_i Y)}{|\phi_i Y||f\phi_i Y|}.$$

On the other hand, we also have

$$(3.14) \qquad cos\theta_i(\phi_i Y) = \frac{|f\phi_i Y|}{|\bar{J}\phi_i Y|}, \quad for\ i = 1, 2.$$

Thus, from Eqs. (3.13) and (3.14), we get

$$(3.15) \qquad cos^2\theta_i(\phi_i Y) = \frac{-\bar{g}(\phi_i Y, f^2\phi_i Y)}{|\phi_i Y|^2}.$$

As we know that, $\theta_i(\phi_i Y)$ is constant on D_i, for $i = 1, 2$. Thus, we conclude that

$$(3.16) \qquad f^2\phi_i Y = -cos^2\theta_i(\phi_i Y),$$

which proves (iii). Next, for $Y \in \Gamma(S(TN))$, we have

$$(3.17) \qquad \bar{J}\phi_i Y = f\phi_i Y + \omega\phi_i Y \quad for \quad i = 1, 2.$$

Further, applying $\bar{J}$, the above equation yields

$$(3.18) \qquad -\phi_i Y = f^2 \phi_i Y + \omega f \phi_i Y + t\omega\phi_i Y + n\omega\phi_i Y.$$

Then, equating the tangential components on both sides, we acquire

$$(3.19) \qquad -\phi_i Y = f^2 \phi_i Y + t\omega\phi_i Y.$$

As N is a screen bi-slant lightlike submanifold, thus using (3.16), we have $f^2\phi_1 Y = -cos^2\theta_1(\phi_1 Y)$ and $f^2\phi_2 Y = -cos^2\theta_2(\phi_2 Y)$, thus we attain

$$(3.20) \qquad t\omega\phi_i Y = -\sin^2\theta_i(\phi_i Y), \quad for \ i = 1, 2,$$

which proves the assertion (iv).
Conversely, we can prove that $Rad(TN)$ is invariant in a similar way that $ltr(TN)$ is invariant. Further, from Lemma 3.1, it is clear that $S(TN)$ is Riemannian; then, for $i = 1, 2$, we have

$$(3.21) \qquad g(f\phi_i Y, f\phi_i Y) = -g(f^2\phi_i Y, \phi_i Y) = cos^2\theta_i(\phi_i Y)g(\phi_i Y, \phi_i Y),$$

for $Y \in \Gamma(S(TN))$, which further gives

$$(3.22) \qquad cos^2\theta_i(\phi_i Y) = \frac{g(f\phi_i Y, f\phi_i Y)}{g(\phi_i Y, \phi_i Y)}.$$

On the other hand, from condition (iv), one has $t\omega\phi_i Y = -\sin^2\theta_i(\phi_i Y)$, for $i = 1, 2$. Further, employing (3.19), we infer

$$(3.23) \qquad f^2\phi_i Y = -(1 - sin^2\theta_i(\phi_i Y)) = -cos^2\theta_i(\phi_i Y).$$

Then, following the similar steps of assertion (iii), the proof follows. $\qquad\square$

Corollary 3.4. *Let N be a screen bi-slant lightlike submanifold of an indefinite Kaehler manifold $\bar{N}$; then one has*

$$(3.24) \qquad g(f\phi_i X, f\phi_i Y) = cos^2\theta_i g(\phi_i X, \phi_i Y)$$

and

$$(3.25) \qquad \bar{g}(\omega\phi_i X, \omega\phi_i Y) = sin^2\theta_i g(\phi_i X, \phi_i Y),$$

for $i = 1, 2$ and $X, Y \in \Gamma(TN)$.

From (2.11), it can be noticed that the induced connection ∇ on a lightlike submanifold is not necessarily a metric connection. Therefore, in the next characterization result, we present a necessary and sufficient condition under which the induced connection ∇ on a screen bi-slant lightlike submanifold of an indefinite Kaehler manifold $\bar{N}$ becomes a metric connection.

