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CONCERNING SOME ARITHMETIC FUNCTIONS WHICH USE EXPONENTIAL DIVISORS

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Abstract. Let $\sigma^{(e)}(n)$ denote the sum of the exponential divisors of n, $\tau^{(e)}(n)$ denote the number of the exponential divisors of n, $\sigma^{(e)*}(n)$ denote the sum of the e-unitary divisors of n and $\tau^{(e)*}(n)$ denote the number of the e-unitary divisors of n. The aim of this paper is to present several inequalities about the arithmetic functions which use exponential divisors. Among these inequalities, we have the following:
$\dfrac{\sigma^{(e)}(n)}{\tau^{(e)}(n)} \geq \gamma(n)+\dfrac{\tau^{(e)}(n)-1}{2}$, for any $n \geq 1$, $\dfrac{\sigma^{(e)*}(n)}{\tau^{(e)*}(n)} \geq \gamma(n)+\dfrac{\tau^{(e)*}(n)-1}{2}$, for any $n \geq 1$ and $\sigma(n)+1 \geq \sigma^{(e)}(n)+\tau(n)$, for any $n \geq 1$, where $\tau(n)$ is the number of the natural divisors of n, $\sigma(n)$ is the sum of the divisors of n and γ is the "core" of n.

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1. Introduction

Some properties of the arithmetic functions which use exponential divisors can be found in the papers [1, 2, 5, 6, 8, 10].

The notion of "exponential divisor" was introduced by M. V. Subbarao in [9], in the following way: if $n > 1$ is an integer of canonical dorm $n = p_1^{a_1} p_2^{a_2} ... p_r^{a_r}$, then the integer $d = \prod_{i=1}^{r} p_i^{b_i}$ is called an *exponential divisor* (or e-divisor) of $n = \prod_{i=1}^{r} p_i^{a_i} > 1$, if $b_i | a_i$ for every $i = \overline{1,r}$. We note $d|_{(e)}n$. Let $\sigma^{(e)}(n)$ denote the sum of the exponential divisors of n and $\tau^{(e)}(n)$ denote the number of the exponential divisors of n.

In [11] L. Tóth and N. Minculete presented several properties for the exponential unitary divisors of a positive integer . The integer $d = \prod_{i=1}^{r} p_i^{b_i}$ is called a e-unitary divisor of $n = \prod_{i=1}^{r} p_i^{a_i} > 1$ if b_i is a unitary divisor of a_i, so $\left(b_i, \dfrac{a_i}{b_i}\right) = 1$, for every $i = \overline{1,r}$. Let $\sigma^{(e)*}(n)$ denote the sum of the e-unitary divisors of n, and $\tau^{(e)*}(n)$ denote the number of the e-unitary divisors of n. By convention, 1 is an exponential divisor of itself, so that $\sigma^{(e)*}(1) = \tau^{(e)*}(1) = 1$.

We notice that 1 is not a e-unitary divisor of $n > 1$, the smallest e-unitary divisor of $n = p_1^{a_1} p_2^{a_2} ... p_r^{a_r} > 1$ is $p_1 p_2 ... p_r = \gamma(n)$.

In [1], J. Fabrykowski and M. V. Subbarao study the maximal order and the average order of the multiplicative function $\sigma^{(e)}(n)$. E. G. Straus and M. V. Subbarao in [8] obtained also several results concerning e-perfect numbers (n is an e-perfect number if $\sigma^{(e)}(n) = 2n$).

In [5], J. Sándor showed that, if n is a perfect square, then

$$2^{\omega(n)} \leq \tau^{(e)}(n) \leq 2^{\Omega(n)}, \tag{1.1}$$

where $\omega(n)$ and $\Omega(n)$ denote the number of the distinct prime factors of n, and the total number of the prime factors of n, respectively. It is easy to see that, for $n = p_1^{a_1} p_2^{a_2} ... p_r^{a_r} > 1$, we have $\omega(n) = r$ and $\Omega(n) = a_1 + a_2 + ... + a_r$.

