HARMONIC UNIVALENT FUNCTIONS DEFINED BY AN INTEGRAL OPERATOR

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Abstract. We define and investigate a new class of harmonic univalent functions defined by an integral operator. We obtain coefficient inequalities, extreme points and distortion bounds for the functions in our classes.

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1. Introduction

A continuous complex-valued function $f = u + iv$ defined in a complex domain D is said to be harmonic in D if both u and v are real harmonic in D. In any simply connected domain we can write $f = h + \overline{g}$, where h and g are analytic in D. A necessary and sufficient condition for f to be locally univalent and sense preserving in D is that $|h'(z)| > |g'(z)|$, $z \in D$. (See Clunie and Sheil-Small [2].)

Denote by H the class of functions $f = h + \overline{g}$ that are harmonic univalent and sense preserving in the unit disc $U = \{z : \ |z| < 1\}$ so that $f = h + \overline{g}$ is normalized by $f(0) = h(0) = f'_z(0) - 1 = 0$.

Let $\mathcal{H}(U)$ be the space of holomorphic functions in U. We let:

$$A_n = \{f \in \mathcal{H}(U),\ f(z) = z + a_{n+1}z^{n+1} + \dots,\ z \in U\},$$

with $A_1 = A$.

We let $\mathcal{H}[a, n]$ denote the class of analytic functions in U of the form

$$f(z) = a + a_n z^n + a_{n+1}z^{n+1} + \dots, \quad z \in U.$$

The integral operator I^n is defined in [5] by:
(i) $I^0 f(z) = f(z)$;
(ii) $I^1 f(z) = If(z) = \int_0^z f(t)t^{-1}dt$;
(iii) $I^n f(z) = I(I^{n-1}f(z))$, $n \in \mathbb{N}$ and $f \in A$.

Ahuja and Jahongiri [3] defined the class $H(n)$ ($n \in \mathbb{N}$) consisting of all univalent harmonic functions $f = h + \overline{g}$ that are sense preserving in U and h and g are of the form

$$h(z) = z + \sum_{k=2}^{\infty} a_k z^k, \quad g(z) = \sum_{k=1}^{\infty} b_k z^k, \quad |b_1| < 1. \tag{1}$$

For $f = h + \overline{g}$ given by (1) the integral operator I^n of f is defined as

$$I^n f(z) = I^n h(z) + (-1)^n \overline{I^n g(z)}, \tag{2}$$

where

$$I^n h(z) = z + \sum_{k=2}^{\infty} (k)^{-n} a_k z^k \text{ and } I^n g(z) = \sum_{k=1}^{\infty} (k)^{-n} b_k z^k.$$

For $0 \leq \alpha < 1$, $n \in \mathbb{N}$, $z \in U$, let $H(n, \alpha)$ the family of harmonic functions f of the form (1) such that

$$\operatorname{Re}\left\{\frac{I^n f(z)}{I^{n+1} f(z)}\right\} > \alpha. \tag{3}$$

The families $H(n+1, n, \alpha)$ and $H^-(n+1, n, \alpha)$ include a variety of well-known classes of harmonic functions as well as many new ones. For example $HS(\alpha) = \overline{H}(1, 0, \alpha)$ is the class of sense-preserving,harmonic univalent functions f which are starlike of order $\alpha \in U$, and $HK(\alpha) = \overline{H}(2, 1, \alpha)$ is the class of sense-preserving,harmonic univalent functions f which are convex of order α in U, and $\overline{H}(n+1, n, \alpha) = \overline{H}(n, \alpha)$ is the class of Sălăgean-type harmonic univalent functions.

Let we denote the subclass $H^-(n, \alpha)$ consist of harmonic functions $f_n = h + \overline{g}_n$ in $H^-(n, \alpha)$ so that h and g_n are of the form

$$h(z) = z - \sum_{k=2}^{\infty} a_k z^k, \quad g_n(z) = (-1)^{n-1} \sum_{k=1}^{\infty} b_k z^k, \tag{4}$$

where $a_k, b_k \geq 0$, $|b_1| < 1$.