Theorem 3.5. *A necessary and sufficient condition for the induced connection ∇ on a screen bi-slant lightlike submanifold of an indefinite Kaehler manifold $\bar{N}$ to be a metric connection is that*

$$\nabla_X \bar{J}Y \in \Gamma(Rad(TN)) \quad and \quad th^s(X, \bar{J}Y) = 0,$$

for $X \in \Gamma(TN)$ and $Y \in \Gamma(Rad(TN))$.

Proof. Assume that N be a screen bi-slant lightlike submanifold of an indefinite Kaehler manifold $\bar{N}$. Since $\bar{J}$ is an almost complex structure on $\bar{N}$, therefore for $X \in \Gamma(TN)$ and $Y \in \Gamma(Rad(TN))$, we have $\bar{\nabla}_X Y = -\bar{\nabla}_X \bar{J}^2 Y$. Further, considering (2.15), we get $\bar{\nabla}_X Y = -\bar{J}\bar{\nabla}_X \bar{J}Y$. Now, employing (2.3) and then equating the tangential components on both sides, we derive

$$(3.26) \qquad\qquad \nabla_X Y = -\bar{J}(\nabla_X \bar{J}Y) - th^s(X, \bar{J}Y).$$

Hence, from (3.26), $\nabla_X Y \in \Gamma(Rad(TN))$ if and only if $\nabla_X \bar{J}Y \in \Gamma(Rad(TN))$ and $th^s(X, \bar{J}Y) = 0$, which proves the result. $\qquad\square$

Theorem 3.6. *Let N be a screen bi-slant lightlike submanifold of an indefinite Kaehler manifold $\bar{N}$; then*

(i) *the slant distributions D_i is integrable, if and only if,*

$$\nabla_X fY - \nabla_Y fX - A_{\omega\phi_i Y}X + A_{\omega\phi_i X}Y \in \Gamma(D_i),$$

for $X, Y \in \Gamma(D_i)$ and $i = 1, 2$.

(ii) *the distribution $Rad(TN)$ is integrable, if and only if,*

$$h^s(X, \bar{J}Y) = h^s(\bar{J}X, Y),$$

for $X, Y \in \Gamma(Rad(TN))$.

Proof. For $X, Y \in \Gamma(D_i)$ for $i = 1, 2$, using (3.9), we derive

$$(3.27) \qquad\qquad \nabla_X fY - A_{\omega\phi_i Y}X - th^s(X, Y) = f\nabla_X Y,$$

by interchanging the role of X and Y in above equation, we get

$$(3.28) \qquad\qquad \nabla_Y fX - A_{\omega\phi_i X}Y - th^s(Y, X) = f\nabla_Y X.$$

Therefore, from Eqs. (3.27) and (3.28), we obtain

$$\nabla_X fY - \nabla_Y fX - A_{\omega\phi_i Y}X + A_{\omega\phi_i X}Y = f[X, Y],$$

which proves the assertion (i).
For $X, Y \in \Gamma(Rad(TN))$, using (3.11), we have

$$(3.29) \qquad\qquad h^s(X, \bar{J}Y) - nh^s(X, Y) = \omega\nabla_X Y.$$

On interchanging the role of X and Y in (3.29), we get

$$(3.30) \qquad\qquad h^s(Y, \bar{J}X) - nh^s(Y, X) = \omega\nabla_Y X.$$

Further, from Eqs. (3.11) and (3.29), we obtain

$$h^s(X, \bar{J}Y) - h^s(\bar{J}X, Y) = \omega[X, Y],$$

from which assertion (ii) follows. $\qquad\square$

Theorem 3.7. *Let N be a screen bi-slant lightlike submanifold of an indefinite Kaehler manifold $\bar{N}$. Then, f is parallel, if and only if, $D^s(X, N') \in \Gamma(\mu)$ and*

$$\bar{g}(h^s(X,Y), \omega Z) = \bar{g}(h^s(X,Z), \omega Y),$$

for $X \in \Gamma(TN)$, $Y, Z \in \Gamma(S(TN))$ and $N' \in \Gamma(ltr(TN))$.

