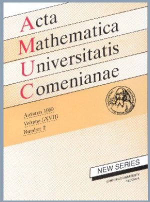

ALMOST STABLE ITERATION SCHEMES FOR LOCAL STRONGLY PSEUDOCONTRACTIVE AND LOCAL STRONGLY ACCRETIVE OPERATORS IN REAL UNIFORMLY SMOOTH BANACH SPACES

ZEQING LIU, YUGUANG XU AND SHIN MIN KANG

Abstract. In this paper we establish the strong convergence and almost stability of the Ishikawa iteration methods with errors for the iterative approximations of either fixed points of local strongly pseudocontractive operators or solutions of nonlinear operator equations with local strongly accretive type in real uniformly smooth Banach spaces. Our convergence results extend some known results in the literature.

1. Introduction

Let X be a real Banach space, X^* be its dual space and $\langle x, f\rangle$ be the generalized duality pairing between $x \in X$ and $f \in X^*$. The mapping $J : X \to 2^{X^*}$ defined by

$$J(x) = \{f \in X^* : \langle x, f\rangle = \|x\|\|f\|, \|f\| = \|x\|\}, \quad \forall\, x \in X,$$

is called the *normalized duality mapping*. In the sequel, we denote by I and $F(T)$ the identity mapping on X and the set of all fixed points of T, respectively.

Received July 4, 2007; revised January 17, 2008.
2000 *Mathematics Subject Classification.* Primary 47H06, 47H10, 47H15, 47H17.
Key words and phrases. Local strongly accretive operator; local strongly pseudocontractive operator; Ishikawa iteration sequence with errors; fixed point; convergence; almost stability; nonempty bounded closed convex subset; real uniformly smooth Banach space.
The authors gratefully acknowledge the financial support from the project (20060467) of the Science Research Foundation of Educational Department of Liaoning Province.

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Let T be an operator on X. Assume that $x_0 \in X$ and $x_{n+1} = f(T, x_n)$ defines an iteration scheme which produces a sequence $\{x_n\}_{n=0}^{\infty} \subset X$. Suppose, furthermore, that $\{x_n\}_{n=0}^{\infty}$ converges strongly to $q \in F(T) \neq \emptyset$. Let $\{y_n\}_{n=0}^{\infty}$ be any sequence in X and put $\varepsilon_n = \|y_{n+1} - f(T, y_n)\|$ for $n \geq 0$.

Definition 1.1. ([13]–[15], [50]).

1. The iteration scheme $\{x_n\}_{n=0}^{\infty}$ defined by $x_{n+1} = f(T, x_n)$ is said to be *T-stable* if $\lim_{n\to\infty} \varepsilon_n = 0$ implies that $\lim_{n\to\infty} y_n = q$.
2. The iteration scheme $\{x_n\}_{n=0}^{\infty}$ defined by $x_{n+1} = f(T, x_n)$ is said to be *almost T-stable* if $\sum_{n=0}^{\infty} \varepsilon_n < \infty$ implies that $\lim_{n\to\infty} y_n = q$.

Note that $\{y_n\}_{n=0}^{\infty}$ is bounded provided that the iteration scheme $\{x_n\}_{n=0}^{\infty}$ defined by $x_{n+1} = f(T, x_n)$ is either T-stable or almost T-stable. Therefore we revise Definition 1.1 as follows:

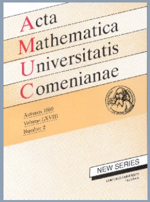
Definition 1.2.

1. The iteration scheme $\{x_n\}_{n=0}^{\infty}$ defined by $x_{n+1} = f(T, x_n)$ is said to be *T-stable* if $\{y_n\}_{n=0}^{\infty}$ is bounded and $\lim_{n\to\infty} \varepsilon_n = 0$ imply that $\lim_{n\to\infty} y_n = q$.
2. The iteration scheme $\{x_n\}_{n=0}^{\infty}$ defined by $x_{n+1} = f(T, x_n)$ is said to be *almost T-stable* if $\{y_n\}_{n=0}^{\infty}$ is bounded and $\sum_{n=0}^{\infty} \varepsilon_n < \infty$ imply that $\lim_{n\to\infty} y_n = q$.

Definition 1.3. ([1]–[12], [18], [19], [53]–[55]). Let X be a real Banach space and $T : D(T) \subseteq X \to X$ be an operator, where $D(T)$ and $R(T)$ denote the domain and range of T, respectively.

1. T is said to be *local strongly pseudocontractive* if for each $x \in D(T)$ there exists $t_x > 1$ such that for all $y \in D(T)$ and $r > 0$

$$\|x - y\| \leq \|(1 + r)(x - y) - rt_x(Tx - Ty)\|. \tag{1.1}$$



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2. T is called *local strongly accretive* if for given $x \in D(T)$ there exists $k_x \in (0,1)$ such that for each $y \in D(T)$ there is $j(x-y) \in J(x-y)$ satisfying

$$\langle Tx - Ty, j(x-y)\rangle \geq k_x \|x-y\|^2. \tag{1.2}$$

3. T is called *strongly pseudocontractive* (respectively, *strongly accretive*) if it is local strongly pseudocontractive (respectively, local strongly accretive) and $t_x \equiv t$ (respectively, $k_x \equiv k$) is independent of $x \in D(T)$.
4. T is said to be *accretive* for if given $x, y \in D(T)$ there is $j(x-y) \in J(x-y)$ satisfying

$$\langle Tx - Ty, j(x-y)\rangle \geq 0.$$

5. T is said to be *m-accretive* if it is accretive and $(I + rT)D(T) = X$ for all $r > 0$.

Clearly, each strongly pseudocontractive operator is local strongly pseudocontractive and each strongly accretive operator is local strongly accretive. It is known (see [54]) that T is local strongly pseudocontractive if and only if $I - T$ is local strongly accretive and $k_x = 1 - \frac{1}{t_x}$, where t_x and k_x are the constants appearing in (1.1) and (1.2), respectively.

The concept of accretive operators was introduced independently by Browder [1] and Kato [17] in 1967. An early fundamental result in the theory of accretive operators, due to Browder, states that the initial value problem

$$\frac{\mathrm{d}u(T)}{\mathrm{d}t} + Tu(T) = 0, \qquad u(0) = u_0,$$

is *solvable* if T is locally Lipschitzian and accretive on X. It is well known that if $T : X \to X$ is strongly accretive and demi-continuous, then for any $f \in X$, the equation

$$Tx = f \tag{1.3}$$

has a solution in X. Martin [50] proved that if T is a continuous accretive operator, then T is m-accretive. Thus for any $f \in X$, the equation

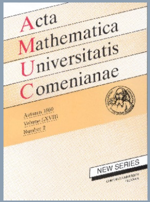
$$x + Tx = f \tag{1.4}$$

has a solution in X.

