A SPLINE APROXIMATION OF THE FACTORS PATH IN MDF

by
Nicoleta Breaz , Daniel Breaz

Abstract: It is known that the factorial index obtained by the factors path method (MDF) depend on the path. Beginning with some particular cases of the factors path we will propose in this paper a spline factors path.

The statistical index is an indicator which measures the variation of a variable, variation that can be registered both in space and time. If the variable *Z* is depending on other variables $X_1, X_2, ..., X_n, Z = f(X_1, X_2, ..., X_n)$ we could speak about a single index of the integral variation and *n* indices of the factorial variations that are generated by the explicative variables X_i .The variable which we are refering to in this paper ,is the value of a panel with *n* goods $Z = \sum_{i=1}^{n} p_i q_i$ where $p = (p_1, p_2, ..., p_n)$ and $q = (q_1, q_2, ..., q_n)$ are the prices and quantities vectors coresponding to those *n* goods. In this case, we speak about an index of the prices which is actually a factorial index of *Z* with respect to *p*. This index expresses the partial variation of the panel value, variation measured from the base situation *j* to the observed situation *k*.The cause of this partial variation of the value is in this case, the variation of prices.

There are many methods of calculus for the factorial indices and implicitly for prices indices.

Here we make a short presentation of four of these methods.

Laspeyres method

$$I_{z/p}^{k/j}(\bullet, L) = \frac{\sum_{i=1}^{m} p_i(k) q_i(j)}{\sum_{i=1}^{m} p_i(j) q_i(j)} \qquad I_{z/q}^{k/j}(L, \bullet) = \frac{\sum_{i=1}^{m} p_i(j) q_i(k)}{\sum_{i=1}^{m} p_i(j) q_i(j)}$$

Paasche method

$$I_{z/p}^{k/j}(\bullet, P) = \frac{\sum_{i=1}^{m} p_i(k) q_i(k)}{\sum_{i=1}^{m} p_i(j) q_i(k)} \qquad I_{z/q}^{k/j}(P, \bullet) = \frac{\sum_{i=1}^{m} p_i(k) q_i(k)}{\sum_{i=1}^{m} p_i(k) q_i(j)}.$$

Edgeworth method

$$I_{z/p}^{k/j}(\bullet, E) = \frac{\sum_{i=1}^{m} p_i(k) \frac{q_i(j) + q_i(k)}{2}}{\sum_{i=1}^{m} p_i(j) \frac{q_i(j) + q_i(k)}{2}}$$

and similary $I_{z/q}^{k/j}(E, \bullet)$.

Factors path method(MDF)
Definition 1.

A factorial index of a variable $z = z(t) = f(x_1(t), x_2(t), \ldots x_m(t))$ with respect to X_i factor and (j,k) time interval, given by the formula

$$\mathrm{I}_{z/x_i}^{k/j} = \exp \int_{(P_j P_k)} \frac{f_x'(x_1, x_2, \ldots x_m)}{f(x_1, x_2, \ldots x_m)} dx_i \qquad (1)$$

is called the MDF index.

In the formula (1) (P_j, P_k) represents the segment of arc i.e. a part of the factors path, which links the points P_j, P_k, in R^m space, $P_j(x_1(j) \ldots x_m(j))$, $P_k(x_1(k) \ldots x_m(k))$ that symbolize the base and the observed situation. The path is given by the equations $x_1 = x_1(t), \ldots x_m = x_m(t)$ where *t* is a time parameter.

Definition 2.

An index given by one of next formula (2) with (2') or (2'')is called the prices index MDF.

$$\mathrm{I}_{z/p}^{k/j} = \prod_{i=1}^{m} \mathrm{I}_{z/p_i}^{k/j} \qquad (2)$$

$$\mathrm{I}_{z/p_i}^{k/j} = \exp \int_{M_j M_k} \frac{z'_{p_i}}{z} dp_i = \exp \int_{M_j M_k} \frac{q_i}{\sum_{i=1}^{m} p_i q_i} dp_i = \exp \int_{t_0}^{t_1} \frac{q_1(t) p_i'(t)}{\sum_{i=1}^{m} q_i(t) p_i(t)} dt \qquad (2')$$

$$\mathrm{I}_{z/p}^{k/j} = \exp\int_{t_0}^{t_1} \frac{\sum_{i=1}^{m} q_i(t)p_1'(t)}{\sum_{i=1}^{m} q_i(t)p_i(t)}dt \qquad (2'')$$

Remark 3.