Proof. Employing (3.9) for $Y \in \Gamma(Rad(TN))$, $X \in \Gamma(TN)$ and $N' \in \Gamma(ltr(TN))$, we obtain $\bar{g}((\nabla_X f)Y, N') = 0$. Further, for $Y \in \Gamma(S(TN))$, we have $\bar{g}((\nabla_X f)Y, N') = \bar{g}(A_{\omega Y}X, N')$. Moreover, using (2.8) for $X \in \Gamma(S(TN))$, we acquire

$$(3.31) \qquad \bar{g}((\nabla_X f)Y, N') = \bar{g}(D^s(X, N'), \omega Y).$$

On the other hand, for $X, Y \in \Gamma(TN)$ and $Z \in \Gamma(S(TN))$, we derive

$$\bar{g}((\nabla_X f)Y, Z) = g(A_{\omega Y}X, Z) - \bar{g}(h^s(X,Y), \omega Z),$$

then employing (2.7), we get

$$(3.32) \qquad \bar{g}((\nabla_X f)Y, Z) = \bar{g}(h^s(X,Z), \omega Y) - \bar{g}(h^s(X,Y), \omega Z).$$

Thus, from Eqs. (3.31) and (3.32), the assertions follow. $\square$

Theorem 3.8. *Let N be a screen bi-slant lightlike submanifold of an indefinite Kaehler manifold $\bar{N}$. If $(\nabla_X f)Y = 0$, for $X \in \Gamma(TN)$ and $Y \in \Gamma(Rad(TN))$, then the induced connection ∇ is a metric connection.*

Proof. Using the hypothesis and (3.9), we have $th^s(X,Y) = 0$ for $X \in \Gamma(TN)$ and $Y \in \Gamma(Rad(TN))$. Therefore, $g(th^s(X,Y), Z) = 0$, for $X, Z \in \Gamma(TN)$ and $Y \in \Gamma(Rad(TN))$. Thus, we attain

$$(3.33) \qquad \bar{g}(\bar{J}h^s(X,Y), Z) = 0,$$

which further gives

$$(3.34) \qquad \bar{g}(h^s(X,Y), \omega\phi_i Z) = 0 \quad for \ i = 1, 2.$$

Moreover, by using (2.4) for $X \in \Gamma(TN)$ and $Y \in \Gamma(Rad(TN))$, we have

$$\bar{g}(\omega\phi_i\nabla_X Y, \bar{J}h^s(X,Y)) = \bar{g}(\omega\phi_i\nabla_X Y, \bar{J}\bar{\nabla}_X Y - \bar{J}\nabla_X Y - \bar{J}h^l(X,Y)).$$

Since $ltr(TN)$ is invariant, then employing Eqs. (2.15) and (3.5), we acquire

$$\bar{g}(\omega\phi_i\nabla_X Y, \bar{J}h^s(X,Y)) = \bar{g}(\omega\phi_i\nabla_X Y, \bar{\nabla}_X \bar{J}Y) - \bar{g}(\omega\phi_i\nabla_X Y, \omega\phi_i\nabla_X Y).$$

Further, employing (2.4), we get

$$\bar{g}(\omega\phi_i\nabla_X Y, \bar{J}h^s(X,Y)) = \bar{g}(\omega\phi_i\nabla_X Y, h^s(X, \bar{J}Y)) - \bar{g}(\omega\phi_i\nabla_X Y, \omega\phi_i\nabla_X Y).$$

Then, using (3.34), we obtain $\bar{g}(\omega\phi_i\nabla_X Y, \bar{J}h^s(X,Y)) = -\bar{g}(\omega\phi_i\nabla_X Y, \omega\phi_i\nabla_X Y)$ and further employing (3.25) for $X \in \Gamma(TN)$ and $Y \in \Gamma(Rad(TN))$, we get

$$(3.35) \qquad \bar{g}(\omega\phi_i\nabla_X Y, \bar{J}h^s(X,Y)) = -\sin^2\theta_i g(\phi_i\nabla_X Y, \phi_i\nabla_X Y).$$