Recently several researches introduced and studied the iterative approximation methods to find either fixed points of ϕ-hemicontractive, strictly hemicontractive, strictly successively hemicontractive, strongly pseudocntractive, generalized asymptotically contractive and generalized hemicontractive, nonexpansive, asymptotically nonexpansive mappings, local strictly pseudocontractive and local strongly pseudocntractive operators or solutions of ϕ-strongly accretive, strongly quasi-accretive, strongly accretive, local strongly accretive and m-accretive operators equations (1.3) and (1.4) (see, for example, [1]–[55]).

Rhoades [52] proved that the Mann and Ishikawa iteration methods may exhibit different behaviors for different classes of nonlinear operators. A few stability results for certain classes of nonlinear operators have been established by several authors in [13]–[15], [23]–[25], [30], [32], [33], [38], [40]–[43], [48], [51]. Harder and Hicks [14] revealed that the importance of investigating the stability of various iteration procedures for various classes of nonlinear operators. Harder [13] obtained applications of stability results to first order differential equations. Osilike [51] obtained the stability of certain Mann and Ishikawa iteration sequences for fixed points of Lipschitz strong pseudocontractions and solutions of nonlinear accretive operator equations in real q-uniformly smooth Banach spaces.

The purpose of this paper is to establish the strong convergence and almost stability of the Ishikawa iteration methods with errors for either fixed point of local strongly pseudocontractive operators or solutions of nonlinear operator equations with local strongly accretive type in uniformly smooth Banach spaces. The convergence results presented in this paper are generalizations and improvements of the results in [3]–[8], [10], [12], [53], [55].



2. Preliminaries

The following results shall be needed in the sequel.

Lemma 2.1. ([56]). *Let X be a real uniformly smooth Banach space. Then there exists a nondecreasing continuous function $b : [0, +\infty) \to [0, +\infty)$ satisfying the conditions*

(a) $b(0) = 0, \quad b(ct) \le cb(t), \quad \forall t \ge 0,\ c \ge 1;$
(b) $\|x + y\|^2 \le \|x\|^2 + 2\langle y, j(x)\rangle + \max\{\|x\|, 1\}\|y\|b(\|y\|), \quad \forall x, y \in X.$

Lemma 2.2. ([4]). *Let X be a real Banach space. Then the following conditions are equivalent.*

(a) *X is uniformly smooth;*
(b) *X^* is uniformly convex;*
(b) *J is single valued and uniformly continuous on any bounded subset of X.*

Lemma 2.3. ([18]). *Suppose that $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$, $\{\gamma_n\}_{n=0}^{\infty}$ and $\{\omega_n\}_{n=0}^{\infty}$ are nonnegative sequences such that*

$$\alpha_{n+1} \le (1 - \omega_n)\alpha_n + \beta_n\omega_n + \gamma_n, \quad \forall n \ge 0$$

with $\{\omega_n\}_{n=0}^{\infty} \subset [0, 1]$, $\sum_{n=0}^{\infty} \omega_n = \infty$, $\sum_{n=0}^{\infty} \gamma_n < \infty$ and $\lim_{n\to\infty} \beta_n = 0$. Then $\lim_{n\to\infty} \alpha_n = 0$.

3. Main results

In this section, put $d_n = b_n + c_n$ and $d'_n = b'_n + c'_n$ for $n \ge 0$. Let b, k_q and t_q are the function and constants appearing in Lemma 2.1 and Definition 1.3, respectively, where $q \in F(T)$.

Theorem 3.1. *Let X be a real uniformly smooth Banach space and $T : X \to X$ be a local strongly pseudocontractive operator. Let $R(T)$ be bounded and $F(T) \ne \emptyset$. Suppose that $\{u_n\}_{n=0}^{\infty}, \{v_n\}_{n=0}^{\infty}$ are arbitrary bounded sequences in X and $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{a'_n\}_{n=0}^{\infty}$,*

$\{b'_n\}_{n=0}^\infty$, $\{c'_n\}_{n=0}^\infty$ *and* $\{r_n\}_{n=0}^\infty$ *are any sequences in* $[0,1]$ *satisfying*

$$a_n + b_n + c_n = a'_n + b'_n + c'_n = 1, \quad \forall n \geq 0; \tag{3.1}$$

$$c_n(1 - r_n) = r_n b_n, \quad \forall n \geq 0; \tag{3.2}$$

$$\lim_{n\to\infty} b(d_n) = \lim_{n\to\infty} r_n = \lim_{n\to\infty} b'_n = \lim_{n\to\infty} c'_n = 0; \tag{3.3}$$

$$\sum_{n=0}^{\infty} d_n = \infty. \tag{3.4}$$

For any $x_0 \in X$*, the Ishikawa iteration sequences with errors* $\{x_n\}_{n=0}^\infty$ *are defined by*

$$z_n = a'_n x_n + b'_n T x_n + c'_n v_n, \quad x_{n+1} = a_n x_n + b_n T z_n + c_n u_n, \quad \forall\, n \geq 0. \tag{3.5}$$

Let $\{y_n\}_{n=0}^\infty$ *be any bounded sequence in* X *and define* $\{\varepsilon_n\}_{n=0}^\infty$ *by*

$$w_n = a'_n y_n + b'_n T y_n + c'_n v_n, \quad \varepsilon_n = \|y_{n+1} - a_n y_n - b_n T w_n - c_n u_n\| \tag{3.6}$$

for all $n \geq 0$*. Then there exist nonnegative sequences* $\{s_n\}_{n=0}^\infty$*,* $\{t_n\}_{n=0}^\infty$ *and a constant* $M > 0$ *such that* $\lim_{n\to\infty} s_n = \lim_{n\to\infty} t_n = 0$ *and*

(a) $\{x_n\}_{n=0}^\infty$ *converges strongly to the unique fixed point* q *of* T *and*

$$\begin{aligned}\|x_{n+1} - q\|^2 &\leq (1 - d_n k_q)\|x_n - q\|^2 \\ &\quad + M d_n (d'_n b(d'_n) + c'_n + s_n + b(d_n) + r_n), \quad \forall\, n \geq 0;\end{aligned}$$

(b) *For all* $n \geq 0$

$$\begin{aligned}\|y_{n+1} - q\|^2 &\leq (1 - d_n k_q)\|y_n - q\|^2 \\ &\quad + M d_n (d'_n b(d'_n) + c'_n + t_n + b(d_n) + r_n) + M\varepsilon_n;\end{aligned}$$

(c) $\sum_{n=0}^\infty \varepsilon_n < \infty$ *implies that* $\lim_{n\to\infty} y_n = q$*, so that* $\{x_n\}_{n=0}^\infty$ *is almost* T*-stable;*

(d) $\lim_{n\to\infty} y_n = q$ *implies that* $\lim_{n\to\infty} \varepsilon_n = 0$.