Let's consider $\tau^*(n)$ the number of the unitary divisors of n and $\sigma_k^*(n)$ the sum of kth powers of the unitary divisors of n. J. Sándor and L. Tóth proved in [7], the inequalities

$$\frac{n^k + 1}{2} \geq \frac{\sigma_k^*(n)}{\tau^*(n)} \geq \sqrt{n^k}, \tag{1.2}$$

and

$$\frac{\sigma_{k+m}^*(n)}{\sigma_m^*(n)} \geq \sqrt{n^k}, \tag{1.3}$$

for all $n \geq 1$ and $k, m \geq 0$, real numbers.

In [3] and [4], it is shown that

$$\sigma^{(e)}(n) \leq \psi(n) \leq \sigma(n), \tag{1.4}$$

where ψ is the function of Dedekind,

$$\tau(n) \leq \frac{\sigma^{(e)}(n)}{\tau^{(e)}(n)}, \tag{1.5}$$

$$\tau(n) + 1 \geq \tau^{(e)}(n) + \tau^*(n) \tag{1.6}$$

and

$$\sigma(n) + n \geq \sigma^{(e)}(n) + \sigma^*(n) \tag{1.7}$$

for all integers $n \geq 1$.

2. Inequalities for several arithmetic functions

In this section we will present several theorems containing some properties of the above functions.

Theorem 2.1. *There are the following inequalities:*

$$\frac{\sigma^{(e)}(n)}{\tau^{(e)}(n)} \geq \gamma(n) + \frac{\tau^{(e)}(n) - 1}{2} \tag{2.1}$$

and

$$\frac{\sigma^{(e)}(n)}{\tau^{(e)}(n)} \geq \gamma(n), \tag{2.2}$$

for all $n \geq 1$.

Proof. For $n = 1$, we obtain $\frac{\sigma^{(e)}(1)}{\tau^{(e)}(1)} = 1 = \gamma(1) + \frac{\tau^{(e)}(1) - 1}{2}$ and $\frac{\sigma^{(e)}(1)}{\tau^{(e)}(1)} = 1 = \gamma(1)$. For $n > 1$, we take the divisors in increasing order. The smallest exponential divisor of $n = p_1^{a_1} p_2^{a_2} ... p_r^{a_r} > 1$ is $p_1 p_2 ... p_r = \gamma(n)$. The second divisor is at least $2p_1 p_2 ... p_r = 2\gamma(n) \geq \gamma(n) + 1$.

Let $d_1, d_2, ..., d_s$ be the exponential divisors of n; it is easy to see that $d_i \geq \gamma(n) + i - 1$, for any $i = \overline{1, s}$. Hence

$$\sigma^{(e)}(n) = \sum_{d|_{(e)} n} d \geq \gamma(n) + \gamma(n) + 1 + \gamma(n) + 2 + ... + \gamma(n) + s - 1 = s\gamma(n) + \frac{s(s-1)}{2}.$$

Since $s = \tau^{(e)}(n)$ is the number of the exponential divisor of n, we deduce the inequality

$$\sigma^{(e)}(n) \geq \tau^{(e)}(n) \cdot \gamma(n) + \frac{\tau^{(e)}(n)(\tau^{(e)}(n) - 1)}{2}.$$

Consequently, we have

$$\frac{\sigma^{(e)}(n)}{\tau^{(e)}(n)} \geq \gamma(n) + \frac{\tau^{(e)}(n) - 1}{2}.$$

On the other hand, we have the inequality, $\tau^{(e)}(n) \geq 1$, which means that

$$\frac{\sigma^{(e)}(n)}{\tau^{(e)}(n)} \geq \gamma(n).$$

□

Remark 1. *If n is a squarefree number, then $\sigma^{(e)}(n) = n = \gamma(n)$ and $\tau^{(e)}(n) = 1$. Therefore, we obtain the equality in relations (2.1) and (2.2).*

If n is not a squarefree number, then in the proof of Theorem 2.1 we use for the second divisor that he is at least $2\gamma(n) \geq \gamma(n) + 1$. But the equality holds for $\gamma(n) = 1$, so $n = 1$. In other words, the equality in relations (2.1) and (2.2) holds, when n is a squarefree number.

