Construction of slant immersions II

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Abstract

In the preceding article [T], we have constructed several examples of slant immersions into complex Euclidean spaces. The purpose of this article is to construct some additional examples to get slant and full immersions, in the sense of slantedness, into complex Euclidean spaces with arbitrary dimension, codimension, and positive slant angle.

1 Introduction

Let f be an immersion of a differentiable manifold M into an almost Hermitian manifold (N, g, J) with an almost complex structure J and an almost Hermitian metric g. For a nonzero vector X tangential to M at $p \in M$, the angle $\theta(X)$ between Jf_*X and $f_*(T_pM)$,

$$\theta(X) = \angle(Jf_*X, f_*(T_pM)) \in [0, \pi/2], \quad 0 \neq X \in T_pM, \quad p \in M, \tag{1.1}$$

is called the Wirtinger angle. The immersion f is called a slant immersion with the slant angle α, or an α-slant immersion, if $\theta(X)$ is constant α ([C1]).

Since the notion of a slant immersion is relatively new, it is necessary to construct as many examples as possible in order to make the argument about slant immersions substantial. In [T], we have proved the following :

Theorem. ([T])

1. *For any natural numbers n and m with $n \leq m$, and for any angle $\alpha \in (0, \pi/2]$, there exists an α-slant spherical homothetic full immersion $f : I^{2n} \to \mathbb{C}^{2m}$ of the standard open $2n$-cube I^{2n} into $\mathbb{C}^{2m}$.*

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2. *For any natural numbers n and m with $n \leq m$, there exists a totally real spherical homothetic full immersion $f : T^{2n} \to \mathbb{C}^{2m}$ of a flat $2n$-torus T^{2n} into $\mathbb{C}^{2m}$.*

We used cartesian product and direct sum of immersions to construct these immersions. The purpose of this article is to improve the theorem above and to show the following :

Theorem

1. *For any natural numbers n and m with $2n \leq m$, and any angle $\alpha \in (0, \pi/2)$, we can construct an immersion $f : I^{2n} \to \mathbb{C}^m$ of the standard open $2n$-cube I^{2n} into $\mathbb{C}^m$ so that f is α-slant, homothetic, and full over $\mathbb{C}$.*

2. *For any natural numbers n and m with $n \leq m$, there exists a totally real spherical homothetic full immersion $f : I^n \to \mathbb{C}^m$ of the standard open n-cube I^n, and also a totally real spherical homothetic full immersion $\widetilde{f} : T^n \to \mathbb{C}^m$ of a flat n-torus T^n into $\mathbb{C}^m$.*

Here in (1), the fullness over $\mathbb{C}$ means fullness in the sense of slant immersions, namely :

Definition *We call an immersion $f : M \to (N, g, J)$ full over $\mathbb{C}$, if and only if the image $f(M)$ is not contained in any totally geodesic almost Hermitian submanifold $(N', g_{|N'}, J_{|N'})$ with $\dim N' < \dim N$.*

We recall here that if $f : M \to \mathbb{C}^m$ is a slant immersion with the slant angle $\alpha < \pi/2$, then M is even dimensional and noncompact, and also that if $\alpha > 0$, then $2 \dim M \leq m$ ([CT 1, 2]). In this sense, the theorem above shows the existence of slant full-over-$\mathbb{C}$ immersions into complex Euclidean spaces with all possible dimensions and codimensions, and with any prescribed positive slant angle.

2 Preliminaries

In this article we consider immersions of connected manifolds into complex Euclidean spaces. We use the notations of [T]. The direct sum and tensor product of immersions into Euclidean spaces were defined in [C2].

