SYMMETRY VS ASYMMETRY

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Abstract. Symmetry, jointly with their ”opposite”, Antisymmetry, may be considered as two sides of the same coin: total symmetry, or total asymmetry, relative to a pattern object, without intermediate situations. But this dychotomical classification suffer a lack of necessary and realistic grades. For this reason, it is convenient the introduction of "shade regions", modulating the degrees (a fuzzy concept). So, we analyze here the Asymmetry Problem, with its different attempts of description (from the Physics, by techniques as Group Theory), searching an efficient algorithm, by the introduction of Asymmetry/Symmetry Level Functions.

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1.Introduction

One of the more fundamental results in Physics, and in any Science, is that obtained by the great mathematician *Emmy Noether* (1882-1935). It was proved in 1915, then published in 1918.

It states [13] that *any differentiable symmetry of the action of a physical system has a corresponding conservation law.* Hence, to every continuous symmetry of a physical theory corresponds a conserved quantity, i.e., a physical quantity that does not change with time.

So, *Symmetry under translation corresponds to conservation of momentum. Symmetry under rotation, to conservation of angular momentum. Symmetry in time, to conservation of energy.* Also it is present in Relativity Theory, Quantum Mechanics and so on.

It is a very important result, because it allows us to derive conserved quantities from the mathematical form of our theories.

Remind that the *action* of a physical system is an integral of a Lagrangian function, from which the behavior of the system can be determined by the Principle of Least Action.

Note that this theorem does not apply to systems that cannot be modeled with a Lagrangian. For instance, to dissipative systems.

The Noether Theorem has become essential not only in modern Theoretical Physics, but in the Calculus of Variations, and therefore, in fields as Modeling and Optimization.

In fact, all modern Physics is based on a bunch of Symmetry Principles, from which the rest follows.

So, we can say that *the Laws of Nature are constrained by Symmetry.*

Such theorem admits distinct, but essentially equivalent, statements, as:

"To every differentiable symmetry generated by local actions, there corresponds a conserved current".

This connect with many today evolving subjects of modern Physics, as Gauge Symmetry, in Quantum Mechanics, the results of Witten (String Theory) and many others.

Noether is remembered not only by this theorem (actually, they are two results, with many consequences), but by many contributions to Abstract Algebra.

There is also a quantum version of this Noether´s theorem. It is known as the:

Ward-Takahasi Identity

The conservation law of a physical quantity is expressed by a continuity equation, where the conserved quantity is named *Noether´s Charge,* and the flow carrying that "charge" is the *Noether´s Current.*

In Quantum Mechanics, invariance under a change of phase of the wave function leads to the *Conservation of Particle Number.*

2.Modeling and Optimization

In the last decades, the expansion of fuzzy mathematics and its applications are very formidable. The parallel version of different mathematical fields, but adapted to degrees of truth, is in advance. The basic idea according which an element not necessarily belongs totally, or does not belongs in absolute, to a set, but it can belongs more or less, that is, in some degree, signifies a modern revolution in the scientific thinking, adapting the some times hieratic mathematics to the features of the real world.

So, it produces new fields, as Fuzzy Measure Theory, which generalizes the classical Measure Theory, of Lebesgue and other authors. It must be very useful as tool in our own papers. And so occurs in every mathematical field.

In *Fuzzy Modeling* we attempt to construct Fuzzy Systems. Many times, it will be a very difficult task, because it is necessary the identification of many parameters.

It offers a great potential for analyzing structures with non-stochastic imprecise input information.

Calyampuchi R. Rao, a very great statistician, said:

$$\textit{Uncertain Knowledge} + \textit{Uncertainty Measure} \Rightarrow \textit{Useful Knowledge}$$

Because uncertainty measures play an important role in A. I. and Reasoning under Uncertainty. So, it is impossible to avoid uncertainty in Decision Making.

Claude Shannon [22] once pointed: *"Information is something that can be used for eliminating the uncertainty"*

In *Fuzzy Optimization,* our objective is to maximize or minimize a fuzzy set submitted to some fuzzy constraints. But we cannot to make this directly, with the "value" of a fuzzy set. For this reason, in areas as Finance, we wish to maximize/minimize the value of a discrete/continue random variable, being restricted by a probability mass/density function.

So, we change the multiobjective problem into a single crisp objective subject to the fuzzy constraints. And it is possible to generate good approximate solutions by Genetic Algorithms.

But also there are different fuzzy optimization problems, which include learning a Fuzzy Neural Network, useful to solve *fuzzy linear programming problems* (FLP), and *fuzzy inventory control,* using such Genetic Algorithms.

3.Entropy vs negentropy

It is essential the axiomatized concept of *entropy* (denoted by H). Many of its seminal ideas proceeds from Claude E. Shannon [21, 22] and Alfred Renyi [17].

It is very related to the coding length of the random variable. In fact, with some simple assumptions, H is the coding length of the random variable.

As you known, *Entropy* is the basic concept of Information Theory.

It can be interpreted, for a random variable, as *the degree of information that the observation of the variable produces.*

The more "randomness" present the variable,
the larger is their entropy.

It is defined, for a *discrete* random variable, Y, as:

$$H(Y) = -\sum P(Y = y_i) \log P(Y = y_i)$$

where the y_i are the possible values of Y.

This may be generalized for the *continuous case,* being then called *Differential Entropy* (also named *continuous entropy*).

It will be defined by:

$$H(y) = -\int f(y) \log f(y)\, dy$$

with $f(y)$ density function for the continuous random variable Y.

There exists a very important result of Information Theory:

A Gaussian random variable has the largest entropy
among all random variables of the same variance.

So, the Normal, or Gaussian, distribution is the "least structured", or equivalently, the "most random" among all distributions.

But we have a second and very important *measure of nongaussianity* (departure of the Normal).

It is called with distinct names, as *Negentropy,* also *Negative Entropy* or *Syntropy,* denoted *by* J.

Actually, it is a slightly modified version of differential entropy, defined by:

$$J(y) = H(y_{gauss}) - H(y)$$

being y_{gauss} a Gaussian random variable of the same covariance matrix as y.

Some of its properties are interesting, as:

$$J\left(y\right) \geq 0, \forall y$$

That is, Negentropy is always non-negative.

And it is null in the case of the Normal distribution:

$$J = 0 \Leftrightarrow Gaussian$$

According Schrödinger classical book *What is Life?* [13]: *"Negentropy of a living system is the entropy that it exports, to maintain its own entropy low".*

And Brillouin [2] says that: *"A living system imports negentropy, and stores it".*

Also recall that in their known Theory of Entropy, *Ilya Prigogine* [15] propose that *Symmetry* may be regarded as *Reduction of Entropy* i. e., as *Order*

The *Curie Principle of Symmetry,* due to Pierre Curie, postulates that: *The symmetry group of the cause is a subgroup of the symmetry group of the effect.*

This idea may produces deep ramifications in Causality Theory, and also analyzing relationships among the foundations of physical theories.

4.Conclusion

So, we conclude that it is possible, in this way, to introduce a new measure [10] quantifying the asymmetry level of shapes, being in general, applicable to fuzzy sets.

This new normal fuzzy measure may improve the precedent results, when it is applied on new situations, so when we analyze Fuzzy Sets, Rough Sets, Rough-Fuzzy Sets, Neuro-Fuzzy Systems, and so on.

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