Now, using Eqs. (2.15) and (3.5), we have

$$\bar{g}(\omega\phi_i\nabla_X Y, \bar{J}h^s(X,Y)) = -\bar{g}(f\phi_i\nabla_X Y, \bar{J}h^s(X,Y)),$$

for $X \in \Gamma(TN)$ and $Y \in \Gamma(Rad(TN))$ and then employing (3.33), we obtain

(3.36) $\bar{g}(\omega\phi_i\nabla_X Y, \bar{J}h^s(X,Y)) = 0.$

Further, from Eqs. (3.35) and (3.36), we attain $\sin^2\theta_i g(\phi_i\nabla_X Y, \phi_i\nabla_X Y) = 0$ for $i = 1, 2$. Since $\theta_i \neq 0$ and $S(TN)$ is Riemannian, therefore we have $\phi_i\nabla_X Y = 0$. Then, $\nabla_X Y \in \Gamma(Rad(TN))$, which means $Rad(TN)$ is parallel. Hence, the theorem is proved following [6]. $\qquad\square$

4   Foliations determined by distributions

Theorem 4.1. *Let N be a screen bi-slant lightlike submanifold of an indefinite Kaehler manifold $\bar{N}$. Then, the screen distribution defines a totally geodesic foliation, if and only if, $\bar{J}A_{\omega\phi_i Y}X - A_{\omega f\phi_i Y}X$ for $i = 1, 2$ has no components in $Rad(TN)$ for $X, Y \in \Gamma(D_i)$.*

Proof. Using Eqs. (2.4) and (2.15), we have $\bar{g}(\nabla_X Y, N') = \bar{g}(\bar{\nabla}_X \bar{J}Y, \bar{J}N')$, for $X, Y \in \Gamma(D_i)$ for $i = 1, 2$ and $N' \in \Gamma(ltr(TN))$. Thus, from Eqs. (2.6) and (3.5), we obtain

$$\bar{g}(\nabla_X Y, N') = \bar{g}(\bar{\nabla}_X f\phi_i Y, \bar{J}N') - \bar{g}(A_{\omega\phi_i Y}X, \bar{J}N').$$

Again using Eqs. (2.4), (2.6), (2.15) and (3.5), we derive

$$\bar{g}(\nabla_X Y, N') = -\bar{g}(\nabla_X f^2\phi_i Y, N') - \bar{g}(A_{\omega f\phi_i Y}X, N') - \bar{g}(A_{\omega\phi_i Y}X, \bar{J}N'),$$

thus in view of Theorem 3.3, we get

$$\bar{g}(\nabla_X Y, N') = \cos^2\theta\bar{g}(\nabla_X Y, N') - \bar{g}(A_{\omega f\phi_i Y}X, N') - \bar{g}(A_{\omega\phi_i Y}X, \bar{J}N')$$
$$\sin^2\theta\bar{g}(\nabla_X Y, N') = -\bar{g}(A_{\omega f\phi_i Y}X, N') - \bar{g}(A_{\omega\phi_i Y}X, \bar{J}N'),$$

for $i = 1, 2$. Hence, the proof is completed. $\qquad\square$

Theorem 4.2. *Let N be a screen bi-slant lightlike submanifold of an indefinite Kaehler manifold $\bar{N}$. Then, $Rad(TN)$ defines a totally geodesic foliation, if and only if,*

$$\bar{g}(h^l(X, f\phi_1 Z) + h^l(X, f\phi_2 Z), \bar{J}Y) = -\bar{g}(D^l(X, \omega\phi_1 Z) + D^l(X, \omega\phi_2 Z), \bar{J}Y),$$

for $X, Y \in \Gamma(Rad(TN))$ and $Z \in \Gamma(S(TN))$.