For an immersion $f : M \to \mathbb{C}^m = (\mathbb{E}^{2m}, <\ ,\ >, J)$, we denote by $\mathcal{T}_{\{}$ the set of all vectors tangential to $f(M)$, identifying tangent vectors with position vectors in $\mathbb{E}^{2m}$,

$$\mathcal{T}_f = \{ f_* X \in \mathbb{E}^{2m} \,|\, X \in T_p M,\ p \in M \} \ \subset\ \mathbb{E}^{2m}. \tag{2.1}$$

$\mathcal{T}_f$ is just a subset of $\mathbb{E}^{2m}$ without any bundle structure. The relation between the usual fullness and the fullness over $\mathbb{C}$ can be stated as in the following lemma:

Lemma 1 *We consider the following eight conditions:*

1. *The immersion f is full, i.e., the image $f(M)$ is not contained in any $(2m-1)$-plane in $\mathbb{E}^{2m}$.*

2. *The set $\mathcal{T}_f$ is not contained in any $(2m-1)$-dimensional linear subspace of $\mathbb{E}^{2m}$, i.e., any $(2m-1)$-plane through the origin O of $\mathbb{E}^{2m}$.*

3. *The image $f(M)$ is not contained in any $(2m-1)$-dimensional linear subspace of $\mathbb{E}^{2m}$.*

4. *The image $f(M)$ is not contained in any $(2m-2)$-plane in $\mathbb{E}^{2m}$.*

5. *The set $\mathcal{T}_f$ is not contained in any $(2m-2)$-dimensional linear subspace of $\mathbb{E}^{2m}$.*

6. *The set $\mathcal{T}_f \cup J(\mathcal{T}_f)$ is not contained in any $(2m-2)$-dimensional linear subspace of $\mathbb{E}^{2m}$.*

7. *The set $\mathcal{T}_f$ is not contained in any J-invariant $(2m-2)$-dimensional linear subspace of $\mathbb{E}^{2m}$.*

8. *The immersion f is full over $\mathbb{C}$, i.e., the image $f(M)$ is not contained in any $(2m-2)$-plane in $\mathbb{E}^{2m}$ whose tangent space, the $(2m-2)$-plane through O parallel to the $(2m-2)$-plane, is invariant under J.*

Then, the relation among these conditions is:

$$(1) \Leftrightarrow (2) \Rightarrow (3) \Rightarrow (4) \Leftrightarrow (5) \Rightarrow (6) \Leftrightarrow (7) \Leftrightarrow (8). \tag{2.2}$$

Proof. Clear. □

As we did in [T], we start from the following fact:

Lemma 2 *For any angle $\alpha \in (0, \pi/2)$ and any positive number κ, we can find a radius r and an α-slant homothetic full immersion $f : I^2 \to \mathbb{C}^2$ of the standard open square I^2 into $\mathbb{C}^2$ with homothetic factor κ, such that the image $f(I^2)$ is contained in the 3-sphere $S^3(r)$ centered at the origin.*

This comes from the existence of spherical slant surfaces in $\mathbb{C}^2$ ([CT2] Lemma 4.4 and Theorem 1.5). Such spherical slant surfaces are flat. Since the slant angle is determined by the immersed image and is invariant under homotheties with respect to the origin, we can reparametrize the surface as stated in the lemma above.

The following lemma gives slant surfaces in $\mathbb{C}^3$, which are full over $\mathbb{C}$, although they may not be full in the ordinary sense.

Lemma 3 *For any angle $\alpha \in (0, \pi/2)$ and any positive number κ, we can find an immersion $f : I^2 \to \mathbb{C}^3$ of the standard open square I^2 into $\mathbb{C}^3$ which is α-slant, homothetic with homothetic factor κ, and whose image is not contained in any 4-plane in $\mathbb{E}^6$. In particular, f is full over $\mathbb{C}$.*

Proof. Pick any $\alpha_1 \in (\alpha, \pi/2)$. We can choose a positive number c such that $\cos\alpha = (\cos\alpha_1 + c)/(1 + c)$, and then positive numbers κ_1 and κ_2 so that $(\kappa_1)^2 +$

$(\kappa_2)^2 = \kappa^2$ and $(\kappa_2/\kappa_1)^2 = c$. Let $f : I^2 \to S^3(r) \subset \mathbb{C}^2$ be an α_1-slant homothetic spherical full immersion with the homothetic factor κ_1 described in Lemma 2. Orient I^2 so that the extended slant angle (cf. [T]) β_1 of f coincides with α_1. Parametrize I^2 by (s,t) and define a 0-slant homothety $h : I^2 \to \mathbb{C}$ by $h(s,t) = \kappa_2(s+it)$.