For the harmonic functions f of the form (1) with $b_1 = 0$ Avci and Zlotkiewich [1] show that if

$$\sum_{k=2}^{\infty} k(|a_k| + |b_k|) \leq 1,$$

then $f \in HS(0)$, where $HS(0) = \overline{H}(1,0,0)$, and if

$$\sum_{k=2}^{\infty} k^2(|a_k| + |b_k|) \leq 1,$$

then $f \in HK(0)$, where $HK(0) = \overline{H}(2,1,0)$.

For the harmonic functions f of the form (4) with $n = 0$, Jahangiri in [4] showed that $f \in HS(\alpha)$ if and only if

$$\sum_{k=2}^{\infty}(k-\alpha)|a_k| + \sum_{k=1}^{\infty}(k+\alpha)|b_k| \leq 1-\alpha$$

and $f \in \overline{H}(2,1,\alpha)$ if and only if

$$\sum_{k=2}^{\infty} k(k-\alpha)|a_k| + \sum_{k=1}^{\infty} k(k+\alpha)|b_k| \leq 1-\alpha.$$

2.Main results

In our first theorem, we deduce a sufficient coefficient bound for harmonic functions in $H(n,\alpha)$.

Theorem 2.1. *Let $f = h + \overline{g}$ be given by (1). If*

$$\sum_{k=1}^{\infty}\{\psi(n,k,\alpha)|a_k| + \theta(n,k,\alpha)|b_k|\} \leq 2, \tag{5}$$

where

$$\psi(n,k,\alpha) = \frac{(k)^{-n} - \alpha(k)^{-(n+1)}}{1-\alpha} \text{ and } \theta(n,k,\alpha) = \frac{(k)^{-n} + \alpha(k)^{-(n+1)}}{1-\alpha},$$

$a_1 = 1$, $0 \leq \alpha < 1$, $n \in \mathbb{N}$. *Then f is sense-preserving in U and $f \in H(n, \alpha)$.*

Proof. According to (2) and (3) we only need to show that

$$\text{Re}\ \left(\frac{I^n f(z) - \alpha I^{n+1} f(z)}{I^{n+1} f(z)}\right) \geq 0.$$

The case $r = 0$ is obvious. For $0 < r < 1$, it follows that

$$\text{Re}\ \left(\frac{I^n f(z) - \alpha I^{n+1} f(z)}{I^{n+1} f(z)}\right)$$

$$= \text{Re}\ \left\{\frac{z(1-\alpha) + \sum_{k=2}^{\infty}\left[\left(\frac{1}{k}\right)^n - \alpha\left(\frac{1}{k}\right)^{n+1}\right] a_k z^k}{z + \sum_{k=2}^{\infty}\left(\frac{1}{k}\right)^{n+1} a_k z^k + (-1)^{n+1}\sum_{k=1}^{n}\left(\frac{1}{k}\right)^n \overline{b}_k \overline{z}^k}\right.$$

$$\left. + \frac{(-1)^n \sum_{k=1}^{\infty}\left[\left(\frac{1}{k}\right)^n + \alpha\left(\frac{1}{k}\right)^{n+1}\right] \overline{b}_k \overline{z}^k}{z + \sum_{k=2}^{\infty}\left(\frac{1}{k}\right)^{n+1} a_k z^k + (-1)^{n+1}\sum_{k=1}^{\infty}\left(\frac{1}{k}\right)^{n+1} \overline{b}_k \overline{z}^k}\right\}$$

$$= \text{Re}\ \left\{\frac{1-\alpha + \sum_{k=2}^{\infty}\left[\left(\frac{1}{k}\right)^n - \alpha\left(\frac{1}{k}\right)^{n+1}\right] a_k z^{k-1}}{1 + \sum_{k=2}^{\infty}\left(\frac{1}{k}\right)^{n+1} a_k z^{k-1} + (-1)^{n+1}\sum_{k=1}^{\infty}\left(\frac{1}{k}\right)^{n+1} \overline{b}_k \overline{z}^k z^{-1}}\right.$$

