MATHEMATICAL MORPHOLOGY BASED ON LOGARITHMIC IMAGE PROCESSING THEORY

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Abstract. The purpose of this paper is to define and to analyze three new sets of logarithmic morphological operators and it is focused on theoretical and practical aspects concerning the enhancement of the transmitted images and the physical absorption/transmission laws expressed within LIP (Logarithmic Image Processing) mathematical framework. Using different logarithmic image representations, an approach of redefining mathematical morphology operators based upon structuring elements with a variable geometrical shape or adaptative structuring elements is presented here. The specific LIP algebraic and functional operations and structures are very well adapted to image representation and processing, and more generally to digital signal processing within a bounded intensity range. This very well structured theory determined us to use the logarithmic image representation in our approach of defining three new categories of mathematical morphology operators: Multiplicative LMO (Logarithmic Morphological Operators), Additive LMO and Additive-Multiplicative LMO.

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1. Introduction

Many representation techniques are currently available in the signal and image processing domain, but most of them require an intensive computation and sequential implementation, disregarding the geometrical information present in signals. By extending mathematical morphology for logarithmic image representation we will take the advantage of a well structured mathematical framework within LIP (Logarithmic Image Processing) theory where we can redefine the entire hierarchy (or pyramid) of morphological operators based upon structuring elements with a variable geometrical shape or adaptative structuring elements. They prove to be very well adapted to the representation and processing of images, and more generally of signals, valued in a bounded intensity range, while reducing the algorithms complexity for efficient parallel implementation. In the past, there were several approaches to define an adaptative structuring element which could change geometrical shape for each image pixel. For example, P. Salembier [4] proposed such an adaptative structuring element where total pixel number varies between an inferior and a superior limit for a given neighborhood using the criteria of minimization of mean squared error calculated from the output image and the desired signal.

More recently, C.-P. Huang and L. F. Chaparro [10] developed a novel signal representation using fuzzy mathematical morphology and achieve geometrical decomposition of a signal by windowing and applying sequentially fuzzy morphological opening with adaptative structuring functions. Before them, T. Kichuchi et al. [9] defined the fuzzy adaptive structuring element (FASE), sensitive to ambiguous images, where the shapes and values of structuring elements are dynamically determined from an input image by searching for the local regions in the image.

Also, using fuzzy morphological operations with adaptive structuring elements, S. Letitia et al. [11] can dynamically modify the shape, size and gray-scale values of the structuring elements, based on the geometric and radiometric properties of the objects to be retained. In this paper we will present several possibilities of defining structuring elements with a variable geometrical shape within the logarithmic image processing theory, i.e. using only vector-oriented operations with images represented as gray-tone functions operands. Finally we will demonstrate that, in fact, all the classical mathematical morphology operators are solely a particular case of the logarithmic mathematical morphology operators, based upon the definition of the structuring elements with a variable geometrical shape or adaptative structuring elements. At least three categories of mathematical morphology operators can be defined within the context of a logarithmic image representation:

1. multiplicative logarithmic morphological operators;
2. additive logarithmic morphological operators;
3. additive-multiplicative logarithmic morphological operators;

In this paper we will only analyze the properties of the first category of derived mathematical morphology operators, i.e. multiplicative logarithmic morphological operators. From now on we will associate both the image and the structuring element with gray-tone functions defined by M. Jourlin and J.C. Pinoli ([1]):

$$F : D \to E, \text{ where } D \subset \Upsilon^2 \text{ and } E = [0, M) \text{ or } E' = (-M, M), M > 0. \quad (1)$$

Also, we will denote by $I(D, E)$ the set of gray-tone functions, defined on a spatial support $D \subset \Upsilon^2$, and taking values within a gray-tone interval $E = [0, M)$ or $E' = (-M, M)$.
First of all, we will use the definition of the specific operations in the logarithmic image processing theory as presented by M. Jourlin, J.C. Pinoli, V. Pătraşcu and V. Buzuloiu [1], [2]. In the set of gray levels E we will define the logarithmic addition $\oplus$:

$$\forall u, v \in E,\ u \oplus v = \frac{u + v}{1 + \frac{uv}{M^2}} \quad (2)$$

where the operations in the right side are meant in Υ. In the same set of gray levels E we will define the real scalar multiplication $\otimes$. For $\forall \lambda^{TM}\Upsilon, \forall u^{TM}E$, we define the logarithmic product between λ and u by:

$$\forall \lambda^{TM}\Upsilon, \forall u \in E, \lambda \otimes u = M \cdot \frac{(M + u)^{\lambda} - (M - u)^{\lambda}}{(M + u)^{\lambda} + (M - u)^{\lambda}} \quad (3)$$

where again the operations in the right hand side of the equality are meant in Υ. The two operations, addition $\oplus$ and scalar multiplication $\otimes$ establish on E a real vector space structure as demonstrated in [2].
A gray level image is a function defined on a bi-dimensional compact D from Υ^2 taking the values in the gray level space E. We denote with $F(D, E)$ the set of gray level images defined on D. The operations and the functions from gray level space E to gray level images $F(D, E)$ can be extended in a very natural way as shown in [1], [2]: Logarithmic addition for gray level images is defined as:

$$\forall f_1, f_2^{TM} F(D, E), \forall (x, y)^{TM} D, (f_1 \oplus f_2)(x, y) = f_1(x, y) \oplus f_2(x, y). \quad (4)$$

Logarithmic scalar multiplication for gray level images is:

$$\forall \lambda^{TM}\Upsilon, \forall f^{TM} F(D, E), \forall (x, y)^{TM} D, (\lambda \otimes f)(x, y) = \lambda \otimes f(x, y). \quad (5)$$

The two operations, addition and scalar multiplication establish on F(D,E) a real vector space structure [2]. For the morphological operators we will use the classical functional definition from [5] and [6], where the morphological erosion and dilation are defined as follows:

$$(f \exists \breve{g})(x) = \inf\{f(y) - g(y-x) | y \in \Upsilon^n\} \tag{6}$$

$$(f \oplus \breve{g})(x) = \sup\{f(y) + g(y-x) | y \in \Upsilon^n\} \tag{7}$$

where f and g are semi-continuous functions from Υ^n to $\Upsilon Y\{-\infty, \infty\}$ and $\breve{g}$ is the symmetrical structuring element defined as follows: $\forall x \in \Upsilon^n, \breve{g}(x) = g(-x)$.

Multiplicative logarithmic morphological operators

A. Multiplicative Logarithmic Morphological Erosion and Dilation

Definition 1: *Multiplicative logarithmic morphological erosion for the image f by structuring element g, represents the gray-tone function defined as follows:*

$$(f \exists_{ML} \breve{g})(x) = \inf\{k \otimes (f(y) - g(y-x)) : y \in \Upsilon^2\} \tag{8}$$

Definition2: *Multiplicative logarithmic morphological dilation for the image f by structuring element g, represents the gray-tone function defined as follows*

$$(f \oplus_{ML} \breve{g})(x) = \sup\{k \otimes (f(y) + g(y-x)) | y \in \Upsilon^2\}. \tag{9}$$

In the definitions (1) and (2) is the symmetrical structuring element ($\forall x \in \Upsilon^2, \breve{g}(x) = g(-x)$) and $\otimes$ represents LIP product of a gray-tone function with a real scalar.

Within different LIP models we will obtain the following definitions for multiplicative logarithmic morphological erosion and dilation, respectively:

1. M. JOURLIN and J.-C. PINOLI [1] logarithmic model:

$$(f \exists_{ML} \breve{g})(x) = \inf\{M - M(1 - \frac{f(y) - g(y-x)}{M})^k | y \in \Upsilon^2\} \tag{10}$$

$$(f \oplus_{ML} \breve{g})(x) = \sup\{M - M(1 - \frac{f(y) - g(y-x)}{M})^k | y \in \Upsilon^2\}. \tag{11}$$

2. . Pătraşcu and V. Buzuloiu [2] logarithmic model:

$$(f\exists_{ML}\breve{(}g))(x) = \inf\{M\cdot\frac{(M+f(y)-g(y-x))^k-(M-f(y)-g(y-x))^k}{(M+f(y)-g(y-x))^k+(M-f(y)-g(y-x))^k}|y\in\Upsilon^2\} \quad (12)$$

$$(f\oplus_{ML}\breve{g})(x) = \sup\{M\cdot\frac{(M+f(y)-g(y-x))^k-(M-f(y)-g(y-x))^k}{(M+f(y)-g(y-x))^k+(M-f(y)-g(y-x))^k}|y\in\Upsilon^2\} \quad (13)$$

Observation: *Scalar k can be a constant for the entire image or can be another gray-tone function (i.e. an adaptative scalar multiplication) defined like this:*

$$\forall x\in\Upsilon^2, k(x) = \frac{f(x)}{M}. \quad (14)$$

Observation: *: As shown in the last section, "Experimental results", the structuring elements have the particular behavior of a variable geometrical shape SE all over the definition domain, i.e. larger or wider when the derivative gray-tone function f'(x) is low and narrower when high.*