By Lemma 2 of [T], the direct sum $f \oplus h : I^2 \to \mathbb{C}^3$ is a homothetic slant immersion with the homothetic factor κ and the extended slant angle β given by

$$\beta = \arccos\left(\frac{(\kappa_1)^2\cos\alpha_1 + (\kappa_2)^2}{(\kappa_1)^2 + (\kappa_1)^2}\right) = \alpha. \tag{2.3}$$

Note that β coincides with the slant angle of $f \oplus h$, since $\alpha < \pi/2$.

To show the fullness over $\mathbb{C}$, we put

$$X_1 = (1,0),\ X_2 = (0,1), \tag{2.4}$$

$$\begin{aligned} \mathcal{A} &= \{(f_*)_p X_1 \mid p \in I^2\} \subset S^3(\kappa_1) \subset \mathbb{E}^4, \\ \mathcal{B} &= \{(f_*)_p X_2 \mid p \in I^2\} \subset S^3(\kappa_1) \subset \mathbb{E}^4, \end{aligned} \tag{2.5}$$

$$\begin{aligned} \widetilde{\mathcal{A}} &= \{((f \oplus h)_*)_p X_1 \mid p \in I^2\} = \mathcal{A} \times (\kappa_2, 0) \subset \mathbb{E}^6, \\ \widetilde{\mathcal{B}} &= \{((f \oplus h)_*)_p X_2 \mid p \in I^2\} = \mathcal{B} \times (0, \kappa_2) \subset \mathbb{E}^6. \end{aligned} \tag{2.6}$$

Then, $\mathcal{A} \cup \mathcal{B} \subset \mathcal{T}_f$ and $\widetilde{\mathcal{A}} \cup \widetilde{\mathcal{B}} \subset \mathcal{T}_{f\oplus h}$. Note that, since f is a spherical immersion, neither $\mathcal{A}$ nor $\mathcal{B}$ is contained in any 1-dimensional linear subspace of $\mathbb{E}^4$.

Case 1. Suppose that $\mathcal{A}$ and $\mathcal{B}$ are contained in 2-dimensional linear subspaces of $\mathbb{E}^4$. Then, all s-curves and t-curves on $f(I^2)$ are plane curves, i. e., portions of circles on the 3-sphere $S^3(r)$. If $\mathcal{A}$ and $\mathcal{B}$ are contained in a common 3-dimensional linear subspace of $\mathbb{E}^4$, then $\mathcal{T}_f$ is also contained in this 3-dimensional linear subspace, which contradicts the fullness of f by Lemma 1. Therefore, $\mathcal{A}$ and $\mathcal{B}$ are contained in two mutually complementary linear subspaces, say V and W, respectively. Hence, the s-curve on $(f \oplus h)(I^2)$ through a point $(f \oplus h)(s_0, t_0)$ is a helicoid in $V(s_0,t_0) \times \mathbb{E} \times \{\kappa_2 t_0\}$, and the t-curve is a helicoid in $W(s_0,t_0) \times \{\kappa_2 s_0\} \times \mathbb{E}$, where $V(s_0,t_0)$ and $W(s_0,t_0)$ are 2-planes in $\mathbb{E}^4$ passing through the point $f(s_0,t_0)$ and parallel to V and W, respectively. This shows that the immersion $f \oplus h$ is full in $\mathbb{E}^6$.

Case 2. Suppose that one of $\mathcal{A}$ and $\mathcal{B}$, say $\mathcal{A}$, is contained in a 3-dimensional linear subspace W in $\mathbb{E}^4$ but not contained in any 2-dimensional linear subspace of $\mathbb{E}^4$. Since f is full, $\mathcal{A} \cup \mathcal{B}$ is not contained in W, and hence we can pick a point b_1 in $\mathcal{B} - W$. $\mathcal{B}$ is connected in $S^3(\kappa_1)$ and not a singleton, so we can pick two other points b_2 and b_3 in $\mathcal{B}$ such that the three points b_1, b_2, and b_3 are not collinear.