Corollary 2.2. *There are the following inequalities:*

$$\frac{\sigma^{(e)*}(n)}{\tau^{(e)*}(n)} \geq \gamma(n) + \frac{\tau^{(e)*}(n) - 1}{2} \tag{2.3}$$

and

$$\frac{\sigma^{(e)*}(n)}{\tau^{(e)*}(n)} \geq \gamma(n), \tag{2.4}$$

for all $n \geq 1$.

Remark 2. *As in remark of Theorem 2.1, the equality in relations (2.3) and (2.4) holds, when n is a squarefree number.*

Theorem 2.3. *For $n = p_1^{a_1} p_2^{a_2} ... p_r^{a_r} > 1$ there is the inequality*

$$\tau(n) \geq \tau^{(e)}(n) + \frac{\tau(n)}{\omega(n)} \left(\frac{1}{a_1 + 1} + \frac{1}{a_2 + 1} + ... + \frac{1}{a_r + 1} \right). \tag{2.5}$$

Equality holds for $n = p$ or for $n = p^2$, where p is a prime number.

Proof. To prove the above inequality, will have to study several cases, namely:
Case I. If $n = p_1^2 p_2^2 ... p_r^2$, then $\tau(n) = 3^r$ and

$$\tau^{(e)}(n) = \tau(a_1) \cdot \tau(a_2) \cdot ... \cdot \tau(a_r) = \tau^r(2) = 2^r.$$

Inequality (2.5) becomes

$$3^r \geq 2^r + \frac{3^r}{r} \cdot \frac{r}{3} = 2^r + 3^{r-1},$$

so, $2 \cdot 3^{r-1} \geq 2^r$, what is true. Equality holds for $r = 1$, so $n = p^2$, where p is a prime number.

Case II. If $a_j \neq 2$ for every $j \in \{1, 2, ..., r\}$, and $a_k = \min\{a_j | a_j \neq 2\}$, then $(a_k - 1) \nmid a_k$.
Therefore, we obtain that
$\frac{n}{p_1^{i_1} \cdot p_2^{i_2} \cdot ... \cdot p_{k-1}^{i_{k-1}} \cdot p_k \cdot p_{k+1}^{i_{k+1}} \cdot ... \cdot p_r^{i_r}} = p_1^{a_1 - i_1} \cdot p_2^{a_2 - i_2} \cdot ... \cdot p_{k-1}^{a_{k-1} - i_{k-1}} \cdot p_k^{a_k - 1} \cdot p_{k+1}^{a_{k+1} - i_{k+1}} \cdot ... \cdot p_r^{a_r - i_r}$

is not exponential divisor of n, for every $i_j = \overline{0, a_j}$, and for every $j \in \{1, ..., r\} \setminus \{k\}$. Thus, the number of divisors of this type, which are not exponential, is $\dfrac{\tau(n)}{a_k + 1}$. Therefore, we have

$$\tau(n) = \sum_{d|_{(e)}n} 1 + \sum_{d\nmid_{(e)}n} 1 = \tau^{(e)}(n) + \sum_{d\nmid_{(e)}n} 1 \geq \tau^{(e)}(n) + \frac{\tau(n)}{a_k + 1},$$

so

$$\tau(n) \geq \tau^{(e)}(n) + \frac{\tau(n)}{a_k + 1} = \tau^{(e)}(n) + \frac{\tau(n)}{\omega(n)} \cdot \frac{\omega(n)}{a_k + 1}.$$

But $\frac{\omega(n)}{a_k+1} \geq \frac{1}{a_1+1} + \frac{1}{a_2+1} + ... + \frac{1}{a_r+1}$, which means that the inequality of the statement is true.