B. Multiplicative Logarithmic Morphological Erosion Properties

Proposition 1:*Multiplicative logarithmic morphological erosion for image f by the structuring element g_k, characterized by the constant scalar parameter $k = 1$ represents the classical morphological erosion defined by the same structuring element.*

Proposition 2: *Let be the gray-tone functions $f, f', g, g' : D \to E$. Then, the following propositions are truthful and can be easily demonstrated like in [6]:*

a) If the origin of the set Υ^2, belongs to the support set of the structuring element g, then $\exists_{ML}$ is anti-extensive : $f\exists_{ML}g \le f$.

b) $\exists_{ML}$ is ascending related to f and descending related to g:

$$f \le f' \Rightarrow f\exists_{ML}g \le f'\exists_{ML}g \quad (15)$$

$$g \le g' \Rightarrow f\exists_{ML}g' \le f\exists_{ML}g. \quad (16)$$

c) $\exists_{ML}$ verifies the following equations (where stands for "sup" and $\wedge$ stands for "inf"):

$$f\exists_{ML}(g\vee g') = (f\exists_{ML}g)\wedge(f\exists_{ML}g') \quad (17)$$

$$f\exists_{ML}(g\wedge g') \ge (f\exists_{ML}g)\vee(f\exists_{ML}g') \quad (18)$$

C. Multiplicative Logarithmic Morphological Dilation Properties

Proposition 3: *Multiplicative logarithmic morphological dilation for image f by the structuring element gk, characterized by the constant scalar parameter $k = 1$, represents the classical morphological erosion defined by the same structuring element. Because dilation is a dual operator for erosion, then all the algebraic properties already presented for erosion can be transposed for dilation based upon duality principle as follows:*

Proposition 4: *Let be the gray-tone functions $f, f', g, g' : D \to E$. Then, the following propositions are truthful and can be easily demonstrated like in [6]:*

a) If the origin of the set Υ^2, belongs to the support set of the structuring element g then $\oplus_{ML}$ is extensive i.e.: $f \leq f \oplus_{ML} g$

b) $\oplus_{ML}$ is ascending related to f and g, i.e.:

$$f \leq f' \Rightarrow f \oplus_{ML} g \leq f \oplus_{ML} g \tag{19}$$

$$g \leq g' \Rightarrow f \oplus_{ML} g \leq f \oplus_{ML} g' \tag{20}$$

c) $\oplus_{ML}$ is distributive related "sup"(where stands for "sup" operator) i.e.:

$$f \oplus_{ML} (g \vee g') = (f \oplus_{ML} g) \vee (f \oplus_{ML} g') \tag{21}$$

d) $\oplus_{ML}$ verifies the following equation(where stands for "inf" operator):

$$f \oplus_{ML} (g \wedge g') \leq (f \oplus_{ML} g) \wedge (f \oplus_{ML} g') \tag{22}$$

e) $\oplus_{ML}$ is associative i.e.:

$$(f \oplus_{ML} g) \oplus_{ML} g' = f \oplus_{ML} (g \oplus_{ML} g') \tag{23}$$

The third property shows the fact that, as structuring element is defined as the "sup" for other elements, it is sufficient to perform the dilations for every element, individually, and then to operate with "sup", finally. Consequently, any more complex dilation can be decomposed in several elementary dilations. The fourth property gives us another rule for decomposing the dilation, very useful in practice. It shows the fact that the dilation of a function f by plane disk with radius $p \in \mathbb{N}^*$ will produce the same result obtained by the p times repeated dilation by plane disk with radius 1.

Obervation: *Multiplicative logarithmic morphological erosion and dilation are compatible with translations.*
Because multiplicative logarithmic morphological erosion and dilation are dual transformations, but not reverse to each other, their functional composition allow us to generate the pyramid of the derivative transformations: multiplicative logarithmic morphological opening and closing.
D. Multiplicative Logarithmic Morphological Opening and Closing
Definition 3: *Multiplicative logarithmic morphological opening for the image f by structuring element g, represents the gray-tone function denoted by ψ_g^{ML} and defined as follows:*

$$\psi_g^{ML} = (f \exists_{ML} \breve{g}) \oplus_{ML} g. \tag{24}$$

Definition 4: *Multiplicative logarithmic morphological closing for the image f by structuring element g, represents the gray-tone function denoted by ϕ_g^{ML} and defined as follows:*