Next, we can pick three points a_1, a_2, and a_3 in $\mathcal{A}$ such that they are linearly independent in W as position vectors, and moreover that the 2-plane passing through these three points is not parallel to the 2-plane passing through b_1, b_2, and b_3. The reason is as follows. If $\mathcal{A}$ is contained entirely in a 2-plane in $\mathbb{E}^4$ parallel to the 2-plane passing through b_1, b_2, and b_3, then $\mathcal{A}$ is contained in a small, not a great, circle on the 2-sphere $S^3(\kappa_1) \cap W$. Let v be a vector in W perpendicular to this small circle. Then, for any point $p_0 \in I^2$ the s-curve $c(s)$ on $f(I^2)$ through $f(p_0)$ is a helicoid in a 3-plane $\tilde{W}$ through $f(p_0)$ in $\mathbb{E}^4$ parallel to W, satisfying $< c'(s), v >= \mathit{const} \neq 0$.

This contradicts the fact that $c(s)$ lies in the 2-sphere $S^3(r) \cap \tilde{W}$. Accordingly, if we put

$$b_2 - b_1 = \lambda_1 a_1 + \lambda_2 a_2 + \lambda_3 a_3 + \lambda_4 b_1, \\ b_3 - b_1 = \mu_1 a_1 + \mu_2 a_2 + \mu_3 a_3 + \mu_4 b_1, \tag{2.7}$$

then at least one of $\lambda_1 + \lambda_2 + \lambda_3, \lambda_4, \mu_1 + \mu_2 + \mu_3$, and μ_4 is nonzero. Put $a_i = (a_{i1}, a_{i2}, a_{i3}, a_{i4}), b_i = (b_{i1}, b_{i2}, b_{i3}, b_{i4})$, and consider the matrix

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & \kappa_2 & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} & \kappa_2 & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} & \kappa_2 & 0 \\ b_{11} & b_{12} & b_{13} & b_{14} & 0 & \kappa_2 \\ b_{21} & b_{22} & b_{23} & b_{24} & 0 & \kappa_2 \\ b_{31} & b_{32} & b_{33} & b_{34} & 0 & \kappa_2 \end{pmatrix}. \tag{2.8}$$

By elementary transformation of matrix, we get

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & \kappa_2 & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} & \kappa_2 & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} & \kappa_2 & 0 \\ b_{11} & b_{12} & b_{13} & b_{14} & 0 & \kappa_2 \\ 0 & 0 & 0 & 0 & -(\lambda_1 + \lambda_2 + \lambda_3)\kappa_2 & -\lambda_4 \kappa_2 \\ 0 & 0 & 0 & 0 & -(\mu_1 + \mu_2 + \mu_3)\kappa_2 & -\mu_4 \kappa_2 \end{pmatrix}. \tag{2.9}$$

The rank of this matrix is at least 5, which means that $\widetilde{\mathcal{A}} \cup \widetilde{\mathcal{B}}$ is not contained in any 4-dimensional linear subspace of $\mathbb{E}^6$.

Case 3. Suppose that one of $\mathcal{A}$ and $\mathcal{B}$, say $\mathcal{A}$, is not contained in any 3-dimensional linear subspace of $\mathbb{E}^4$. We pick up four points $a_1, ..., a_4$ in $\mathcal{A}$ whose position vectors are linearly independent in $\mathbb{E}^4$, and two points b_1, b_2 in $\mathcal{B}$ whose position vectors are linearly independent in $\mathbb{E}^4$. If we put $a_i = (a_{i1}, a_{i2}, a_{i3}, a_{i4})$ and $b_i = (b_{i1}, b_{i2}, b_{i3}, b_{i4})$, then the rank of the matrix

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & \kappa_2 & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} & \kappa_2 & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} & \kappa_2 & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} & \kappa_2 & 0 \\ b_{11} & b_{12} & b_{13} & b_{14} & 0 & \kappa_2 \\ b_{21} & b_{22} & b_{23} & b_{24} & 0 & \kappa_2 \end{pmatrix} \tag{2.10}$$

is at least 5. Therefore, $\widetilde{\mathcal{A}} \cup \widetilde{\mathcal{B}}$ is not contained in any 4-dimensional linear subspace of $\mathbb{E}^6$.