Proof. Since T is local strongly pseudocontractive and $F(T) \neq \emptyset$, it follows from (1.1) that $F(T)$ is a singleton, say $F(T) = \{q\}$. Thus there exists $k_q \in (0,1)$ such that

$$\langle Tx - Tq, j(x-q)\rangle \leq (1-k_q)\|x-q\|^2, \quad \forall\, x \in X. \tag{3.7}$$

Set, for all $n \geq 0$,

$$p_n = a_n y_n + b_n S w_n + c_n u_n, \tag{3.8}$$

$$\begin{aligned} D = 2 + 2\|x_0 - q\| &+ 2\sup\{\|Tx - q\| : x \in X\} \\ &+ \sup\{\|y_n - q\| : n \geq 0\} + \sup\{\|u_n - q\| : n \geq 0\} \\ &+ \sup\{\|v_n - q\| : n \geq 0\}, \end{aligned} \tag{3.9}$$

$$s_n = \|j(x_n - q) - j(z_n - q)\|, \quad t_n = \|j(y_n - q) - j(w_n - q)\|. \tag{3.10}$$

It is easy to show that for all $n \geq 0$

$$\max\{\|x_n - q\|, \|z_n - q\|, \|p_n - q\|, \|y_n - q\|, \|w_n - q\|,\} \leq \frac{D}{2} < D, \tag{3.11}$$

$$\varepsilon_n \leq \|y_{n+1} - q\| + \|p_n - q\| \leq D. \tag{3.12}$$

In view of Lemma 2.1, (3.1), (3.5), (3.7) and (3.11), we infer that

$$
\begin{aligned}
\|z_n-q\|^2 &\\
&= \|(1-d'_n)(x_n-q)+d'_n(Tx_n-q)+c'_n(v_n-Tx_n)\|^2\\
&\le \|(1-d'_n)(x_n-q)+d'_n(Tx_n-q)\|^2\\
&\quad + 2c'_n\langle v_n-Tx_n, j((1-d'_n)(x_n-q)+d'_n(Tx_n-q))\rangle\\
&\quad + \max\{\|(1-d'_n)(x_n-q)+d'_n(Tx_n-q)\|,1\}\\
&\qquad \times c'_n\|v_n-Tx_n\|b(c'_n\|v_n-Tx_n\|)\\
&\le (1-d'_n)^2\|x_n-q\|^2+2d'_n\langle Tx_n-q, j((1-d'_n)(x_n-q))\rangle\\
&\quad + \max\{(1-d'_n)\|x_n-q\|,1\}d'_n\|Tx_n-q\|b(d'_n\|Tx_n-q\|)\\
&\quad + 2c'_n\|v_n-Tx_n\|\|(1-d'_n)(x_n-q)+d'_n(Tx_n-q)\|+D^3c'_nb(c'_n)\\
&\le (1-d'_n)^2\|x_n-q\|^2+2d'_n(1-d'_n)\langle Tx_n-q, j((x_n-q))\rangle\\
&\quad + D^3d'_nb(d'_n)+2D^2c'_n+D^3c'_nb(c'_n)\\
&\le \{(1-d'_n)^2+2d'_n(1-d'_n)(1-k_q)\}\|x_n-q\|^2+2D^3(c'_n+d'_nb(d'_n))\\
&= \{1-k_qd'_n+{d'_n}^2(k_q-1)+k_qd_n(d_n-1)\}\|x_n-q\|^2\\
&\quad + 2D^3(c'_n+d'_nb(d'_n))\\
&\le (1-k_qd'_n)\|x_n-q\|^2+2D^3(c'_n+d'_nb(d'_n))
\end{aligned}
\tag{3.13}
$$

for all $n\ge 0$. Observe that

$$
\begin{aligned}
\|(x_n-q)-(z_n-q)\| &\le b'_n\|x_n-Tx_n\|+c'_n\|x_n-v_n\|\\
&\le Dd'_n\to 0 \quad \text{as } n\to\infty
\end{aligned}
\tag{3.14}
$$

and

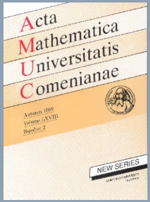
$$\|(y_n - q) - (w_n - q)\| \le b'_n\|y_n - Ty_n\| + c'_n\|y_n - v_n\| \le Dd'_n \to 0 \quad \text{as } n \to \infty. \tag{3.15}$$

Using Lemma 2.2, (3.14) and (3.15), we have

$$s_n, t_n \to 0 \quad \text{as } n \to \infty. \tag{3.16}$$

Using again Lemma 2.1, (3.1), (3.2), (3.5), (3.7), (3.11) and (3.13), we obtain that

$$\begin{aligned}
&\|x_{n+1} - q\|^2 \\
&\quad = \|(1-d_n)(x_n - q) + d_n(Tz_n - q) + c_n(u_n - Tz_n)\|^2 \\
&\quad \le (1-d_n)^2\|x_n - q\|^2 + 2d_n(1-d_n)\langle Tz_n - q, j(x_n - q)\rangle \\
&\qquad + \max\{(1-d_n)\|x_n - q\|, 1\} d_n\|Tz_n - q\| b(d_n\|Tz_n - q\|) \\
&\qquad + 2c_n\langle u_n - Tz_n, j((1-d_n)(x_n - q) + d_n(Ty_n - q))\rangle \\
&\qquad + \max\{\|(1-d_n)(x_n - q) + d_n(Tz_n - q)\|, 1\} \\
&\qquad\quad \times c_n\|u_n - Tz_n\| b(c_n\|u_n - Tz_n\|) \\
&\quad \le (1-d_n)^2\|x_n - q\|^2 + 2d_n(1-d_n)[\langle Tz_n - q, j(z_n - q)\rangle \\
&\qquad + \langle Tz_n - q, j(x_n - q) - j(z_n - q)\rangle] + D^3(d_n b(d_n) + c_n b(c_n)) \\
&\qquad + 2c_n\|u_n - Tz_n\|\|(1-d_n)(x_n - q) + d_n(Ty_n - q)\|
\end{aligned}$$