Proof. Let N be a screen bi-slant lightlike submanifold of an indefinite Kaehler manifold $\bar{N}$. To prove that $Rad(TN)$ defines a totally geodesic foliation, it is sufficient to show that $\nabla_X Y \in \Gamma(Rad(TN))$, for every $X, Y \in \Gamma(Rad(TN))$. Since $\bar{\nabla}$ is a metric connection, using Eqs. (2.4) and (2.15), for every $X, Y \in \Gamma(Rad(TN))$ and for every $Z \in \Gamma(S(TN))$, we obtain

$$g(\nabla_X Y, Z) = \bar{g}(\bar{\nabla}_X Y, Z) = -\bar{g}(Y, \bar{\nabla}_X Z)$$
(4.1) $= -\bar{g}(\bar{\nabla}_X \bar{J}Z, \bar{J}Y).$

In view of (3.5), we get

$$\bar{g}(\nabla_X Y, Z) = -\bar{g}(\bar{\nabla}_X(f\phi_1 Z + \omega\phi_1 Z + f\phi_2 Z + \omega\phi_2 Z), \bar{J}Y)$$
$$= \bar{g}(h^l(X, f\phi_1 Z), \bar{J}Y) - \bar{g}(D^l(X, \omega\phi_1 Z), \bar{J}Y)$$

(4.2)
$$- \bar{g}(h^l(X, f\phi_2 Z), \bar{J}Y) - \bar{g}(D^l(X, \omega\phi_2 Z), \bar{J}Y),$$

which completes the proof. □

Theorem 4.3. *Let N be a screen bi-slant lightlike submanifold of an indefinite Kaehler manifold $\bar{N}$. Then, D_i for=1,2 defines a totally geodesic foliation, if and only if,*

$$\bar{g}(fY, A_{\bar{J}N'}X) = \bar{g}(A_{\omega Y}X, \bar{J}N')$$

for $X, Y \in \Gamma(D_i)$ where $i = 1, 2$ and $N' \in \Gamma(ltr(TN))$.

Proof. Let N be a screen bi-slant lightlike submanifold of an indefinite Kaehler manifold $\bar{N}$. To prove that D_i for $i = 1, 2$ defines a totally geodesic foliation, it is sufficient to show that $\nabla_X Y \in D_i$ for $i = 1, 2$, for every $X, Y \in \Gamma(D_i)$ for $i = 1, 2$. Since $\bar{\nabla}$ is a metric connection, using Eqs. (2.4) and (2.15), for every $X, Y \in \Gamma(D_i)$ for $i = 1, 2$ and $N' \in \Gamma(ltr(TN))$, we obtain

$$\bar{g}(\nabla_X Y, N') = \bar{g}(\bar{\nabla}_X Y, N') = \bar{g}(\bar{\nabla}_X \bar{J}Y, \bar{J}N')$$
$$= -\bar{g}(\bar{J}Y, \bar{\nabla}_X \bar{J}N')$$

(4.3)
$$= g(fY, A_{\bar{J}N'}X) - \bar{g}(\omega Y, D^s(X, \bar{J}N')).$$

Further, employing (2.8), we obtain

(4.4)
$$\bar{g}(\nabla_X Y, N') = g(fY, A_{\bar{J}N'}X) - \bar{g}(A_{\omega Y}X, \bar{J}N'),$$

which completes the proof. □

5 Totally umbilical screen bi-slant lightlike submanifolds

Definition 5.1. [7] A lightlike submanifold (N, g) of a semi-Riemannian manifold $(\bar{N}, \bar{g})$ is called totally umbilical, if there exists transversal curvature vector field $H \in \Gamma(tr(TN))$ on N such that

(5.1)
$$h(Y_1, Y_2) = H\bar{g}(Y_1, Y_2),$$

for $Y_1, Y_2 \in \Gamma(TN)$. Using Eqs. (2.4) and (2.6), clearly N is totally umbilical, if and only if, there exist smooth vector fields $H^l \in \Gamma(ltr(TN))$ and $H^s \in \Gamma(S(TN^\perp))$ such that

(5.2) $\quad h^l(Y_1, Y_2) = H^l g(Y_1, Y_2), \quad h^s(Y_1, Y_2) = H^s g(Y_1, Y_2), \quad and \quad D^l(Y_1, W) = 0,$

for $Y_1, Y_2 \in \Gamma(TN)$ and $W \in \Gamma(S(TN^\perp))$.
On the other hand, a lightlike submanifold is totally geodesic, if and only if, $h(Y_1, Y_2) = 0$, for $Y_1, Y_2 \in \Gamma(TN)$. Thus, a lightlike submanifold is totally geodesic, if and only if, $H^l = 0$ and $H^s = 0$.