Contrary to the first methods which don't take into account the real data of prices and quantities in *j* and *k* situations, the MDF method realizes all these. The only inconvenient of this method is that the index depends on the path described by the price and quantity factors between the situations nominated by P_j and P_k . This fact, although well sustained by the economic reality, makes impossible the calculus of index whenever the path is unknown.

We present below some particular paths of factors.

The linear path of the factors

In particular cases of a linear path given by the equations

$$x_i(t) = x_i(j) + t[x_i(k) - x_i(j)] = x_i(j) + t\Delta x_i \qquad \begin{matrix} i = \overline{1,m} \\ t \in [0,1] \end{matrix}$$

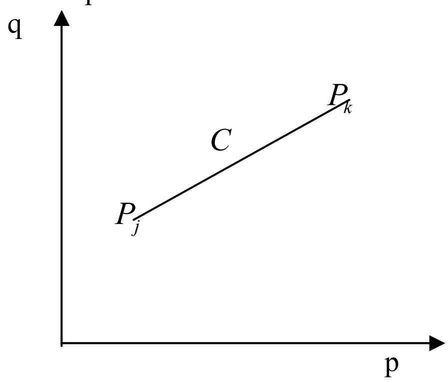
the index (1) becomes

$$\mathrm{I}_{z/x_i}^{k/j} = \exp\int_0^1 \frac{f_x'(t)\Delta x_i}{f(t)}dt \qquad (3)$$

For an index of prices we use the equations

$$\begin{cases} p_i(t) = p_i(j) + t(p_i(k) - p_i(j)) = p_i(j) + t\Delta p_i \\ q_i(t) = q_i(j) + t(q_i(k) - q_i(j)) = q_i(j) + t\Delta q_i \end{cases} \quad \text{with } i = \overline{1,m}\ , t \in [0,1]$$

This situation can be represented in a space of *2m* dimension:



So the index (2) becomes

$$\mathrm{I}_{z/p_i}^{k/j} = \exp\int_0^1 \frac{[q_i(j)+t\Delta q_i]\Delta p_i}{\sum_{i=1}^{m}[q_i(j)+t\Delta q_i](p_i(j)+t\Delta p_i)}dt \qquad (4)$$

or

$$\mathrm{I}_{z/p}^{k/j} = \exp\int_0^1 \frac{\sum_{i=1}^{m}[q_i(j)+t\Delta q_i(t)]\Delta p_i}{\sum_{i=1}^{m}[q_i(j)+t\Delta q_i](p_i(j)+t\Delta p_i)}dt \qquad (4')$$

Examples of polygonal paths

Proposition 4.

The Laspeyres index $\mathrm{I}_{z/p}^{k/j}(\bullet, L)$ is MDF index on Δ_1 path,(figure no.1)

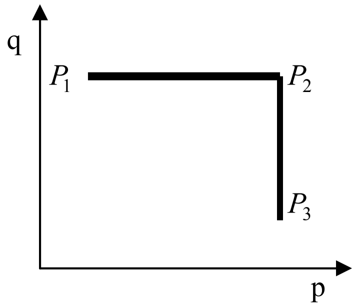


Fig. 1

$P_1(p(j), q(j))$
$P_2(p(k), q(j))$
$P_3(p(k), q(k))$
$\Delta_1 = P_1P_2 \cup P_2P_3$

Proof: We have $\mathrm{I}_{z/p}^{k/j}(MDF,\ \Delta_1) = \mathrm{I}_{z/p}(MDF,\ P_1P_2)\cdot \mathrm{I}_{z/p}(MDF,\ P_2P_3)$