$$
(3.17)\quad
\begin{aligned}
&\le (1-d_n)^2\|x_n-q\|^2 + 2d_n(1-d_n)(1-k_q)\|z_n-q\|^2\\
&\quad + 2d_n(1-d_n)\|Tz_n-q\|\|j(x_n-q)-j(z_n-q)\|\\
&\quad + D^3(d_n b(d_n)+c_n b(c_n)) + 2c_n D^2\\
&\le \{(1-d_n)^2 + 2d_n(1-d_n)(1-k_q)(1-k_q d'_n)\}\|x_n-q\|^2\\
&\quad + 2Dd_n(1-d_n)s_n + 4D^3 d_n(1-d_n)(1-k_q)(c'_n + d'_n b(d'_n))\\
&\quad + D^3(d_n b(d_n)+c_n b(c_n)) + 2D^2 c_n\\
&\le (1-k_q d_n)\|x_n-q\|^2 + D^5 d_n(d'_n b(d'_n) + c'_n + s_n + b(d_n)) + 2D^2 c_n\\
&\le (1-k_q d_n)\|x_n-q\|^2 + M d_n(d'_n b(d'_n) + c'_n + s_n + b(d_n) + r_n)
\end{aligned}
$$

for all $n \ge 0$, where $M = D^5$. Let

$$
\begin{aligned}
&\alpha_n = \|x_n-q\|^2, \quad \omega_n = k_q d_n, \quad \gamma_n = 0,\\
&\beta_n = k_q^{-1} M(d'_n b(d'_n) + c'_n + s_n + b(d_n) + r_n)
\end{aligned}
$$

for all $n \ge 0$. Thus (3.17) can be written as

$$
(3.18)\qquad \alpha_{n+1} \le (1-\omega_n)\alpha_n + \omega_n\beta_n + \gamma_n, \quad \forall n \ge 0.
$$

It follows from (3.3), (3.4), (3.16), (3.18) and Lemma 2.3 that $\alpha_n \to 0$ as $n \to \infty$. That is, $x_n \to q$ as $n \to \infty$.

Observe that Lemma 2.1 and (3.1), (3.6), (3.7) and (3.9) ensure that

$$
\begin{aligned}
\|w_n - q\|^2 \\
&= \|(1-d'_n)(y_n - q) + d'_n(Ty_n - q) + c'_n(v_n - Ty_n)\|^2 \\
&\le (1-d'_n)^2\|y_n - q\|^2 + 2d'_n(1-d'_n)\langle Ty_n - q, j(y_n - q)\rangle \\
&\quad + \max\{(1-d'_n)\|y_n - q\|, 1\}d'_n\|Ty_n - q\|b(\|Ty_n - q\|) \\
&\quad + 2c'_n\langle v_n - Ty_n, j((1-d'_n)(y_n - q) + d'_n(Ty_n - q))\rangle \\
&\quad + \max\{\|(1-d'_n)(y_n - q) + d'_n(Ty_n - q)\|, 1\} \\
&\qquad \times c'_n\|v_n - Ty_n\|b(c'_n\|v_n - Ty_n\|) \\
&\le \{(1-d'_n)^2 + 2d'_n(1-d'_n)(1-k_q)\}\|y_n - q\|^2 + D^3 d'_n b(d'_n) \\
&\quad + 2c'_n\|v_n - Ty_n\|\|(1-d'_n)(y_n - q) + d'_n(Ty_n - q)\| + D^3 c'_n b(c'_n) \\
&\le (1 - k_q d'_n)\|y_n - q\|^2 + 2D^3 d'_n b(d'_n) + 2D^2 c'_n
\end{aligned}
\tag{3.19}
$$

for all $n \ge 0$. In view of Lemma 2.1, (3.1), (3.8) and (3.11), we get that

$$
\begin{aligned}
\|p_n - q\|^2 \\
&= \|(1-d_n)(y_n - q) + d_n(Tw_n - q) + c_n(u_n - Tw_n)\|^2 \\
&\le (1-d_n)^2\|y_n - q\|^2 + 2\langle d_n(Tw_n - q), j((1-d_n)(y_n - q))\rangle \\
&\qquad \times \max\{(1-d_n)\|y_n - q\|, 1\}d_n\|Tw_n - q\|b(d_n\|Tw_n - q\|) \\
&\quad + 2\langle c_n(u_n - Tw_n), j((1-d_n)(y_n - q) + d_n(Tw_n - q))\rangle \\
&\quad + \max\{\|(1-d_n)(y_n - q) + d_n(Tw_n - q)\|, 1\} \\
&\qquad \times c_n\|u_n - Tw_n\|b(c_n\|u_n - Tw_n\|)
\end{aligned}
$$

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$$
\begin{aligned}
&\le (1-d_n)^2\|y_n-q\|^2 + 2d_n(1-d_n)(1-k_q)\|w_n-q\|^2 \\
&\quad + 2d_n(1-d_n)\langle Tw_n - q, j(y_n-q) - j(w_n-q)\rangle \\
&\quad + D^3 d_n b(d_n) + 2c_n\|u_n - Tw_n\|\|(1-d_n)(y_n-q) \\
&\quad + d_n(Tw_n - q)\| + D^3 c_n b(c_n) \\
&\le \{(1-d_n)^2 + 2d_n(1-d_n)(1-k_q)(1-k_q d'_n)\}\|y_n-q\|^2 \\
&\quad + 2d_n(1-d_n)Dt_n + 2d_n(1-d_n)(1-k_q)[2D^3 d'_n b(d'_n) + 2D^2 c'_n] \\
&\quad + 2D^3 d_n b(d_n) + 2c_n D^2 \\
&\le (1-k_q d_n)\|y_n-q\|^2 + M d_n(d'_n b(d'_n) + c'_n + t_n + b(d_n) + r_n)
\end{aligned}
\tag{3.20}
$$

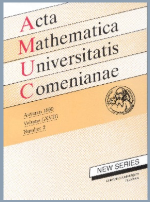
for any $n \ge 0$ It follows from (3.2), (3.12) and (3.20) that

$$
\begin{aligned}
\|y_{n+1}-q\|^2 &\le (\|p_n - q\| + \varepsilon_n)^2 \le \|p_n-q\|^2 + M\varepsilon_n \\
&\le (1-k_q d_n)\|y_n-q\|^2 \\
&\quad + M d_n(d'_n b(d'_n) + c'_n + t_n + b(d_n) + r_n) + M\varepsilon_n
\end{aligned}
\tag{3.21}
$$

for any $n \ge 0$.