$$\left. + \frac{(-1)^n \sum_{k=1}^{\infty}\left[\left(\frac{1}{k}\right)^n + \alpha\left(\frac{1}{k}\right)^{n+1}\right] \overline{b}_k \overline{z}^k z^{-1}}{1 + \sum_{k=2}^{\infty}\left(\frac{1}{k}\right)^{n+1} a_k z^{k-1} + (-1)^{n+1}\sum_{k=1}^{\infty}\left(\frac{1}{k}\right)^{n+1} \overline{b}_k \overline{z}^k z^{-1}}\right\}$$

$$= \text{Re}\ \left[\frac{1-\alpha + A(z)}{1 + B(z)}\right].$$

For $z = re^{i\theta}$ we have

$$A(re^{i\theta}) = \sum_{k=2}\left[\left(\frac{1}{k}\right)^n - \alpha\left(\frac{1}{k}\right)^{n+1}\right] a_k r^{k-1} e^{(k-1)\theta i}$$

$$+(-1)^n \sum_{k=1}^{\infty} \left[\left(\frac{1}{k}\right)^n + \alpha \left(\frac{1}{k}\right)^{n+1} \right] \overline{b}_k r^{k-1} e^{-(k+1)\theta i}$$

$$B(re^{i\theta}) = \sum_{k=2}^{\infty} \left(\frac{1}{k}\right)^{n+1} a_k r^{k-1} e^{(k-1)\theta i}$$

$$+(-1)^{n+1} \sum_{k=1}^{\infty} \left(\frac{1}{k}\right)^{n+1} \overline{b}_k r^{k-1} e^{-(k+1)\theta i}$$

Setting

$$\frac{1-\alpha+A(z)}{1+B(z)} = (1-\alpha)\frac{1+w(z)}{1-w(z)},$$

the proof will be complete if we can show that $|w(z)| \leq 1$. This is the case since, by the condition (5), we can write

$$|w(z)| = \left| \frac{A(z)-(1-\alpha)B(z)}{A(z)+(1-\alpha)B(z)+2(1-\alpha)} \right|$$

$$= \left| \frac{\sum_{k=2}^{\infty} \left[\left(\frac{1}{k}\right)^n - \left(\frac{1}{k}\right)^{n+1} \right] a_k r^{k-1} e^{(k-1)\theta i}}{2(1-\alpha) + \sum_{k=2}^{\infty} C(n,k,\alpha) a_k r^{k-1} e^{(k-1)\theta i} + (-1)^n \sum_{k=1}^{\infty} D(n,k,\alpha) \overline{b}_k r^{k-1} e^{-(k+1)\theta i}} \right.$$

$$\left. + \frac{(-1)^n \sum_{k=1}^{\infty} \left[\left(\frac{1}{k}\right)^n + \left(\frac{1}{k}\right)^{n+1} \right] \overline{b}_k r^{k-1} e^{-(k+1)\theta i}}{2(1-\alpha) + \sum_{k=2}^{\infty} C(n,k,\alpha) a_k r^{k-1} e^{(k-1)\theta i} + (-1)^n \sum_{k=1}^{\infty} D(n,k,\alpha) \overline{b}_k r^{k-1} e^{-(k+1)\theta i}} \right|$$

$$\leq \frac{\sum_{k=2}^{\infty} \left[\left(\frac{1}{k}\right)^n - \left(\frac{1}{k}\right)^{n+1} \right] |a_k| r^{k-1}}{2(1-\alpha) - \sum_{k=2}^{\infty} C(n,k,\alpha) |a_k| r^{k-1} - \sum_{k=1}^{\infty} D(n,k,\alpha) |b_k| r^{k-1}}$$

$$+\frac{\sum_{k=1}^{\infty}\left[\left(\frac{1}{k}\right)^{n}+\left(\frac{1}{k}\right)^{n+1}\right]|b_k|r^{k-1}}{2(1-\alpha)-\sum_{k=2}^{\infty}C(n,k,\alpha)|a_k|r^{k-1}-\sum_{k=1}^{\infty}D(n,k,\alpha)|b_k|r^{k-1}}$$