Case III. If there is at least a number $a_j \neq 2$, and at least a number $a_i = 2$, where $j, l \in \{1, 2, ..., r\}$, then without decreasing the generality, we renumber the prime factors from the factorization of n and we obtain $n = p_1^2 p_2^2 ... p_s^2 p_{s+1}^{a_{s+1}} ... p_r^{a_r}$, with $a_{s+1}, a_{s+2}, ..., a_r \neq 2$, and $a_k = \min\{a_j | a_j \neq 2, j \in \{s+1, ..., r\}\}$. If $a_k \neq 2$, then $(a_k - 1) \nmid a_k$, so $\frac{n}{p_1^{i_1} \cdot p_2^{i_2} \cdot ... \cdot p_{k-1}^{i_{k-1}} \cdot p_k \cdot p_{k+1}^{i_{k+1}} \cdot ... \cdot p_r^{i_r}} = p_1^{a_1 - i_1} \cdot p_2^{a_2 - i_2} \cdot ... \cdot p_{k-1}^{a_{k-1} - i_{k-1}} \cdot p_k^{a_k - 1} \cdot p_{k+1}^{a_{k+1} - i_{k+1}} \cdot ... \cdot p_r^{a_r - i_r}$ is not exponential divisor of n, for every $i_j = \overline{0, a_j}$ and for every $j \in \{1, ..., r\} \setminus \{k\}$. Thus, the number of divisors of this type is $\dfrac{\tau(n)}{a_k + 1}$, and the number $\dfrac{n}{p_1^2 p_2^{i_2} \cdot ... \cdot p_r^{i_r}} = p_2^{2-i_2} \cdot ... \cdot p_s^{2-i_s} \cdot p_{s+1}^{a_{s+1} - i_{s+1}} \cdot ... \cdot p_r^{a_r - i_r}$ is not exponential divisor of n, for all $i_2, ..., i_s \in \{0, 1, 2\}$ and $i_j = \overline{0, a_j}$, for every $j \in \{s+1, ..., r\}$. The second type of divisors are different from those of the above, and their number is $\dfrac{\tau(n)}{3}$. Therefore

$$\tau(n) = \sum_{d|_{(e)}n} 1 + \sum_{d\nmid_{(e)}n} 1 = \tau^{(e)}(n) + \sum_{d\nmid_{(e)}n} 1 \geq \tau^{(e)}(n) + \frac{\tau(n)}{a_k + 1} + \frac{\tau(n)}{3},$$

so

$$\tau(n) \geq \tau^{(e)}(n) + \frac{\tau(n)}{\omega(n)} \left(\frac{\omega(n)}{a_k + 1} + \frac{\omega(n)}{3} \right) \geq \tau^{(e)}(n) + \frac{\tau(n)}{\omega(n)} \left(\frac{r - s}{a_k + 1} + \frac{s}{3} \right) \geq$$

$$\geq \tau^{(e)}(n) + \frac{\tau(n)}{\omega(n)} \left(\frac{1}{a_{s+1} + 1} + \frac{1}{a_{s+2} + 1} + ... + \frac{1}{a_r + 1} + \frac{1}{2+1} + ... + \frac{1}{2+1} \right) =$$

$$= \tau^{(e)}(n) + \frac{\tau(n)}{\omega(n)} \left(\frac{1}{a_1 + 1} + \frac{1}{a_2 + 1} + ... + \frac{1}{a_r + 1} \right),$$

where $\omega(n) = r$, which means that the inequality of the statement is true. Thus, the proof is complete.

□

Corollary 2.4. *For every $n > 1$ there are the following inequalities:*

$$\tau(n) \geq \tau^{(e)}(n) + \frac{\tau(n)\omega(n)}{\Omega(n) + \omega(n)} \tag{2.6}$$

and

$$\tau(n) \geq \tau^{(e)}(n) + \sqrt[\omega(n)]{\tau^{\omega(n)-1}(n)}. \tag{2.7}$$

Proof. From Cauchy's inequality, we have

$$(a_1 + 1 + a_2 + 1 + ... + a_r + 1)\left(\frac{1}{a_1 + 1} + \frac{1}{a_2 + 1} + ... + \frac{1}{a_r + 1}\right) \geq r^2.$$

But $a_1 + a_2 + ... + a_r = \Omega(n)$, so, according to above inequality, we deduce

$$\frac{1}{a_1 + 1} + \frac{1}{a_2 + 1} + ... + \frac{1}{a_r + 1} \geq \frac{\omega^2(n)}{\Omega(n) + \omega(n)}.$$

Therefore, by using theorem 2.3, we obtain inequality (2.6).
Combining inequality (2.5) with the inequality

$$\frac{1}{a_1 + 1} + \frac{1}{a_2 + 1} + ... + \frac{1}{a_r + 1} \geq r \sqrt[r]{\frac{1}{(a_1 + 1)(a_2 + 1)...(a_r + 1)}} = \frac{r}{\sqrt[r]{\tau(n)}},$$

it follows inequality (2.7).