Suppose that $\sum_{n=0}^{\infty}\varepsilon_n < \infty$. Put $\alpha_n = \|y_n - q\|^2$, $\omega_n = k_q d_n$, $\gamma_n = M\varepsilon_n$ $\beta_n = M(d'_n b(d'_n) + c'_n + t_n + b(d_n) + r_n)k_q^{-1}$ for all $n \ge 0$. Using Lemma 2.3, (3.3), (3.4), (3.16) and (3.21), we conclude immediately that $\alpha_n \to 0$ as $n \to \infty$. That is, $y_n \to q$ as $n \to \infty$. Therefore $\{x_n\}_{n=0}^{\infty}$ is almost S-stable. Suppose that $\lim_{n\to\infty} y_n = q$. It follows from (3.20), (3.16) and (3.3) that

$$
\begin{aligned}
\varepsilon_n &\le \|y_{n+1} - q\| + \|p_n - q\| \\
&\le \|y_{n+1} - q\| + \Big[(1-k_q d_n)\|y_n - q\|^2
\end{aligned}
$$



$$+ Md_n(d'_n b(d'_n) + c'_n + t_n + b(d_n) + r_n)\big]^{1/2} \to 0$$

as $n \to \infty$. That is, $\varepsilon_n \to 0$ as $n \to \infty$. This completes the proof. □

Theorem 3.2. *Let X, T, $R(T)$, q, $\{u_n\}_{n=0}^\infty$, $\{v_n\}_{n=0}^\infty$, $\{x_n\}_{n=0}^\infty$, $\{z_n\}_{n=0}^\infty$, $\{y_n\}_{n=0}^\infty$, $\{w_n\}_{n=0}^\infty$ and $\{\varepsilon_n\}_{n=0}^\infty$ be as in Theorem 3.1. Suppose that $\{a_n\}_{n=0}^\infty$, $\{b_n\}_{n=0}^\infty$, $\{c_n\}_{n=0}^\infty$, $\{a'_n\}_{n=0}^\infty$, $\{b'_n\}_{n=0}^\infty$ and $\{c'_n\}_{n=0}^\infty$ are any sequences in $[0,1]$ satisfying (3.1) and*

$$\lim_{n\to\infty} b(d_n) = \lim_{n\to\infty} b'_n = \lim_{n\to\infty} c'_n = 0; \tag{3.22}$$

$$\sum_{n=0}^{\infty} c_n < \infty; \tag{3.23}$$

$$\sum_{n=0}^{\infty} b_n = \infty. \tag{3.24}$$

Then the conclusions of Theorem 3.1 hold.

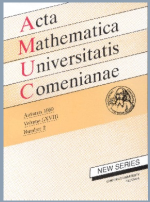
Proof. Let

$$\alpha_n = \|x_n - q\|^2, \quad \omega_n = k_q d_n, \quad \gamma_n = 2D^2 + r_n,$$
$$\beta_n = k_q^{-1} M(d'_n b(d'_n) + c'_n + s_n + b(d_n))$$

for all $n \geq 0$. As in the proof of (3.17), we conclude that $x_n \to q$ as $n \to \infty$.

Put $\alpha_n = \|y_n - q\|^2$, $\omega_n = k_q d_n$, $\gamma_n = M(r_n + \varepsilon_n)$ and $\beta_n = M(d'_n b(d'_n) + c'_n + t_n + b(d_n))k_q^{-1}$ for all $n \geq 0$. It follows from (3.21) that $y_n \to q$ as $n \to \infty$. The rest of the proof is similar to that of Theorem 3.1, and is omitted. This completes the proof. □

The proof of Theorem 3.3 below is similar to that of Theorem 3.1, so we omit the details.



Theorem 3.3. *Let K be a nonempty bounded closed convex subset of a real uniformly smooth Banach space X and $T : K \to K$ be a local strongly pseudocontractive operator. Let $q \in K$ be a fixed point of T and $\{u_n\}_{n=0}^{\infty}$, $\{v_n\}_{n=0}^{\infty}$ be arbitrary sequences in K. Suppose that $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{a'_n\}_{n=0}^{\infty}$, $\{b'_n\}_{n=0}^{\infty}$, $\{c'_n\}_{n=0}^{\infty}$ and $\{r_n\}_{n=0}^{\infty}$ are any sequences in $[0,1]$ satisfying (3.1)–(3.4). If $\{x_n\}_{n=0}^{\infty}$ is the Ishikawa iteration sequence with errors generated from an arbitrary $x_0 \in K$ by (3.5), then it converges strongly to the unique fixed point q of T.*

Theorem 3.4. *Let X, K, T, q, $\{u_n\}_{n=0}^{\infty}$, $\{v_n\}_{n=0}^{\infty}$, $\{x_n\}_{n=0}^{\infty}$, $\{z_n\}_{n=0}^{\infty}$ be as in Theorem 3.2 and $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{a'_n\}_{n=0}^{\infty}$, $\{b'_n\}_{n=0}^{\infty}$ and $\{c'_n\}_{n=0}^{\infty}$ be any sequences in $[0,1]$ satisfying (3.1) and (3.22)–(3.24). Then $\{x_n\}_{n=0}^{\infty}$ converges strongly to the unique fixed point q of T.*

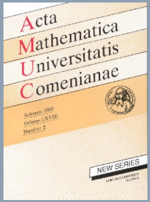
Remark. Theorem 3.3 extends, improves and unifies Theorems 3.2 and 4.1 of Chang [3], Theorems 3.3 and 4.1 of Chang et al. [4], the Theorem Chidume [5], Theorems 1 and 2 of Chidume [6], Theorems 3 and 4 of Chidume [7], Theorem 4 of Chidume and Osilike [10], Theorem 4.2 of Tan and Xu [53] and Theorem 3.3 of Xu [55].

Theorem 3.5. *Let X be a real uniformly smooth Banach space and $T : X \to X$ be a local strongly accretive operator. Define $G : X \to X$ by $Gx = f - Tx$ for all $x \in X$. Suppose that $R(T)$ is bounded and the equation $x + Tx = f$ has a solution q for some $f \in X$. Suppose that $\{u_n\}_{n=0}^{\infty}$, $\{v_n\}_{n=0}^{\infty}$ are arbitrary bounded sequences in X and $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{a'_n\}_{n=0}^{\infty}$, $\{b'_n\}_{n=0}^{\infty}$, $\{c'_n\}_{n=0}^{\infty}$ and $\{r_n\}_{n=0}^{\infty}$ are any sequences in $[0,1]$ satisfying (3.1)–(3.4). For arbitrary $x_0 \in X$, the Ishikawa iteration sequence with errors $\{x_n\}_{n=0}^{\infty}$ is defined by*

$$z_n = a'_n x_n + b'_n G x_n + c'_n v_n, \quad x_{n+1} = a_n x_n + b_n G z_n + c_n u_n \tag{3.25}$$

for all $n \geq 0$. Let $\{y_n\}_{n=0}^{\infty}$ be any bounded sequence in X and define $\{\varepsilon_n\}_{n=0}^{\infty}$ by

$$\begin{aligned} w_n &= a'_n y_n + b'_n G y_n + c'_n v_n, \\ \varepsilon_n &= \|y_{n+1} - a_n y_n - b_n G w_n - c_n u_n\| \end{aligned} \tag{3.26}$$



for all $n \geq 0$*. Then there exist nonnegative sequences* $\{s_n\}_{n=0}^{\infty}$*,* $\{t_n\}_{n=0}^{\infty}$ *and a constant* $M > 0$ *such that* $\lim_{n\to\infty} s_n = \lim_{n\to\infty} t_n = 0$ *and*