$$\phi_g^{ML} = (f \oplus_{ML} \breve{g}) \exists_{ML} g. \tag{25}$$

In the definitions (3) and (4) $\breve{g}$ is the symmetrical structuring element ($\forall x \in \Upsilon^2, \breve{g}(x) = g(-x)$. Also, $\exists_{ML}$ and $\oplus_{ML}$ represent multiplicative logarithmic morphological erosion and dilation, respectively, already defined in (1) and (2).
E. Multiplicative Logarithmic Morphological Opening and Closing Properties
Proposition 5: *: Multiplicative logarithmic morphological opening and closing for image f by the structuring element gk, characterized by the constant scalar parameter $k = 1$, represents the classical morphological opening and closing defined by the same structuring element.*
Proof: This result can be easily achieved using the propositions (1) and (3) which proof the same property for the fundamental operators, multiplicative logarithmic morphological erosion and dilation.
Observation: *Multiplicative logarithmic morphological opening and closing are compatible with translation.*
Proposition 6: *Multiplicative logarithmic morphological opening and closing are idempotent transforms, i.e.:*

$$\forall g \in I(D,E), \psi_g^{ML} \circ \psi_g^{ML} = \psi_g^{ML} \text{ and } \phi_g^{ML} \circ \phi_g^{ML} = \phi_g^{ML} \tag{26}$$

where we have denoted by $I(D,E)$ the set of gray-tone functions, defined onto a spatial support $D \subset \Upsilon^2$ and taking values in a gray-tone interval $E = [0, M)$ or $E' = (-M, M)$.

Proposition 7: *Multiplicative logarithmic morphological opening and closing are ascending transforms related to the image, i.e.:*

$$\forall f, f', g \in I(D,E) \text{ with } f \leq f' \Rightarrow \psi_g^{ML}(f) \leq \psi_g^{ML}(f') \text{ and } \phi_g^{ML}(f) \leq \phi_g^{ML}(f') \tag{27}$$

Proposition 8: *Multiplicative logarithmic morphological opening and closing are anti-extensive (or extensive) transforms, respectively, related to the image, i.e.:*

$$\forall f, g \in I(D,E) \quad \psi_g^{ML}(f) \leq f \text{ and } \phi_g^{ML}(f) \geq f. \tag{28}$$

Logarithmic top-hat transforms

In the context of the transmitted signals through different optical mediums, the objects observed on a dark background generate smaller peaks associated with segmentation threshold r, then the peaks generated in the case of a light background. This phenomenon occurs due to the non-linear physics laws for the absorption associated to the optical mediums. Using the logarithmic contrast M. Jourlin and J.C. Pinoli have introduced the notion of Logarithmic Top-Hat (LTH) [1]. In this new logarithmic morphological context we can introduce two new different logarithmic top-hat transforms:

Definition 5: *Logarithmic White Top-Hat Transform of an image f represents the logarithmic contrast between the gray-tone function and its logarithmic opening:*

$$LWTH(f)(x,y) = \frac{f(x,y) - \delta_g[\varepsilon_g(f(x,y))]}{1 - \frac{\delta_g[\varepsilon_g(f(x,y))]}{M}} \tag{29}$$

where $\varepsilon_g(f(x,y))$ stands for the logarithmic morphological erosion, $\delta_g(f(x,y))$ stands for the logarithmic morphological dilation and obviously, in the end, we can rewrite the definition of logarithmic opening with this new notation like this:

$$\psi_g^L = \delta_g[\varepsilon_g(f(x,y))]. \tag{30}$$

Analog, like in the classic morphology, we can introduce its complementary transform associated:

Definition 6: *Logarithmic Black Top-Hat Transform of an image f represents the logarithmic contrast between the logarithmic closing of a gray-tone function and function itself:*

$$LBTH(f)(x,y) = \frac{\varepsilon_g[\delta_g(f(,y))] - f(x,y)}{1 - \frac{f(x,y)}{M}} \tag{31}$$

where $\phi_g^L = \varepsilon_g[\delta_g(f(x,y))]$ *stands for the logarithmic morphological closing.*

These transforms are used in image segmentation by detecting the interesting objects associated with the peaks of the gray-tone function f (in the case of logarithmic white top-hat transform defined by opening) or by detecting the local minimums (in the case of logarithmic black top-hat transform defined by closing) [8].