The path P_1 P_2 is given by the equation $q_i = q_i(j)$, $p_i = p_i(j)+t\Delta p_i$, $t \in [0,1]$ so the index $\mathrm{I}_{z/p}(MDF,\ P_1P_2)$ becomes

$$\mathrm{I}_{z/p}(MDF,\ P_1P_2) = \exp\int_0^1 \frac{\sum_{i=1}^{m} q_i(j)\Delta p_i}{\sum_{i=1}^{m} q_i(j)[p_i(j)+t\Delta p_i]}dt = \exp\int_0^1 \frac{\sum_{i=1}^{m} q_i(j)\Delta p_i \quad dt}{\sum_{i=1}^{m}(q_i(j)p_i(j)+tq_i(j)\Delta p_i)} =$$

$$= \exp \ln \sum_{i=1}^{m} \left(q_i(j) p_i(j) + t q_i(j) \Delta p_i \right) \Big|_0^1 = \frac{\sum_{i=1}^{m} \left(q_i(j) p_i(j) + q_i(j) \Delta p_i \right)}{\sum_{i=1}^{m} q_i(j) p_i(j)} = \frac{\sum_{i=1}^{m} q_i(j) p_i(k)}{\sum_{i=1}^{m} q_i(j) p_i(j)} =$$

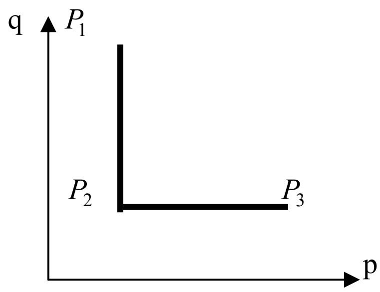
$$= \mathrm{I}_{z/p}^{k/j}(\bullet, L)$$

The path P_2 P_3 is given by the equation $\begin{cases} q_i = q_i(j) + t\Delta q_i \\ p_i = p_i(k) \Rightarrow \Delta p_i = 0 \end{cases}$ so the index $\mathrm{I}_{z/p}(MDF,\ P_2P_3)$ becomes

Finally, we have $\mathrm{I}_{z/p}^{k/j}(MDF,\ \Delta_1) = \mathrm{I}_{z/p}(MDF,\ P_1P_2) = \mathrm{I}_{z/p}^{k/j}(\bullet, L)$.

Proposition 5.

The Paasche index $\mathrm{I}_{z/p}^{k/j}(\bullet, P)$ is MDF index on Δ_2 path,(figure no.2)



$P_1(p(j), q(j))$
$P_2(p(j), q(k))$
$P_3(p(k), q(k))$
$\Delta_2 = P_1P_2 \cup P_2P_3$

Fig. 2

Proof: We have $\mathrm{I}_{z/p}^{k/j}(MDF,\ \Delta_2) = \mathrm{I}_{z/p}(MDF,\ P_1P_2) \cdot \mathrm{I}_{z/p}(MDF,\ P_2P_3)$

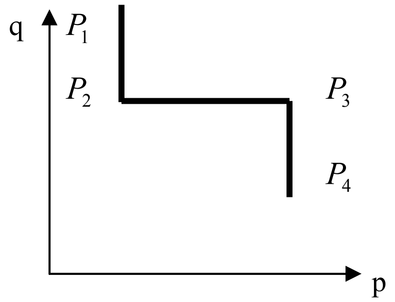
Similary with the previous proposition we proove $\mathrm{I}_{z/p}(MDF,\ P_1P_2) = 1$.

The path P_2 P_3 is given by the equation $q_i = q_i(k),\ \ p_i = p_i(j) + t\Delta p_i,\ \ t \in [0,1]$ so the index $\mathrm{I}_{z/p}(MDF,\ P_2P_3)$ becomes

$$\mathrm{I}_{z/p}(MDF,\ P_2P_3) = \frac{\sum_{i=1}^{m} p_i(k) q_i(k)}{\sum_{i=1}^{m} p_i(j) q_i(k)} = \mathrm{I}_{z/p}^{k/j}(\bullet, P)$$

Proposition 6.