(a) $\{x_n\}_{n=0}^{\infty}$ *converges strongly to the unique solution* q *of the equation* $x+Tx=f$ *and*

$$\begin{aligned}\|x_{n+1} - q\|^2 \leq (1 - d_n k_q)\|x_n - q\|^2 + M d_n (d'_n b(d'_n) \\ + c'_n + s_n + b(d_n) + r_n), \quad \forall\, n \geq 0;\end{aligned}$$

(b) *for all* $n \geq 0$

$$\begin{aligned}\|y_{n+1} - q\|^2 \leq (1 - d_n k_q)\|y_n - q\|^2 + M d_n (d'_n b(d'_n) + c'_n \\ + t_n + b(d_n) + r_n) + M\varepsilon_n;\end{aligned}$$

(c) $\sum_{n=0}^{\infty} \varepsilon_n < \infty$ *implies that* $\lim_{n\to\infty} y_n = q$*, so that* $\{x_n\}_{n=0}^{\infty}$ *is almost G-stable;*

(d) $\lim_{n\to\infty} y_n = q$ *implies that* $\lim_{n\to\infty} \varepsilon_n = 0$.

Proof. It follows from (1.2) that for given $x \in X$ there exists $k_x \in (0,1)$ such that

$$\langle Tx - Ty, j(x-y)\rangle \geq k_x \|x-y\|^2, \quad \forall\, y \in X,$$

which implies that

$$\begin{aligned}\langle (I-G)x - (I-G)y, j(x-y)\rangle &= \|x-y\|^2 - \langle Gx - Gy, j(x-y)\rangle \\ &= \|x-y\|^2 + \langle Tx - Ty, j(x-y)\rangle \\ &\geq k_x\|x-y\|^2, \quad \forall\, y \in X.\end{aligned}$$

That is, $I - G$ is local strongly accretive. Thus G is local strongly pseudocontractive. It is easy to see that q is a unique fixed point of G. Therefore, q is the unique solution of the equation $x + Tx = f$. The rest of the argument uses the same ideas as that of Theorem 3.1 and is thus omitted. This completes the proof. □

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Theorem 3.6. *Let X, T, G, $R(T)$, f, q, $\{u_n\}_{n=0}^{\infty}$, $\{v_n\}_{n=0}^{\infty}$, $\{x_n\}_{n=0}^{\infty}$, $\{z_n\}_{n=0}^{\infty}$, $\{y_n\}_{n=0}^{\infty}$, $\{w_n\}_{n=0}^{\infty}$ and $\{\varepsilon_n\}_{n=0}^{\infty}$ be as in Theorem 3.3. Suppose that $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{a'_n\}_{n=0}^{\infty}$, $\{b'_n\}_{n=0}^{\infty}$ and $\{c'_n\}_{n=0}^{\infty}$ are any sequences in $[0,1]$ satisfying (3.1) and (3.22)–(3.24). Then the conclusions of Theorem 3.5 hold.*

Remark. The convergence result in Theorem 3.6 generalizes Theorems 11 and 12 of Chidume [8].

Theorem 3.7. *Let X be a real uniformly smooth Banach space and $T : X \to X$ be a local strongly accretive operator. Define $S : X \to X$ by $Sx = f + x - Tx$ for all $x \in X$. Suppose that the equation $Tx = f$ has a solution q for some $f \in X$ and either $R(T)$ or $R(I - T)$ is bounded. Assume that $\{u_n\}_{n=0}^{\infty}$, $\{v_n\}_{n=0}^{\infty}$ are arbitrary bounded sequences in X and $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{a'_n\}_{n=0}^{\infty}$, $\{b'_n\}_{n=0}^{\infty}$, $\{c'_n\}_{n=0}^{\infty}$ and $\{r_n\}_{n=0}^{\infty}$ are any sequences in $[0,1]$ satisfying (3.1)–(3.4). For arbitrary $x_0 \in X$, the Ishikawa iteration sequence with errors $\{x_n\}_{n=0}^{\infty}$ is defined by*

$$z_n = a'_n x_n + b'_n S x_n + c'_n v_n, \quad x_{n+1} = a_n x_n + b_n S z_n + c_n u_n, \quad \forall\, n \ge 0. \tag{3.27}$$

Let $\{y_n\}_{n=0}^{\infty}$ be any bounded sequence in X and define $\{\varepsilon_n\}_{n=0}^{\infty}$ by

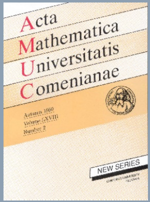
$$\begin{aligned} w_n &= a'_n y_n + b'_n S y_n + c'_n v_n, \\ \varepsilon_n &= \|y_{n+1} - a_n y_n - b_n S w_n - c_n u_n\| \end{aligned} \tag{3.28}$$

for all $n \ge 0$. Then there exist nonnegative sequences $\{s_n\}_{n=0}^{\infty}$, $\{t_n\}_{n=0}^{\infty}$ and a constant $M > 0$ such that $\lim_{n\to\infty} s_n = \lim_{n\to\infty} t_n = 0$ and

(a) *$\{x_n\}_{n=0}^{\infty}$ converges strongly to the unique solution q of the equation $Tx = f$ and*

$$\|x_{n+1} - q\|^2 \le (1 - d_n k_q)\|x_n - q\|^2 + M d_n (d'_n b(d'_n) + c'_n + s_n + b(d_n) + r_n)$$

for all $n \ge 0$;



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(b) *for all* $n \geq 0$

$$\|y_{n+1}-q\|^2 \leq (1-d_nk_q)\|y_n-q\|^2 + Md_n(d'_nb(d'_n) + c'_n + t_n + b(d_n) + r_n) + M\varepsilon_n;$$

(c) $\sum_{n=0}^{\infty} \varepsilon_n < \infty$ *implies that* $\lim_{n\to\infty} y_n = q$*, so that* $\{x_n\}_{n=0}^{\infty}$ *is almost* S*-stable;*

(d) $\lim_{n\to\infty} y_n = q$ *implies that* $\lim_{n\to\infty} \varepsilon_n = 0$.