$$=\frac{\sum_{k=1}^{\infty}\left[\left(\frac{1}{k}\right)^{n}-\left(\frac{1}{k}\right)^{n+1}\right]|a_k|r^{k-1}}{4(1-\alpha)-\sum_{k=1}^{\infty}\{C(n,k,\alpha)|a_k|+D(n,k,\alpha)|b_k|\}r^{k-1}}$$

$$+\frac{\sum_{k=1}^{\infty}\left[\left(\frac{1}{k}\right)^{n}+\left(\frac{1}{k}\right)^{n+1}\right]|b_k|r^{k-1}}{4(1-\alpha)-\sum_{k=1}^{\infty}\{C(n,k,\alpha)|a_k|+D(n,k,\alpha)|b_k|\}r^{k-1}}$$

$$<\frac{\sum_{k=1}^{\infty}\left[\left(\frac{1}{k}\right)^{n}-\left(\frac{1}{k}\right)^{n+1}\right]|a_k|}{4(1-\alpha)-\sum_{k=1}^{\infty}\{C(n,k,\alpha)|a_k|+D(n,k,\alpha)|b_k|\}}$$

$$+\frac{\sum_{k=1}^{\infty}\left[\left(\frac{1}{k}\right)^{n}+\left(\frac{1}{k}\right)^{n+1}\right]|b_k|}{4(1-\alpha)-\sum_{k=1}^{\infty}\{C(n,k,\alpha)|a_k|+D(n,k,\alpha)|b_k|\}}\le 1,$$

where

$$C(n,k,\alpha)=\left(\frac{1}{k}\right)^{n}+(1-2\alpha)\left(\frac{1}{k}\right)^{n+1}$$

and

$$D(n,k,\alpha)=\left(\frac{1}{k}\right)^{n}-(1-2\alpha)\left(\frac{1}{k}\right)^{n+1}.$$

The harmonic univalent functions

$$f(z)=z+\sum_{k=2}^{\infty}\frac{1}{\psi(n,k,\alpha)}x_k z^k+\sum_{k=1}^{\infty}\frac{1}{\theta(n,k,\alpha)}\overline{y_k z^k}, \tag{6}$$

where $n \in \mathbb{N}$ and $\sum_{k=2}^{\infty} |x_k| + \sum_{k=1}^{\infty} |y_k| = 1$, show that the coefficient bound given by (5) is sharp. The functions of the form (6) are in $H(n, \alpha)$ because

$$\sum_{k=1}^{\infty} \{\psi(n,k,\alpha)|a_k| + \theta(n,k,\alpha)|b_k|\} = 1 + \sum_{k=2}^{\infty} |x_k| + \sum_{k=1}^{\infty} |y_k| = 2.$$

In the following theorem it is shown that the condition (5) is also necessary for functions $f_n = h + \overline{g}_n$ where h and g_n are of the form (4).

Theorem 2.2. *Let $f_n = h + \overline{g}_n$ be given by (4). Then $f_n \in H^-(n, \alpha)$ if and only if*

$$\sum_{k=1}^{\infty} \{\psi(n,k,\alpha)a_k + \theta(n,k,\alpha)b_k\} \leq 2 \tag{7}$$

where $a_1 = 1$, $0 \leq \alpha < 1$, $n \in \mathbb{N}$.

Proof. Since $H^-(n, \alpha) \subset H(n, \alpha)$ we only need to prove the "only if" part of the theorem. For functions f_n of the form (4), we note that the condition

$$\text{Re}\left\{\frac{I^n f_n(z)}{I^{n+1} f_n(z)}\right\} > \alpha$$

is equivalent to

$$\text{Re}\left\{\frac{(1-\alpha)z - \sum_{k=2}^{\infty}\left[\left(\frac{1}{k}\right)^n - \alpha\left(\frac{1}{k}\right)^{n+1}\right] a_k z^k}{z - \sum_{k=2}^{\infty}\left(\frac{1}{k}\right)^{n+1} a_k z^k + (-1)^{2n}\sum_{k=1}^{\infty}\left(\frac{1}{k}\right)^{n+1} b_k \overline{z}^k}\right. \tag{8}$$

$$\left. + \frac{(-1)^{2n-1}\sum_{k=1}^{\infty}\left[\left(\frac{1}{k}\right)^n + \alpha\left(\frac{1}{k}\right)^{n+1}\right] b_k \overline{z}^k}{z - \sum_{k=2}^{\infty}\left(\frac{1}{k}\right)^{n+1} a_k z^k + (-1)^{2n}\sum_{k=1}^{\infty}\left(\frac{1}{k}\right)^{n} b_k \overline{z}_k}\right\} \geq 0.$$