In any of *Cases 1* to *3*, the image of $f \oplus h$ is not contained in any 4-plane in $\mathbb{E}^6$ by Lemma 1, and hence $f \oplus h$ is full over $\mathbb{C}$. $\square$

We alter the statements of Propositions 1 and 2 in [T] from the view point of fullness over $\mathbb{C}$, and get the following lemmas 4 to 6.

Lemma 4 *If the two immersions $f : M \to \mathbb{C}^m$ and $h : M' \to \mathbb{C}^l$ are full over $\mathbb{C}$, then the cartesian product $f \times h : M \times M' \to \mathbb{C}^{m+l}$ is also full over $\mathbb{C}$.*

Proof. Since

$$\mathcal{T}_{f\times h} \supset \mathcal{T}_f \cup \mathcal{T}_h,\ J(\mathcal{T}_{f\times h}) \supset J(\mathcal{T}_f) \cup J(\mathcal{T}_h), \tag{2.11}$$

the equivalence of (6) and (8) of Lemma 1 means our claim. Note that in (6) the minimal linear subspace containing $\mathcal{T}_f \cup J(\mathcal{T}_f)$ is invariant under J. □

Lemma 5 *Let $f : I^n \to \mathbb{E}^m$ be an immersion such that the image $f(I^n)$ is not contained in any l-plane in $\mathbb{E}^m$ ($l < m$). Then, there is a spherical full immersion $h : I^n \to \mathbb{E}^{2n}$ such that the image $(f \oplus h)(I^n)$ of the direct sum $f \oplus h : I^n \to \mathbb{E}^{m+2n}$ is not contained in any $(l + 2n)$-plane in $\mathbb{E}^{m+2n}$.*

Proof. The proof of Proposition 2 (ii) of [T] is valid also here after a small adjustment, just choosing $(l+1)$ points $p_0, ..., p_l$ in *Step 2* instead of choosing $(m+1)$ points, and accordingly $(l + 2n + 1)$ points in *Step 5.* □

Lemma 6 *Let $f : I^n \to \mathbb{E}^m$ be an immersion of the standard open n-cube I^n into $\mathbb{E}^m$. Assume that the image $f(I^n)$ is not contained in any $(m-1)$-dimensional linear subspace of $\mathbb{E}^m$. Then, we can choose a spherical full immersion $h : I^n \to \mathbb{E}^{2n}$ such that the image $(f \otimes h)(I^n)$ of the tensor product $f \otimes h : I^n \to \mathbb{E}^m \otimes \mathbb{E}^{2n} = \mathbb{E}^{2mn}$ is not contained in any $(2mn - 1)$-dimensional linear subspace of $\mathbb{E}^{2mn}$.*

Proof. The idea is basically the same as that of Proposition 2 (ii) of [T].

Step 1. Let $T^n(r)$ be the same as in *Step 1* of Proposition 2 (ii) of [T]. Pick $2n$ points $q_1, ..., q_{2n}$ in $T^n(r)$ whose position vectors are linearly independent in $\mathbb{E}^{2n}$.

Step 2. Pick up m points $p_1, ..., p_m$ in I^n such that $f(p_1), ..., f(p_m)$ are linearly independent in $\mathbb{E}^m$. Choose $\epsilon > 0$ such that, if we pick any point x_i out of each ϵ-neighborhood V_i of $f(p_i)$ in $\mathbb{E}^m$, then $x_1, ..., x_m$ are linearly independent in $\mathbb{E}^m$.

Step 3. Divide I^n into small cubes and pick A_i for each $i = 1, ..., m$ in the same way as in *Step 3* of Proposition 2 (ii) of [T], except for the range of the index i and replacing the U_i's with V_i's.

Step 4. Let the h and p_{ij}'s be the same as in *Step 4* of Proposition 2 (ii) of [T], except for the ranges of the indices i, j and that $h(p_{ij}) = q_j$.