Theorem 5.1. *Consider N be a totally umbilical screen bi-slant lightlike submanifold of an indefinite Kaehler manifold $\bar{N}$. Then, at least one of the following statements is true:*

(a) N is an anti-invariant submanifold.

(b) $D_i = \{0\}$ for $i = 1, 2$.

(c) If N is a proper screen bi-slant lightlike submanifold, then $H^s \in \Gamma(\mu)$.

Proof. For a totally umbilical screen bi-slant lightlike submanifold N of $\bar{N}$, employing (5.1), for $Z \in \Gamma(D_i)$ for $i = 1, 2$, $\Phi Z \in D_1$ or D_2, we have

$$(5.3) \qquad\qquad h(\phi Z, \phi Z) = g(\phi Z, \phi Z) H.$$

Then, using Eqs. (2.3) and (3.24), we obtain

$$(5.4) \qquad\qquad \bar{\nabla}_{\phi Z} \phi Z - \nabla_{\phi Z} \phi Z = \cos^2 \theta g(Z, Z) H,$$

which yields

$$(5.5) \qquad\qquad \bar{J} \bar{\nabla}_{\phi Z} Z - \bar{\nabla}_{\phi Z} \omega Z - \nabla_{\phi Z} \phi Z = \cos^2 \theta g(Z, Z) H.$$

Further, using Eqs. (2.4) and (2.6), we derive

$$\cos^2 \theta g(Z, Z) H = \bar{J} \nabla_{\phi Z} Z + \bar{J} h^l(\phi Z, Z) + \bar{J} h^s(\phi Z, Z) + A_{\omega Z} \phi Z$$
$$- \nabla^s_{\phi Z} \omega Z - D^l(\phi Z, \omega Z) - \nabla_{\phi Z} \phi Z.$$

$\Phi Z \in \Gamma(D_1)$ or D_2 and employing (3.2), (3.3) and (5.4), we obtain

$$\cos^2 \theta g(Z, Z) H = \phi \nabla_{\phi Z} Z + \omega \nabla_{\phi Z} Z + g(\phi Z, Z) \bar{J} H^l + g(\phi Z, Z) t H^s + g(\phi Z, Z) n H^s$$
$$(5.6) \qquad\qquad + A_{\omega Z} \phi Z - \nabla^s_{\phi Z} \omega Z - D^l(\phi Z, \omega Z) - \nabla_{\phi Z} \phi Z.$$

Taking the inner product with respect to ωZ in above equation, we get

$$(5.7) \qquad \cos^2 \theta g(Z, Z) \bar{g}(H^s, \omega Z) \quad = \bar{g}(\omega \nabla_{\phi Z} Z, \omega Z) - \bar{g}(\nabla^s_{\phi Z} \omega Z, \omega Z).$$

Since (3.24) holds for $i = 1, 2$, $\phi_i X = \phi_i Y \in \Gamma(D_i)$ and taking the covariant derivative with respect to ϕZ, we get

$$(5.8) \qquad\qquad \bar{g}(\nabla^s_{\phi Z} \omega Z, \omega Z) = \sin^2 \theta g(\nabla_{\phi Z} Z, Z).$$

Now, using Eqs. (3.25) and (5.8) in (5.7), we obtain

$$(5.9) \qquad\qquad \cos^2 \theta g(Z, Z) \bar{g}(H^s, \omega Z) = 0.$$

Thus, (5.9) implies that either $Z = 0$ or $\theta = \pi/2$ or $H^s \in \Gamma(\mu)$. Hence, the result follows. $\qquad\square$

Theorem 5.2. *Every totally umbilical proper screen bi-slant lightlike submanifold of an indefinite Kaehler manifold $\bar{N}$ is totally geodesic.*