The above required condition (8) must hold for all values of z in U. Upon choosing the values of z on the positive real axis where $0 \leq z = r < 1$, we must have

$$\frac{1-\alpha-\sum_{k=2}^{\infty}\left[\left(\frac{1}{k}\right)^{n}-\alpha\left(\frac{1}{k}\right)^{n+1}\right]a_k r^{k-1}}{1-\sum_{k=2}^{\infty}\left(\frac{1}{k}\right)^{n+1}a_k r^{k-1}+\sum_{k=1}^{\infty}\left(\frac{1}{k}\right)^{n+1}b_k r^{k-1}} \tag{9}$$

$$-\frac{\sum_{k=1}^{\infty}\left[\left(\frac{1}{k}\right)^{n}+\alpha\left(\frac{1}{k}\right)^{n+1}\right]b_k r^{k-1}}{1-\sum_{k=2}^{\infty}\left(\frac{1}{k}\right)^{n+1}a_k r^{k-1}+\sum_{k=1}^{\infty}\left(\frac{1}{k}\right)^{n+1}b_k r^{k-1}}\geq 0.$$

If the condition (7) does not hold, then the expression in (9) is negative for r sufficiently close to 1. Hence there exist $z_0 = r_0$ in $(0,1)$ for which the quotient in (9) is negative. This contradicts the required condition for $f_n \in H^-(n,\alpha)$. So the proof is complete.

Next we determine the extreme points of the closed convex hull of $H^-(n,\alpha)$ denoted by $clcoH^-(n,\alpha)$.

Theorem 2.3. *Let f_n be given by (4). Then $f_n \in H^-(n,\alpha)$ if and only if*

$$f_n(z)=\sum_{k=1}^{\infty}[x_k h_k(z)+y_k g_{n_k}(z)],$$

where

$$h(z)=z,\quad h_k(z)=z-\frac{1}{\psi(n,k,\alpha)}z^k,\quad (k=2,3,\dots)$$

and

$$g_{n_k}(z)=z+(-1)^{n-1}\frac{1}{\theta(n,k,\alpha)}\overline{z}^k\quad (k=1,2,3,\dots)$$

$$x_k\geq 0,\quad y_k\geq 0,\quad x_p=1-\sum_{k=2}^{\infty}x_k-\sum_{k=1}^{\infty}y_k.$$

In particular, the extreme points of $H^-(n,\alpha)$ are $\{h_k\}$ and $\{g_{n_k}\}$.

Proof. For functions f_n of the form (5)

$$f_n(z)=\sum_{k=1}^{\infty}[x_k h_k(z)+y_k g_{n_k}(z)]=\sum_{k=1}^{\infty}(x_k+y_k)z-\sum_{k=2}^{\infty}\frac{1}{\psi(n,k,\alpha)}x_k z^k$$

$$+(-1)^{n-1}\sum_{k=1}^{\infty}\frac{1}{\theta(n,k,\alpha)}y_k\overline{z}^k.$$

Then

$$\sum_{k=2}^{\infty}\psi(n,k,\alpha)\left(\frac{1}{\psi(n,k,\alpha)}x_k\right)+\sum_{k=1}^{\infty}\theta(n,k,\alpha)\left(\frac{1}{\theta(n,k,\alpha)}y_k\right)$$

$$=\sum_{k=2}^{\infty}x_k+\sum_{k=1}^{\infty}y_k=1-x_1\leq 1$$

and so $f_n(z)\in clcoH^-(n,\alpha)$.