Step 5. The $2mn$ points $(f \otimes h)(p_{ij}), i = 1, ..., m, j = 1, ..., 2n$ are linearly independent in $\mathbb{E}^m \otimes \mathbb{E}^{2n}$, and so the image $(f \otimes h)(I^n)$ is not contained in any $(2mn - 1)$-dimensional linear subspace of $\mathbb{E}^{2mn}$. □

3 Construction

We construct the wanted immersions by combining the slant surfaces of Lemmas 2 and 3 by means of cartesian product $\times$, direct sum $\oplus$, and tensor product $\otimes$ of immersions.

Case 1. Assume $\alpha \in (0, \pi/2)$.

Let $\varphi_j : I^2 \to \mathbb{C}^2$, $j = 0, 1, ..., k$ be spherical homothetic full slant immersions described in Lemma 2, and let $\psi_0 : I^2 \to \mathbb{C}^3$ be a slant full immersion over $\mathbb{C}$ obtained by Lemma 3.

By Theorem (1) of [T], we already have α-slant spherical homothetic full immersions of types

$$f = \varphi_1 \times \ \cdots \ \times \varphi_k : I^{2k} \to \mathbb{C}^{2k} \tag{3.1}$$

and

$$f' = \varphi_0 \oplus h_1 \oplus \ \cdots \ \oplus h_{k'-1} : I^2 \to \mathbb{C}^{2k'} \tag{3.2}$$

by a suitable choice of $h_1, ..., h_{k'-1}$.

Replacing φ_0 in (3.2) with ψ_0 and choosing $\widehat{h}_1, ..., \widehat{h}_{\widehat{k}-1}$ suitably, we get by Lemma 5 above and Lemma 2 of [T] a slant homothetic immersion

$$\widehat{f} = \psi_0 \oplus \widehat{h}_1 \oplus \ \cdots \ \oplus \widehat{h}_{\widehat{k}-1} : I^2 \to \mathbb{C}^{2\widehat{k}+1} \tag{3.3}$$

whose image $\widehat{f}(I^2)$ is not contained in any $4\widehat{k}$-plane in $\mathbb{E}^{4\widehat{k}+2}$. By Lemma 6 above and Lemma 3 of [T], we also have a slant homothetic immersion

$$\widetilde{f} = \varphi_0 \otimes \widetilde{h}_1 \otimes \ \cdots \ \otimes \widetilde{h}_{\widetilde{k}} : I^2 \to \mathbb{C}^{2\times 4^{\widetilde{k}}} \tag{3.4}$$

whose image $\widetilde{f}(I^2)$ is not contained in any $(4 \times 4^{\widetilde{k}} - 1)$-dimensional linear subspace of $\mathbb{E}^{4\times 4^{\widetilde{k}}}$. In choosing ψ_0, $\widehat{h}_i$'s, φ_0 and $\widetilde{h}_i$'s of (3.3) and (3.4), we have to be careful about the change of slant angles so that $\widehat{f}$ and $\widetilde{f}$ have eventually the prescribed slant angle α. This can be done in the similar way as we did in the proof of Proposition of [T], but we don't state the detail here.

The immersions of the types (3.1),...,(3.4) are all full over $\mathbb{C}$ by Lemma 1. Therefore, for any combination $f_1, ..., f_l$ of these immersions such that the sum of the dimensions of the immersed manifolds is equal to $2n$, and at the same time, the sum of the real dimensions of the target spaces is equal to $2m$, the cartesian product

$$f_1 \times \ \cdots \ \times f_l : I^{2n} \to \mathbb{C}^m \tag{3.5}$$

is also α-slant and full over $\mathbb{C}$ by Lemma 4 above and Proposition 1 of [T] .

Case 2. Assume that $\alpha = \pi/2$.

Note that the dimension of immersed manifolds does not have to be even in this case, and that the statements in Propositions 1 and 2 of [T] hold also for $n = 1$. Let φ be a totally real immersion defined by

$$\varphi : I \to \mathbb{C} \ ; \ \varphi(t) = e^{2\pi i t}. \tag{3.6}$$

Replacing the immersions φ_j's in (3.1) and (3.2) by this φ, we get the first half of our conclusion.

Since $T^n = S^1 \times \ \cdots \ \times S^1$ and $S^1 = \bar{I}/\{0, 1\}$, the second half of *Case 2* is obtained immediately.

This completes our construction.

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