Image segmentation is realized by selecting the pixels (x, y) in the spatial domain, satisfying the following criteria:

$$LWTH(f)(x,y) > r. \tag{32}$$

The purpose of the structuring element g and the segmentation threshold r is to select the important peaks (higher and wider then the unimportant noise peaks) [8].

4. Experimental results

The experimental results, subsequently presented in this paper, used the multiplicative version of the logarithmic top-hat transforms:

Definition 7: *Multiplicative Logarithmic White Top-Hat Transform of an image f represents the logarithmic difference between the gray-tone function and its multiplicative logarithmic opening:*

$$WTH_{ML}(f)(x,y) = f \exists \psi_g^{ML} = f \exists ((f \exists_{ML} \breve{g}) \oplus_{ML} g) \tag{33}$$

Definition 8: *Multiplicative Logarithmic Black Top-Hat Transform of an image f represents the logarithmic difference between multiplicative logarithmic closing of the gray-tone function and function itself:*

$$BTH_{ML}(f)(x,y) = \phi_g^{ML} \exists f = ((f \oplus_{ML} \breve{g}) \exists_{ML} g) \exists f \tag{34}$$

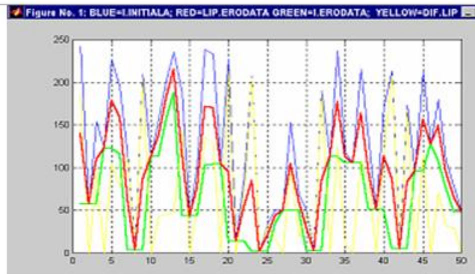

Fig. 1. Multiplicative logarithmic morphological erosion.
Blue=gray-tone function; Red=multiplicative logarithmic morphological erosion; Green=classical morphological erosion; Yellow=multiplicative logarithmic difference between gray-tone function and its eroded

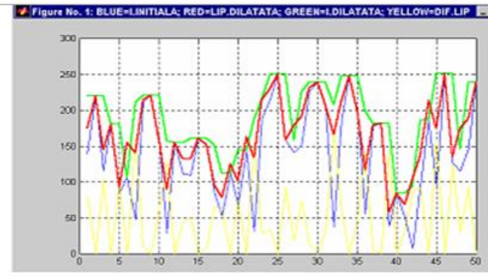

Fig. 2. Multiplicative logarithmic morphological dilation.
Blue=gray-tone function; Red=multiplicative logarithmic morphological dilation; Green=classical morphological dilation; Yellow=multiplicative logarithmic difference between dilated gray-tone function and function itself.

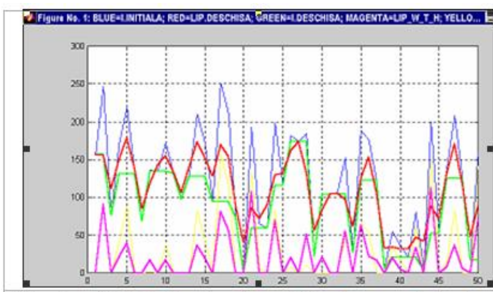

Fig. 3. Multiplicative logarithmic morphological opening.
Blue=gray-tone function; Red=multiplicative logarithmic morphological opening; Green=classical morphological opening; Magenta= multiplicative logarithmic white top-hat; Yellow= classical morphological white top-hat

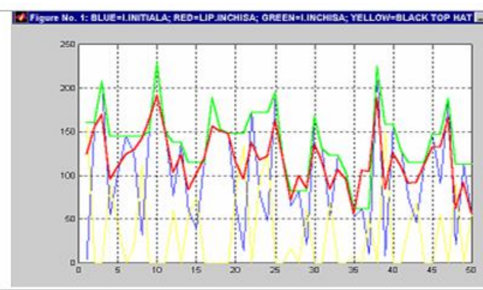

Fig. 4. Multiplicative logarithmic morphological closing.
Blue=gray-tone function; Red=multiplicative logarithmic morphological closing; Green=classical morphological closing; Yellow= classical morphological black top-hat transform

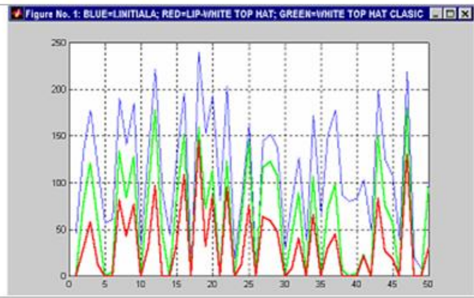

Fig. 5. Multiplicative logarithmic white top-hat transform.
Blue=gray-tone function;
Red= multiplicative logarithmic white top-hat;
Green= classical morphological white top-hat