The Edgeworth index $\mathrm{I}_{z/p}^{k/j}(\bullet, E)$ is MDF index on Δ_3 path,(figure no.3)



$$P_1\left(p(j), q(j)\right)$$
$$P_2\left(p(j), \frac{q(j)+q(k)}{2}\right)$$
$$P_3\left(p(k), \frac{q(j)+q(k)}{2}\right)$$
$$P_4\left(p(k), q(k)\right)$$
$$\Delta_3 = P_1P_2 \cup P_2P_3 \cup P_3P_4$$

Fig. 3

Proof: The path P_2P_3 is given by the equations

$$q_i = \frac{q_i(j)+q_i(k)}{2} \text{ and } p_i = p_i(j)+t\Delta p_i, \quad t\in[0,1]$$

so the index $\mathrm{I}_{z/p}(MDF, \; P_2P_3)$ becomes

$$\mathrm{I}_{z/p}(MDF, \; P_2P_3) = \exp\int_0^1 \frac{\sum_{i=1}^{m}\frac{q_i(j)+q_i(k)}{2}\Delta p_i}{\sum_{i=1}^{m}\frac{q_i(j)+q_i(k)}{2}\cdot\left(p_i(j)+t\Delta p_i\right)}dt = \exp\int_0^1 \frac{\sum_{i=1}^{m}\frac{q_i(j)+q_i(k)}{2}\Delta p_i}{\sum_{i=1}^{m}\frac{q_i(j)+q_i(k)}{2}\cdot p_i(j)+t\Delta p_i\frac{q_i(j)+q_i(k)}{2}}dt =$$

$$\exp\ln\left(\sum_{i=1}^{m}\frac{q_i(j)+q_i(k)}{2}\cdot p_i(j)+t\Delta p_i\frac{q_i(j)+q_i(k)}{2}\right)\Bigg|_0^1 = \frac{\sum_{i=1}^{m}\frac{q_i(j)+q_i(k)}{2}p_i(k)}{\sum_{i=1}^{m}\frac{q_i(j)+q_i(k)}{2}\cdot p_i(j)} = \mathrm{I}_{z/p}^{k/j}(\bullet, E)$$

Finally we have:

$$\mathrm{I}_{z/p}^{k/j}(MDF, \; \Delta_3) = \mathrm{I}_{z/p}(MDF, \; P_1P_2)\cdot\mathrm{I}_{z/p}(MDF, \; P_2P_3)\cdot\mathrm{I}_{z/p}(MDF, \; P_3P_4) =$$
$$= 1\cdot\mathrm{I}_{z/p}(\cdot E)\cdot 1 = \mathrm{I}_{z/p}^{k/j}(\cdot E)$$

Corollary 7. The MDF price index on the path which is presented in the figure no.4 is a product of Laspeyres indices.

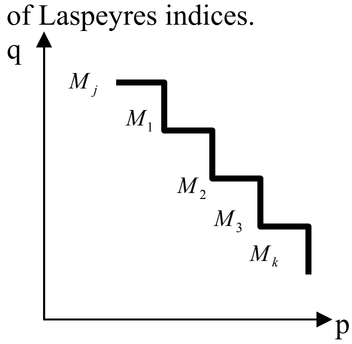


Fig. 4

$$I_{z/p}^{k/j}(MDF) = I_{z/p}(MDF, M_jM_1)\cdot I_{z/p}(MDF, M_1M_2)\cdot I_{z/p}(MDF, M_2M_3)\cdot I_{z/p}(MDF, M_3M_k) = \prod I_{z/p}(\cdot L)$$

The problem is to determine the path that aproximate the most the empiric path of the prices and quantities between *j* and *k*

The case of a spline path

Remark 8.

At the beginning ,we define the factorial indices of prices MDF with respect to a family of parabolas. We consider the parametric equations $q_i = a_i + b_i t + c_i t^2$, $p_i = a_i' + b_i' t + c_i' t^2$, $t \in [0,1]$ (5)

where $q_i(0) = q_i(j)$, $q_i(1) = q_i(k)$, $p_i(0) = p_i(j)$, $p_i(1) = p_i(k)$ that become

$$\begin{cases} q_i = q_i(j) + \Delta q_i t + c_i t^2 \\ p_i = p_i(j) + \Delta p_i t + c_i' t^2 \end{cases} t \in [0,1], c_i c_i' \in R$$

If $c_i = c_i' = 0$ we have the factorial index of prices on the linear path .Otherwise we obtain

$$I_{z/p}^{k/j} = \exp \int_0^1 \frac{\sum_{i=1}^{n} \left(q_i(j) + \Delta q_i t + c_i t^2\right)\cdot\left(\Delta p_i + 2tc_i'\right)}{\sum_{i=1}^{n} \left(q_i(j) + \Delta q_i t + c_i t^2\right)\cdot\left(p_i(j) + \Delta p_i t + c_i' t^2\right)} dt \qquad (6)$$

where the choice of constants c_i, c_i', n .could be made taking into account the new information for the situation $t \in [j,k]$ (moment of time).