Proof. Since T is local strongly accretive and q is a solution of the equation $Tx = f$, it follows that q is a unique solution of the equation $Tx = f$ and there exists $k_q \in (0,1)$ such that

$$\langle Tx - Tq, j(x-q)\rangle \geq k_q\|x-q\|^2, \quad \forall\, x \in X,$$

which implies that

$$\|x-q\| \leq k_q^{-1}\|Tx-Tq\|, \quad \forall\, x \in X \tag{3.29}$$

and

$$\langle Sx - Sq, j(x-q)\rangle \leq (1-k_q)\|x-q\|^2, \quad \forall\, x \in X. \tag{3.30}$$

We now claim that $R(S)$ is bounded. Suppose that $R(I-T)$ is bounded. It is clear that $R(S)$ is bounded. Suppose that $R(T)$ is bounded. From (3.29), we have

$$\begin{aligned}\|Sx - Sy\| &\leq \|x-y\| + \|Tx - Ty\| \\ &\leq \|x-q\| + \|y-q\| + \|Tx-Tq\| + \|Ty-Tq\| \\ &\leq (1+k_q^{-1})(\|Tx-Tq\| + \|Ty-Tq\|), \quad \forall\, x,y \in X,\end{aligned}$$

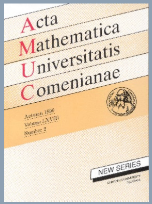
which implies that $R(S)$ is bounded. Note that S is local strongly pseudocontractive and $F(S) = \{q\}$. The rest of the proof follows immediately as in the proof of Theorem 3.1, and is therefore omitted. This completes the proof. □

Theorem 3.8. *Let X, T, S, $R(T)$, $R(I-T)$, f, q, $\{u_n\}_{n=0}^{\infty}$, $\{v_n\}_{n=0}^{\infty}$, $\{x_n\}_{n=0}^{\infty}$, $\{z_n\}_{n=0}^{\infty}$, $\{y_n\}_{n=0}^{\infty}$, $\{w_n\}_{n=0}^{\infty}$ and $\{\varepsilon_n\}_{n=0}^{\infty}$ be as in Theorem 3.4. Suppose that $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{a'_n\}_{n=0}^{\infty}$, $\{b'_n\}_{n=0}^{\infty}$ and $\{c'_n\}_{n=0}^{\infty}$ are any sequences in $[0,1]$ satisfying (3.1) and (3.22)–(3.24). Then the conclusions of Theorem 3.4 hold.*

Remark. The convergence result in Theorem 3.8 extends Theorem 1 of Chidume [7], Theorems 7 and 8 of Chidume [8], Theorem 3.2 of Ding [12], Theorem 4.1 of Tan and Xu [53] and Theorem 3.1 of Xu [55].

Acknowledgement. The authors thank the referee for his valuable suggestion for the improvement of the paper.

1. Browder F. E., *Nonlinear mappings of nonexpansive and accretive type in Banach spaces*, Bull. Amer. Math. Soc. **73** (1967), 875–882.
2. ______, *Nonlinear operations and nonlinear equations of evolution in Banach spaces*, Proc. Sympos. Pure Math. **18(2)** (1976).
3. Chang S. S., *Some problems and results in the study of nonlinear analysis*, Nonlinear Anal. **30(7)** (1997), 4197–4208.
4. Chang S. S., Cho Y. J., Lee B. S., Jung J. S. and Kang S. M., *Iterative approximations of fixed points and solutions for strongly accretive and strongly pseudo-contractive mappings in Banach spaces*, J. Math. Anal. Appl. **224** (1998), 149–165.
5. Chidume C. E., *Iterative approximation of fixed points of Lipschitzian strictly pseudo-contractive mappings*, Proc. Amer. Math. Soc. **99(2)** (1987), 283–288.
6. ______, *Approximation of fixed points of strongly pseudo-contractive mappings*, Proc. Amer. Math. Soc. **120(2)** (1994), 545–551.
7. ______, *Iterative solutions of nonlinear equations with strongly accretive operators*, J. Math. Anal. Appl. **192** (1995), 502–518.



8. ———, *Iterative solutions of nonlinear equations in smooth Banach spaces*, Nonlinear Anal. **26(11)** (1996), 1823–1834.

9. Chidume C. E. and Moore C., *The solution by iteration of nonlinear equations in uniformly smooth Banach spaces*, J. Math. Anal. Appl. **215** (1997), 132–146.

10. Chidume C. E. and Osilike M. O., *Ishikawa iteration process for nonlinear Lipschitz strongly accretive mappings*, J. Math. Anal. Appl. **192** (1995), 727–741.

11. Deng L. and Ding X. P., *Iterative process for Lipschitz local strictly pseudocontractive mappings*, Appl. Math. Mech. **15(2)** (1994), 119–123.

12. Ding X. P., *Iterative process with errors to locally strictly pseudocontractive maps in Banach spaces*, Comput. Math. Appl. **32(10)** (1996), 91–97.

13. Harder A. M., *Fixed point theory and stability results for fixed point iteration procedures*, Ph. D. Thesis, University of Missouri-Rolla, 1987.

14. Harder A. M. and Hicks T. L., *A stable iteration procedure for nonexpansive mappings*, Math. Japon. **33** (1988), 687–692.

15. ———, *Stability results for fixed point iteration procedures*, Math. Japon. **33** (1988), 693–706.

16. Ishikawa S., *Fixed points by a new iteration method*, Proc. Amer. Math. Soc. **44** (1974), 147–150.

17. Kato T., *Nonlinear semigroups and evolution equations*, J. Math. Soc. Japan. **19** (1967), 509–519.

18. Liu L. S., *Fixed points of local strictly pseudo-contractive mappings using Mann and Ishikawa iteration with errors*, Indian J. Pure Appl. Math. **26(7)** (1995), 649–659.

19. ———, *Approximation of fixed points of a strictly pseudocontractive mapping*, Proc. Amer. Math. Soc. **125(5)** (1997), 1363–1366.

20. Liu Z., Agarwal R. P., Feng C. and Kang S. M., *Weak and strong convergence theorems of common fixed points for a pair of nonexpansive and asymptotically nonexpansive mappings*, Acta. Univ. Palacki. Olomuc., Fac. rer. nat. Mathematica **44** (2005), 83–96.

21. Liu Z., An Z., Li Y. and Kang S. M., *Iterative approximation of fixed points for ϕ-hemicontractive operators in Banach spaces*, Commun. Korean. Math. Soc. **19** (2004), 63–74.

22. Liu Z., Bouxias M. and Kang S. M., *Iterative approximation of solutions to nonlinear equations of ϕ-strongly accretive operators in Banach spaces*, Rocky Mountain J. Math. **32** (2002), 981–997.

23. Liu Z., Feng C., Kang S. M. and Kim K. H., *Convergence and stability of modified Ishikawa iterative procedures with errors for some nonlinear mappings*, Panamer. Math. J. **13** (2003), 19–33.