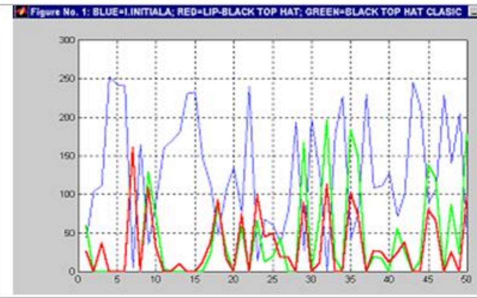

Fig. 6. Multiplicative logarithmic black top-hat transform
Blue=gray-tone function;
Red= multiplicative logarithmic black top-hat;
Green= classical morphological black top-hat

Definition 9: *Logarithmic contrast of an image f represents the logarithmic addition between the gray-tone function and its multiplicative logarithmic white top-hat transform followed by the logarithmic difference with its multiplicative logarithmic black top-hat transform:*

$$Contrast_{ML}(f) = f \oplus WTH_{ML}(f) \exists BTH_{ML}(f). \tag{35}$$

All these experiments were first made in a MATLAB environment and finally translated in a high-level level programming language as presented in [7].

Fig. 7. Multiplicative logarithmic morphological contrast
Logarithmic contrast works better for the very narrow peaks in the original signal or gray-tone function as the structuring element has a variable geometric shape.

Algorithm Complexity

A classical kernel based image processing algorithm (e.g. rank and morphology filters) replaces the pixel at the kernel origin with the result of a function of all pixels defined by the kernel. Direct implementations of such algorithm typically involve visiting all pixels defined by the kernel in order to evaluate the function. Such an approach leads to an algorithm complexity proportional to the number of pixels in the kernel (or $O(n^d)$, where n is the kernel size and d is the dimensionality). The most typically used approaches to reduce complexity of kernel based image processing algorithm are separability and recursive computation. The kernel is usually referred to as the structuring element in mathematical morphology. The separability implementation uses a multidimensional structuring element by cascading several one dimensional structuring elements, therefore reducing complexity from $O(n^d)$ to $O(nd)$.
The second approach, recursive computation, exploits redundancy that might be present in the computations of kernel functions at neighboring locations, leading, in some cases, to a complexity independent of n. Multiplicative logarithmic morphological operators (e.g. erosions and dilations) are separable as we have presented in the properties section. For example, successive dilations by orthogonal lines are equivalent to dilation by a rectangle with sides equal to the line lengths. This means that any hyper-rectangular structuring element can be constructed using several orthogonal lines, typically parallel to the axes. The implemented algorithm uses decomposition of structuring elements in order to achieve the reduced complexity of O(nd). The implemented algorithm relies on the simple concept of an up-datable histogram or "moving histogram" approach. A histogram is computed for a structuring element located at the first pixel. The histogram at the neighboring pixel can then be computed by including newly included pixels and removing newly excluded pixels. The list of included and excluded pixels corresponding to movement in any direction can be computed when the structuring element is created, and the direction with the smallest number of changes should be selected as the direction for sweeping the kernel across the image. The erosion or dilation at each location is computed by selecting the minimum or maximum from the histogram. This approach is very efficient when 8 or 16 bit pixels are used because the histogram can be represented as an array and the histogram updated by incrementing or decrementing the appropriate bins. Using place holders to track the current maximum or minimum increases performance.
The "moving histogram" approach reduces the algorithm complexity from

$O(n^d)$ to $O(n^{d-1})$, while keeping the structuring element identical to the direct implementation Execution times for the multiplicative logarithmic morphological dilation and opening are shown in table 1. The predicted linear complexity for the number of neighbors is observed for the basic algorithm, as well as the constant complexity of the van Herk/Gil Werman and the anchor algorithm. The linear complexity for the number of pixels added and removed per translation is observed for the moving histogram algorithm, as expected.

TABLE I
EXECUTION TIMES (IN MILISECONDS) FOR
MULTIPLICATIVE LOGARITHMIC MORPHOLOGICAL EROSION / DILATION

Algorithm	Adaptive structuring element size			
	11x11	15x15	19x19	23x23
Basic	50	100	150	200
Moving histogram	10	20	30	40
Anchor	4	4	4	4
Van Herk / Gil Werman	3	3	3	3
Iterative basic	10	20	30	40
Separable basic	10	15	20	25
Separable moving histogram	5	5	5	5

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