The formula (6) can be generated when in (5) we have the polynoms of m degree, so the index will depend on a number of *2n(m-1)* constants.

Remark 9.

We suppose that we know the empiric data

$p(j)$, $q(j)$, $p(l)$, $q(l)$, $p(k)$, $q(k)$, $l \in [j,k]$ and we want to calculate the index $I_{z/p}^{k/j}(MDF)$.

a)If we consider that between every two points the path is linear then from the point $(p(j), q(j))$ to the point $(p(k), q(k))$ the path is a polygonal curve, so: $I_{z/p}^{k/j}(MDF) = I_{z/p}^{l/j}(MDF - liniar)\cdot I_{z/p}^{k/l}(MDF - liniar)$

b)The parametric equations $\begin{cases} q_i = a_i + b_i t + c_i t^2 \\ p_i = a_i' + b_i' t + c_i' t^2 \end{cases}$, $i = \overline{1,n}$ having the conditions

$t \in [0,2]$

$q_i(0) = q_i(j)$ $\quad p_i(0) = p_i(j)$

$q_i(1) = q_i(l)$ $\quad p_i(1) = p_i(l)$

$q_i(2) = q_i(k)$ $\quad p_i(2) = p_i(k)$

become $q_i = q_i(j) + \frac{4\Delta q_i^{l/j} - \Delta q_i^{k/j}}{2} t + \frac{\Delta q_i^{k/j} - 2\Delta q_i^{l/j}}{2} t^2$ and similarly for p_i.

So, one could observe that in both cases, although the index measures the variation from the moment *j* to moment *k* , it also depends of the intermediar moment *l*. Therefore, we calculate $I_{z/p}^{k/j}(MDF, \varphi)$ where φ is the curve which links the points *j,k,l*.

Remark 10.

If we generalize the remark (9) we obtain that the index MDF for the points *j,k* and for the empiric path between *j* and *k* which passed by the intermediar points coresponding to *m* moments of time $t_1, t_2, ...t_{m-1}$ and $t_0 = j,\ t_m = k$ can be calculate either as a product of *m* linear path indices or by reducing the problem to one of Lagrange polynomial interpolation on the nodes *(0,...,m)* with $p_i(s) = p_i(t_s),\ s = \overline{0,m}\ ,i = \overline{1,n}$ .Thus it can be determined a number of *2n* Lagrange polynoms of *m* degrees at the most $q_i = (p_m)_i,\ p_i = (p'_m)_i, i = \overline{1,n}$.

The choosing one method in spite of the other is a meter of comparison of the errors resulted in each case.

Remark 11.

For a better comparison of errors and taking into account that the polygonal line is the graphic representation of a particular spline function we define the factorial index of prices attached to M_j,M_k points and the linear path $M_j, M_{t_1}, M_{t_2} ....M_{t_m}, M_k$ by using the spline polynomial interpolation problem.

We use the next notions and results from the spline functions theory, the interpolation properties of these functions being the reason to recomand a spline path in MDF:

Definition 12. Let $\Delta : -\infty < x_1 < x_2 < ... < x_N < \infty,\ x_0 = -\infty, x_{N+1} = \infty$ .The function $s : R \to R$ is called *m* degrees spline with nodes $x_1 < x_2 < ... < x_N$ if

a) $s / I_i \in P_m(I_i),\ I_i = (x_i, x_{i+1}),\ i = \overline{0.N}$

b) $s \in c^{m-1}(R)$

Definition 13. The function define in (12) which satisfies $s(t_i) = y_i,\ i = \overline{1, N + m + 1}$ is called the interpolation polynomial spline function attached to the vector $\{y_i\}_{i=1}^{m+N+1}$ and the points $t_1 < t_2 < ... < t_{m+N+1}$.

Proposition 14. Any polynomial spline function of m degrees can be expressed uniquely $s(x) = p_0(x) + \sum_{i=1}^{N} c_i (x - x_i)_+^m,\ p_0 \in P_m$.