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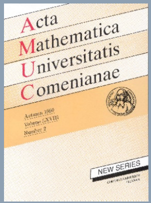
24. Liu Z. and Kang S. M., *Stability of Ishikawa iteration methods with errors for strong pseudocontractions and nonlinear equations involving accretive operators in arbitrary real Banach spaces*, Math. Comput. Modelling **34** (2001), 319–330.
25. ———, *Convergence and stability of the Ishikawa iteration procedures with errors for nonlinear equations of the ϕ-strongly accretive type*, Neural Parallel Sci. Comput. **9** (2001), 103–118.
26. ———, *Weak and strong convergence for fixed points of ssymptotically nonexpansive mappings*, Acta Math. Sin. (Engl. Ser.) **20** (2004), 1009–1018.
27. ———, *Convergence theorems for ϕ-strongly accretive and ϕ-hemicontractive operators*, J. Math. Anal. Appl. **253** (2001), 35–49.
28. ———, *Iterative solutions of nonlinear equations with ϕ-strongly accretive operators in uniformly smooth Banach spaces*, Comput. Math. Appl. **45** (2003), 623–634.
29. ———, *Iterative process with errors for nonlinear equations of local $\phi-$strongly accretive operators in arbitrary Banach Spaces*, Int. J. Pure Appl. Math. **12** (2004), 229–246.
30. ———, *Stable and almost stable iteration schemes for nonlinear accretive operator equations in arbitrary Banach spaces*, Panamer. Math. J. **13** (2003), 91–102.
31. ———, *Iterative approximation of fixed points for ϕ-hemicontractive operators in arbitrary Banach spaces*, Acta Sci. Math. (Szeged) **67** (2001), 821–831.
32. Liu Z., Kang S. M. and Cho Y. J., *Convergence and almost stability of Ishikawa iterative scheme with errors for m-accretive operators*, Comput. Math. Appl. **47** (2004), 767–778.
33. Liu Z., Kang S. M. and Shim S. H., *Almost stability of the Mann iteration method with errors for strictly hemi-contractive operators in smooth Banach spaces*, J. Korean Math. Soc. **40** (2003), 29–40.
34. Liu Z., Kang S. M. and Ume J. S., *General principles for Ishikawa iterative process for multi-valued mappings*, Indian J. Pure Appl. Math. **34** (2003), 157–162.
35. ———, *Iterative solutions of ϕ-positive definite operator equations in real uniformly smooth Banach spaces*, Int. J. Math. Math. Sci. **27** (2001), 155–160.
36. ———, *Error bounds of the iterative approximations of Ishikawa iterative schemes with errors for strictly hemicontractive and strongly quasiaccretive operators*, Comm. Appl. Nonlinear Anal. **9** (2002), 33–46.
37. Liu Z., Kim J. K. and Chun S. A., *Iterative approximation of fixed points for generalized asymptotically contractive and generalized hemicontractive mappings*, Panamer. Math. J. **12** (2002), 67–74.
38. Liu Z., Kim J. K. and Kim K. H., *Convergence theorems and stability problems of the modified Ishikawa iterative sequences for strictly successively hemicontractive mappings*, Bull. Korean Math. Soc. **39** (2002), 455–469.



39. Liu Z., Kim J. K. and Kang S. M., *Necessary and sufficient conditions for convergence of Ishikawa Iterative schemes with errors to ϕ-hemicontractive mappings*, Commun. Korean Math. Soc. **18** (2003), 251–261.

40. Liu Z., Nam Y. M., Kim J. K. and Ume J. S., *Stable iteration schemes for nonlinear strongly quasi-accretive and strictly hemicontractive operators in Banach spaces*, Nonlinear Funct. Anal. Appl. **7** (2002), 313–328.

41. ______, *Stability of Ishikawa iterative schemes with errors for nonlinear accretive operators in arbitrary Banach spaces*, Nonlinear Funct. Anal. Appl. **7** (2002), 55–67.

42. Liu Z. and Ume J. S., *Stable and almost stable iteration schemes for strictly hemi-contractive operators in arbitrary Banach spaces*, Numer. Funct. Anal. Optim. **23** (2002), 833–848.

43. Liu Z., Ume J. S. and Kang S. M., *Strong convergence and pseudo stability for operators of the $\phi-$accretive type in uniformly smooth Banach spaces*, Rostock. Math. Kolloq. **59** (2005), 29–40.

44. ______, *Approximation of a solution for a K-positive definite operator equation in real uniformly smooth Banach spaces*, Int. J. Pure Appl. Math. **25** (2005), 135–143.

45. Liu Z., Wang L., Kim H. G. and Kang S. M., *The equivalence of Mann and Ishikawa iteration schemes with errors for $\phi-$strongly accretive operators in uniformly smooth Banach spaces*, Math. Sci. Res. J. **9** (2005), 47–57.

46. Liu Z., Xu Y. and Cho Y. J., *Iterative solution of nonlinear equations with ϕ-strongly accretive operators*, Arch. Math. **77** (2001), 508–516.

47. Liu Z., Zhang L. and Kang S. M., *Iterative solutions to nonlinear equations of the accretive type in Banach spaces*, East Asian Math. J. **17** (2001), 265–273.

48. ______, *Convergence theorems and stability results for Lipschitz strongly pseudocontractive operators*, Int. J. Math. Math. Sci. **31** (2002), 611–617.

49. Mann W. R., *Mean value methods in iteration*, Proc. Amer. Math. Soc. **4** (1953), 506–510.

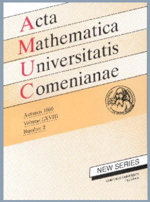
50. Martin R. H., *A global existence theorem for autonomous differential equations in Banach spaces*, Proc. Amer. Math. Soc. **26** (1970), 307–314.

51. Osilike M. O., *Stable iteration procedures for strong pseudocontractions and nonlinear operator equations of the accretive type*, J. Math. Anal. Appl. **204** (1996), 677–692.

52. Rhoades B. E., *Comments on two fixed point iteration methods*, J. Math. Anal. Appl. **56** (1976), 741–750.

53. Tan K. K. and Xu H. K., *Iterative solutions to nonlinear equations of strongly accretive operators in Banach spaces*, J. Math. Anal. Appl. **178** (1993), 9–21.

54. Weng X., *Fixed point iteration for local strictly pseudo-contractive mapping*, Proc. Amer. Math. Soc. **113(3)** (1991), 727–731.



55. Xu Y., *Ishikawa and Mann iterative processes with errors for nonlinear strongly accretive operator equations*, J. Math. Anal. Appl. **224** (1998), 91–101.

56. Xu Z. B. and Roach G. F., *Characteristic inequalities of uniformly convex and uniformly smooth Banach spaces*, J. Math. Anal. Appl. **157** (1991), 189–210.

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