□

Lemma 2.5. *For any $x_i > 0$ with $i \in \{1, 2, ..., n\}$, there is the following inequality:*

$$\prod_{i=1}^{n}(1 + x_i + x_i^2) + \prod_{i=1}^{n} x_i^2 \geq \prod_{i=1}^{n}(x_i + x_i^2) + \prod_{i=1}^{n}(1 + x_i^2). \tag{2.8}$$

Proof. We consider

$$p(n) : \prod_{i=1}^{n}(1 + x_i + x_i^2) + \prod_{i=1}^{n} x_i^2 \geq \prod_{i=1}^{n}(x_i + x_i^2) + \prod_{i=1}^{n}(1 + x_i^2), \text{ for any } n \geq 1.$$

We check that $p(1)$ is true, so

$$1 + x_1 + x_1^2 + x_1^2 \geq x_1 + x_1^2 + 1 + x_1^2,$$

and we suppose that $p(k)$ is true, then we prove that $p(k+1)$ is true, so

$$\prod_{i=1}^{k+1}(1+x_i+x_i^2)+\prod_{i=1}^{k+1}x_i^2\geq\prod_{i=1}^{k+1}(x_i+x_i^2)+\prod_{i=1}^{k+1}(1+x_i^2),$$

which is equivalent to the inequality

$$x_{k+1}^2\left(\prod_{i=1}^{k}(1+x_i+x_i^2)+\prod_{i=1}^{k}x_i^2-\prod_{i=1}^{k}(x_i+x_i^2)-\prod_{i=1}^{k}(1+x_i^2)\right)+$$

$$+x_{k+1}\left(\prod_{i=1}^{k}(1+x_i+x_i^2)-\prod_{i=1}^{k}(x_i+x_i^2)\right)+\prod_{i=1}^{k}(1+x_i+x_i^2)-\prod_{i=1}^{k}(1+x_i^2)\geq 0.$$

According to the principle of mathematical induction, $p(n)$ is true for any $n\geq 1$.

□

Theorem 2.6. *For every $n\geq 1$, the inequality*

$$\sigma(n)+1\geq\sigma^{(e)}(n)+\tau(n),\tag{2.9}$$

holds.

Proof. If $n=1$, then we obtain $\sigma(1)+1=2=\sigma^{(e)}(1)+\tau(1)$.
Let's consider $n>1$. To prove the above inequality will be a study on more cases namely:
Case I. If $n=p_1^2p_2^2...p_r^2$, then $\sigma(n)=\prod_{i=1}^{r}(1+p_i+p_i^2)$, $\sigma^{(e)}(n)=\prod_{i=1}^{r}(p_i+p_i^2)$ and $\tau(n)=3^r$, which means that inequality (2.9) is equivalent to the inequality

$$\prod_{i=1}^{r}(1+p_i+p_i^2)+1\geq\prod_{i=1}^{r}(p_i+p_i^2)+3^r.$$

Apply lemma 2.5, for $n=r$ and $x_i=p_i$, thus, we obtain the inequality

$$\prod_{i=1}^{r}(1+p_i+p_i^2)+\prod_{i=1}^{r}p_i^2\geq\prod_{i=1}^{r}(p_i+p_i^2)+\prod_{i=1}^{r}(1+p_i^2).$$

Since $\prod_{i=1}^{r}(1+p_i^2)\geq 5^r-4^r+\sum_{i=1}^{r}p_i^2$, and $5^r-4^r\geq 3^r-1$, it follows that the inequality of statement is true.