Proposition 15. Let $m > 0$, $t_1 < t_2 < ... < t_{m+N+1}$, the real numbers $y_i, i = \overline{1, m+N+1}$, and the division Δ, with the nodes $x_1 < x_2 < ... < x_N$ which satisfy the relations $t_i < x_i < t_{i+m+1}, i = \overline{1, N}$.
Then, a unique polynomial spline function of m degree exists so that $s(t_i) = y_i, i = \overline{1, m+N+1}$.
Proposition 16. Any of odd degrees with the nodes $\Delta : a = x_1 < x_2 < ... < x_N = b$, can be written as $s_{2m-1}(x) = p_{2m-1}(x) + \sum_{i=1}^{N} c_i (x - x_i)_+^{2m-1}$ with $p_{2m-1} \in P_{2m-1}$, $c_i \in R$ and $s_{2m-1} \in c^{2m-2}[a, b]$.
Definition 17. The polynomial spline function of *2m-1* degree which satisfy the conditions $s_{2m-1}(x_i) = y_i$, $\forall_i = \overline{1, N}$
$s_{2m-1}^{(j)}(x_1) = s_{2m-1}^{(j)}(x_N) = 0$, $j = m$, $m+1, ...2\text{m} - 1$. $(s \in P_{m-1} \quad x < x_1 \text{ şi } x > x_n)$ is called the natural spline function.
Theorem 18. Any natural spline function can be written uniquely
$s(x) = \sum_{i=0}^{m-1} a_i x^i + \sum_{i=1}^{N} c_i (x - x_i)_+^{2m-1}$, where $\sum_{i=0}^{N} c_i x_i^r = 0$, $r = 0, \ 1, \ ...m-1$
Theorem 19. Let $m \geq 1$, $N \geq m$ and y_j, $j = \overline{1, N}$. Then only a natural spline function exists, so that $s(x_j) = y_j$, $j = \overline{1, N}$.
Remark 20.
In the case when the degree of polynomial spline function is three then we speak about cubic spline function.
Remark 21.
With respect to the index problem we use the empiric data for *m* time moments and we interpolate them by *2m* spline functions p_i and q_i.
Remark 22.
Let the time moments *j* and *k* and N-2 intermediar moments $j_2, ... j_{N-1}$ for which we know the prices and quantities.
We consider a parameter $t \in [a, b]$, $q_i = q_i(t)$, $p_i = p_i(t)$ so that $q_i(t_1) = q_i(j)$, $q_i(t_2) = q_i(j_2)$, $...q_i(t_{N-1}) = q_i(j_{N-1})$, $q_i(t_N) = q_i(k)$ and similar for p, $\forall_i = \overline{1, n}$, t_s being the division points $\Delta : a = t_1 < t_2 < ... < t_N = b$.
a)To determine the cubic spline function q_i that satisfies the interpolation conditions $q_i(t_s) = q_i(j_s)$, $s = \overline{1, N}$ we must have with respect to proposition (4) a number of N+2 conditions because there are N+2 coefficients (the interpolation conditions are imposed on nodes).
There are more ways to determine the coefficients :

-To the interpolations conditions (in number of N) we add $q_i''(t_1)=0$, $q_i''(t_N)=0$ and thus unique natural cubic spline functions results.

-If we consider two nodes of the function we have four coefficients and after the use of this N conditions both in nodes and in points $t_2,...t_{N-1}$, a Lagrange interpolation polynom it results.

-At last, if we consider N-2 nodes $x_1 < x_2 < ... < x_{N-2}$ between the points $t_1,...t_N$ it results a function with N-2 coefficient defined on the interval $(x_1, x_2),...(x_{N-3}, x_{N-2})$. The coefficients result from N-2 interpolation conditions in nodes and the two conditions in t_{j_1} and t_{j_2} that are not nodes.

-Generally speaking, we can determine such an interpolation cubic spline given by

$$\begin{aligned} q_i(t) &= \sum_{l=o}^{2} b_l^i (t-t_1)_+^l + \sum_{j=1}^{N-1} c_j^i (t-t_j)_+^3 \\ p_i(t) &= \sum_{l=o}^{2} b_l^{i'} (t-t_1)_+^l + \sum_{j=1}^{N-1} c_j^{i'} (t-t_j)_+^3 \end{aligned}, i=\overline{1,n}.$$

where q_i and p_i are polynoms of three degree at the most in the interval generated by nodes and q_i , $p_i \in C^2[a,b]$.