Conversely, suppose $f_n(z)\in clcoH^-(n,\alpha,\beta)$. Letting

$$x_1=1-\sum_{k=2}^{\infty}x_k-\sum_{k=1}^{\infty}y_k,$$

$$x_k=\psi(n,k,\alpha)a_k,\quad (k=2,3,\dots)\text{ and } y_k=\theta(n,k,\alpha)b_k,\quad (k=1,2,3,\dots)$$

we obtain the required representation, since

$$f_n(z)=z-\sum_{k=2}^{\infty}a_kz^k+(-1)^{n-1}\sum_{k=1}^{\infty}b_k\overline{z}^k$$

$$=z-\sum_{k=2}^{\infty}\frac{1}{\psi(n,k,\alpha)}x_kz^k+(-1)^{n-1}\sum_{k=1}^{\infty}\frac{1}{\theta(n,k,\alpha)}y_k\overline{z^k}$$

$$=z-\sum_{k=2}^{\infty}[z-h_k(z)]x_k-\sum_{k=1}^{\infty}[z-g_{n_k}(z)]y_k$$

$$=\left[1-\sum_{k=2}^{\infty}x_k-\sum_{k=1}^{\infty}y_k\right]z+\sum_{k=2}^{\infty}x_kh_k(z)+\sum_{k=1}^{\infty}y_kg_{n_k}(z)$$

$$=\sum_{k=1}^{\infty}[x_kh_k(z)+y_kg_{n_k}(z)].$$

The following theorem gives the distortion bounds for functions in $H^-(n,\alpha)$ which yields a covering results for this class.

Theorem 2.4. *Let $f_n \in H^-(n,\alpha)$. Then for $|z| = r < 1$ we have*

$$|f_n(z)| \le (1+b_1)r + \{\phi(n,k,\alpha) - \Omega(n,k,\alpha)b_1\}r^{n+1}$$

and

$$|f_n(z)| \ge (1-b_1)r - \{\phi(n,k,\alpha) - \Omega(n,k,\alpha)b_1\}r^{n+1},$$

where

$$\phi(n,k,\alpha) = \frac{1-\alpha}{\left(\frac{1}{2}\right)^n - \alpha\left(\frac{1}{2}\right)^{n+1}}$$

and

$$\Omega(n,k,\alpha) = \frac{1+\alpha}{\left(\frac{1}{2}\right)^n - \alpha\left(\frac{1}{2}\right)^{n+1}}.$$

Proof. We prove the right hand side inequality for $|f_n|$. The proof for the left hand inequality can be done using similar arguments. Let $f_n \in H^-(n,\alpha)$. Taking the absolute value of f_n then by Theorem 2.2, we obtain:

$$|f_n(z)| = \left| z - \sum_{k=2}^{\infty} a_k z^k + (-1)^{n-1} \sum_{k=1}^{\infty} b_k \overline{z}^k \right|$$

$$\le r + \sum_{k=2}^{\infty} a_k r^k + \sum_{k=1}^{\infty} b_k r^k = r + b_1 r + \sum_{k=2}^{\infty} (a_k + b_k) r^k$$

$$\le r + b_1 r + \sum_{k=2}^{\infty} (a_k + b_k) r^2 = (1+b_1)r + \phi(n,k,\alpha) \sum_{k=2}^{\infty} \frac{1}{\phi(n,k,\alpha)} (a_k + b_k) r^2$$

$$\le (1+b_1)r + \phi(n,k,\alpha) r^{n+2} \left[\sum_{k=2}^{\infty} \psi(n,k,\alpha) a_k + \theta(n,k,\alpha) b_k \right]$$

$$\le (1+b_1)r + \{\phi(n,k,\alpha) - \Omega(n,k,\alpha) b_1\} r^{n+2}.$$

The following covering result follows from the left hand inequality in Theorem 2.4.

Corollary 2.1. *Let $f_n \in H^-(n,\alpha)$, then for $|z| = r < 1$ we have*

$$\{w : \ |w| < 1 - b_1 - [\phi(n,k,\alpha) - \Omega(n,k,\alpha)b_1] \subset f_n(U)\}.$$

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