Proof. As $\bar{N}$ is an indefinite Kaehler manifold therefore for $Z \in \Gamma(D_i)$ where $i = 1, 2$, employing (2.15), we acquire $\bar{\nabla}_Z \bar{J} Z = \bar{J} \bar{\nabla}_Z Z$, which implies that

$$\nabla_Z \phi Z + h^l(Z, \phi Z) + h^s(Z, \phi Z) - A_{\omega Z} Z + \nabla_Z^s \omega Z + D^l(Z, \omega Z)$$
$$(5.10) \qquad = \phi \nabla_Z Z + \omega \nabla_Z Z + \bar{J} h^l(Z, Z) + t h^s(Z, Z) + n h^s(Z, Z).$$

Using (5.2) and equating the tangential components on both sides, the above equation becomes

$$(5.11) \qquad \nabla_Z \phi Z - A_{\omega Z} Z = \phi \nabla_Z Z + \bar{J} h^l(Z, Z) + t h^s(Z, Z).$$

Taking the inner product with respect to $\bar{J}\xi \in \Gamma(Rad(TN))$ on both sides of (5.10), we obtain

$$(5.12) \qquad g(A_{\omega Z} Z, \bar{J}\xi) + \bar{g}(h^l(Z, Z), \xi) = 0.$$

Then, employing (2.7), we have

$$(5.13) \qquad \bar{g}(h^s(Z, \bar{J}\xi), \omega Z) + \bar{g}(\bar{J}\xi, D^l(Z, \omega Z)) + \bar{g}(h^l(Z, Z), \xi) = 0.$$

In view of (5.2), the above equation reduces to

$$(5.14) \qquad \bar{g}(H^s, \omega Z) g(Z, \bar{J}\xi) + \bar{g}(H^l, \xi) g(Z, Z) = 0.$$

From Theorem 5.1, we have $H^s \in \Gamma(\mu)$; therefore from (5.14), we obtain

$$(5.15) \qquad \bar{g}(H^l, \xi) g(Z, Z) = 0.$$

Since D_i for $i = 1, 2$ is non-degenerate, therefore $\bar{g}(H^l, \xi) = 0$, which implies

$$(5.16) \qquad H^l = 0.$$

Moreover, $H^s \in \Gamma(\mu)$ for a proper totally umbilical screen bi-slant lightlike submanifold of $\bar{N}$. Therefore, equating the transversal components on both sides of (5.10), we have

$$\omega \nabla_Z Z + n h^s(Z, Z) = h^l(Z, \phi Z) + h^s(Z, \phi Z) + \nabla_Z^s \omega Z + D^l(X, \omega Z).$$

Then, employing (5.2), we derive

$$\omega \nabla_Z Z + g(Z, Z) n H^s = g(Z, \phi Z) H^l + g(Z, \phi Z) H^s + \nabla_Z^s \omega Z.$$

By taking the inner product of (5.10) with respect to $\bar{J} H^s$, we obtain

$$(5.17) \qquad g(Z, Z) \bar{g}(H^s, H^s) = \bar{g}(\nabla_Z^s \omega Z, \bar{J} H^s).$$

Furthermore, one has $\bar{\nabla}_Z \bar{J} H^s = \bar{J} \bar{\nabla}_Z H^s$ and it implies

$$-A_{\bar{J} H^s} Z + \nabla_Z^s \bar{J} H^s + D^l(Z, \bar{J} H^s) = -\phi A_{H^s} Z - \omega A_{H^s} Z + t \nabla_Z^s H^s$$
$$(5.18) \qquad\qquad\qquad\qquad\qquad + n \nabla_Z^s H^s + \bar{J} D^l(Z, H^s).$$

Since μ is invariant and by taking the inner product on both sides with respect to ωZ, we infer

$$(5.19) \qquad \bar{g}(\nabla^s_Z \bar{J} H^s, \omega Z) = -\bar{g}(\omega A_{H^s} Z, \omega Z) = -\sin^2 \theta g(A_{H^s} Z, Z).$$