We have $\mathrm{I}_{z/p}^{k/j}(MDF,\varphi) = \exp\int_\varphi \frac{\sum q_i}{\sum p_i q_i} dp_i$ where φ will be the spline curve that passes through *j,k* and through the intermediar points, being in fact the graphic representations of interpolation spline functions.

$$\int_\varphi = \int_{t_1}^{t_N} = \int_{t_1}^{t_2} + \int_{t_2}^{t_3} + ... + \int_{t_{N-1}}^{t_N}$$

In each case of these integrals the spline parametrization becomes a polynomial parametrization.

In the third case mentioned above, when only $x_1 < x_2 < ... < x_{N-2}$ nodes exist we have $\int_\varphi = \int_{t_1}^{t_N} = \int_{x_1}^{x_2} + \int_{x_2}^{x_3} + ... + \int_{x_{N-3}}^{x_{N-2}}$.

b)In a similar way we use a spline function of *m* degree.

If interpolation is not made in nodes but in points $t_1 < t_2 < ... < t_{m+N+1}$ and we know the values $q_i(t_j)$, $p_i(t_j)$, $j=\overline{1,m+N+1}$ we will consider, for example, the quantity q_i as a spline polynomial function of *m* degree with nodes $x_1 < x_2 < ... < x_N$ choosed so that $t_l < x_l < t_{l+m+1}$, $l=\overline{1,N}$.

According to proposition (15) there is only a spline polynomial function of *m* degree which solves the interpolation conditions in points t_j, $j=\overline{1,m+N+1}$ and it has the form $q_i(t) = h_i(t) + \sum_{l=1}^{N} c_l^i (t - t_l)_+^m$, where $h_i \in P_m$ and similary

$p_i(t) = h_i'(t) + \sum_{l=1}^{N} c_l^{i'} (t - t_l)_+^m$

Then the index MDF leads to an integral of type

$\int_{\varphi} = \int_{t_1}^{t_{m+N+1}} = \int_{t_1}^{x_1} + \int_{x_1}^{x_2} + \ldots + \int_{x_{N-1}}^{x_N} + \int_{x_N}^{t_{m+N+1}}$

It is more practically if we use the spline natural functions. In this case we consider N data regarding to *p* and *q* as interpolation conditions on nodes $t_1 < t_2 < \ldots < t_N$.

In respect with proposition (19) if $m \geq 1$, $N \geq m$ and we have interpolation conditions in nodes there is only an interpolation spline functions. Taking to account the proposition (18) theorem we can write

$$q_i(t) = \sum_{l=0}^{m-1} a_l^i t^l + \sum_{j=1}^{N} c_j^i (t - t_j)_+^{2m-1}$$

$$p_i(t) = \sum_{l=0}^{m-1} a_l^{i'} t^l + \sum_{j=1}^{N} c_j^{i'} (t - t_j)_+^{2m-1}$$

where the coefficients will be determined from the interpolation conditions(N conditions for every q_i and p_i) plus the conditions

$$\sum_{j=1}^{N} c_j^i t_j^r = 0 \quad , r = \overline{0,1,...m-1} \ , \ \sum_{j=1}^{N} c_j^{i'} t_j^r = o, \ i = \overline{1,n}$$

$$q_i^{(j)}(t_1) = q_i^{(j)}(t_N) = 0 \ , \ \forall j = m, \ m+1, \ 2m-1$$

$$p_i^{(j)}(t_1) = p_i^{(j)}(t_N) = 0$$

Remark 23.

If we take into account that the empiric data (p_i, q_i), $i = 1, n$ obtained for N different moments of time comprise errors then we speak rather of a numerical adjustment.

Analogous, we have the spline functions $q = f(p)$ respectively regression model of spline type $q = f(p) + \varepsilon$.

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Authors:

Nicoleta Breaz - „ 1 Decembrie 1918" University of Alba Iulia, str. N.Iorga, No.13, Departament of Mathematics and Computer Science, nbreaz@lmm.uab.ro

Daniel Breaz - „ 1 Decembrie 1918" University of Alba Iulia, str. N.Iorga, No.13, Departament of Mathematics and Computer Science, dbreaz@lmm.uab.ro