Case II. If there is a number $a_k\geq 3$, then $(a_k-1)\nmid a_k$, so
$\frac{n}{p_1^{i_1}\cdot p_2^{i_2}\cdot...\cdot p_{k-1}^{i_{k-1}}\cdot p_k\cdot p_{k+1}^{i_{k+1}}\cdot...\cdot p_r^{i_r}}=p_1^{a_1-i_1}\cdot p_2^{a_2-i_2}\cdot...\cdot p_{k-1}^{a_{k-1}-i_{k-1}}\cdot p_k^{a_k-1}\cdot p_{k+1}^{a_{k+1}-i_{k+1}}\cdot...\cdot p_r^{a_r-i_r}$
is not exponential divisors of n, for all $i_j=\overline{0,a_j}$ and for all $j\in\{1,...,r\}\setminus\{k\}$.

Thus, the number of divisors of this type is $\dfrac{\tau(n)}{a_k+1}$, and the sum of these divisors non-exponential is

$$p_k^{a_k-1}\sigma\left(\frac{n}{p_k^{a_k}}\right).$$

Hence
$\sigma(n) = \sum_{d|_{(e)}n} d + \sum_{d\nmid_{(e)}n} d = \sigma^{(e)}(n) + \sum_{d\nmid_{(e)}n} d \geq \sigma^{(e)}(n) + p_k^{a_k-1}\sigma\left(\frac{n}{p_k^{a_k}}\right) \geq \sigma^{(e)}(n) + \frac{n}{p_k} + p_k^{a_k-1}$,
so, using Sierpinski's inequality, $2\sqrt{n} > \tau(n)$, we have

$$\sigma(n) \geq \sigma^{(e)}(n) + \frac{n}{p_k} + p_k^{a_k-1} \geq \sigma^{(e)}(n) + \frac{n}{p_k} + p_k - 1 \geq \sigma^{(e)}(n) + 2\sqrt{n} - 1 >$$

$$\sigma^{(e)}(n) + \tau(n) - 1.$$

Case III. If there is at least a number $a_i = 1$, at least a number $a_j = 2$ and at least a number $a_k \geq 3$, where $i, j, k \in \{1, 2, ..., r\}$, then without decreasing the generality, we renumber the prime factors from the factorization of n and we obtain

$$n = p_1p_2...p_sp_{s+1}^2p_{s+2}^2...p_t^2p_t^{a_{t+1}}...p_r^{a_r}, \text{ with } a_{t+1}, a_{t+2}, ..., a_r \geq 3.$$

Therefore, we can write $n = n_1 \cdot n_2 \cdot n_3$, where $n_1 = p_1p_2...p_s$, $n_2 = p_1^2p_2^2...p_s^2$ and $n_3 = p_{t+1}^{a_{t+1}}...p_r^r$, which means that $(n_1, n_2, n_3) = 1$, and it is easy to see, using the multiplicativity of these functions, that

$$\begin{gathered}
\sigma(n) = \sigma(n_1 \cdot n_2 \cdot n_3) = \sigma(n_1) \cdot \sigma(n_2) \cdot \sigma(n_3) \geq \\
(\sigma^{(e)}(n_1) + \tau(n_1) - 1)(\sigma^{(e)}(n_2) + \tau(n_2) - 1)(\sigma^{(e)}(n_3) + \tau(n_3) - 1) = \\
= (\sigma^{(e)}(n_1n_2) + \sigma^{(e)}(n_1)(\tau(n_2) - 1) + \tau(n_1)(\sigma^{(e)}(n_2) - 1) + \tau(n_1n_2) - \sigma^{(e)}(n_2) \\
-\tau(n_2) + 1) \\
(\sigma^{(e)}(n_3) + \tau(n_3) - 1) \geq \\
(\sigma^{(e)}(n_1n_2) + \tau(n_1n_2) - 1)(\sigma^{(e)}(n_3) + \tau(n_3) - 1) = \\
= \sigma^{(e)}(n_1n_2n_3) + \sigma^{(e)}(n_1n_2)(\tau(n_3) - 1) + \tau(n_1n_2)(\sigma^{(e)}(n_3) - 1) + \\
\tau(n_1n_2n_3) - \sigma^{(e)}(n_3) - \tau(n_3) + 1 \geq \sigma^{(e)}(n) + \tau(n) - 1,
\end{gathered}$$

because

$$\sigma^{(e)}(n_1), \tau(n_1), \sigma^{(e)}(n_1n_2), \tau(n_1n_2) \geq 1.$$

Thus, the demonstration is complete.

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