As $\bar{\nabla}$ is a metric connection, thus we have $(\bar{\nabla}_Z \bar{g})(\omega Z, \bar{J} H^s) = 0$, which implies that $\bar{g}(\nabla^s_Z \omega Z, \bar{J} H^s) = -\bar{g}(\nabla^s_Z \bar{J} H^s, \omega Z)$; therefore (5.19) becomes

$$(5.20) \qquad \bar{g}(\nabla^s_Z \omega Z, \bar{J} H^s) = \sin^2 \theta g(A_{H^s} Z, Z).$$

Then, using (5.20) in (5.17), we derive

$$(5.21) \qquad g(Z, Z)\bar{g}(H^s, H^s) = \sin^2 \theta g(A_{H^s} Z, Z).$$

Now, employing (2.7), the above equation yields

$$g(Z, Z)\bar{g}(H^s, H^s) = \sin^2 \theta \bar{g}(h^s(Z, Z), H^s) = \sin^2 \theta g(Z, Z)\bar{g}(H^s, H^s),$$

it implies that

$$(1 - \sin^2 \theta)g(Z, Z)\bar{g}(H^s, H^s) = 0.$$

Since N is a proper screen bi-slant lightlike submanifold, therefore $\sin^2 \theta \neq 1$ and from the non-degeneracy of D_i for $i = 1, 2$, we derive

$$(5.22) \qquad\qquad\qquad H^s = 0.$$

Hence, the result follows from Eqs. (5.16) and (5.22). □

Theorem 5.3. *For a proper totally umbilical screen bi-slant lightlike submanifold N of an indefinite Kaehler manifold $\bar{N}$, ∇ is always a metric connection.*

Proof. The proof follows directly from Eqs. (5.16) and (2.11). □

Theorem 5.4. *There does not exist any proper totally umbilical screen bi-slant lightlike submanifold in $\bar{N}(c)$, provided, $c \neq 0$.*

Proof. For $N' \in \Gamma(ltr(TN)), Y \in \Gamma(S(TN))$ and $\xi \in \Gamma(Rad(TN))$, using (2.16), we infer

$$(5.23) \qquad \bar{g}(\bar{R}(Y, \bar{J}Y)N', \xi) = -\frac{c}{2}\bar{g}(\bar{J}N', \xi)g(Y, Y).$$

Moreover, using (2.12), we attain

$$(5.24) \qquad -\bar{g}(\bar{R}(Y, \bar{J}Y)N', \xi) = \bar{g}((\nabla_{\bar{J}Y} h^l)(Y, N'), \xi) - \bar{g}((\nabla_Y h^l)(\bar{J}Y, N'), \xi).$$

Then, from (5.23) and (5.24), we get

$$(5.25) \qquad -\frac{c}{2}\bar{g}(\bar{J}N', \xi)g(Y, Y) = -\bar{g}((\nabla_{\bar{J}Y} h^l)(Y, N'), \xi) + \bar{g}((\nabla_Y h^l)(\bar{J}Y, N'), \xi).$$

Since N is totally umbilical; therefore using (5.2), we have

$$(5.26) \qquad (\nabla_Y h^l)(\bar{J}Y, N') = -\{g(\nabla_Y \bar{J}Y, N') + g(\bar{J}Y, \nabla_Y N')\}H^l.$$

For $N' \in \Gamma(ltr(TN))$ and $Y \in \Gamma(S(TN))$, one has $\bar{g}(\bar{J}Y, N') = 0$; then taking covariant derivative with respect to Y, we get $g(\bar{J}Y, \nabla_Y N') = -g(\nabla_Y \bar{J}Y, N')$. Thus (5.26) reduces to

$$(5.27) \qquad (\nabla_Y h^l)(\bar{J}Y, N') = 0.$$

Similarly, it follows that

$$(5.28) \qquad (\nabla_{\bar{J}Y} h^l)(Y, N') = 0.$$

Then, using (5.27) and (5.28), the relation (5.25) yields

$$-\frac{c}{2} g(Y, Y) g(\bar{J}N', \xi) = 0.$$

Hence, the non-degeneracy of $S(TN)$ implies that $c \neq 0$, which proves the result. $\square$

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