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ON ALMOST PSEUDO BOCHNER SYMMETRIC GENERALIZED COMPLEX SPACE FORMS

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ABSTRACT. In the present paper, we study when the Bochner curvature tensor is almost pseudo symmetric, almost pseudo Ricci symmetric and flat almost pseudo Ricci symmetric in generalized complex space forms.

1. INTRODUCTION

A Kaehler manifold with constant holomorphic sectional curvature is a complex space form and it has a specific form of its curvature tensor. More generally an almost Hermitian manifold M is called a *generalized complex space form* $M(f_1, f_2)$ if its Riemannian curvature tensor R satisfies,

$$(1.1)\quad \begin{aligned} R(X,Y)Z = f_1\{g(Y,Z)X - g(X,Z)Y\} + f_2\{&g(X,JZ)JY \\ &- g(Y,JZ)JX + 2g(X,JY)JZ\} \end{aligned}$$

for all $X, Y, Z \in TM$ where f_1 and f_2 are smooth functions on M [18, 19]. In [18], an important obstruction for such a space was presented by F. Tricerri and L. Vanhecke: If M is connected, dim$\geq$ 6, and f_2 is not identically zero, then M is a complex-space-form(in particular, f_1 and f_2 must be constant). In 1989 the author Z. Olszak [10] has worked on existence of generalized complex space form.

A non flat Riemannian manifold (M, g) is said to be almost pseudo symmetric manifold [6] if its curvature tensor satisfies the condition

$$(1.2)\quad \begin{aligned} &(\nabla_X R)(Y,Z,U,V) \\ &\quad = [A(X) + B(X)]R(Y,Z,U,V) + A(Y)R(X,Z,U,V) \\ &\quad + A(Z)R(Y,X,U,V) + A(U)R(Y,Z,X,V) + A(V)R(Y,Z,U,X), \end{aligned}$$

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where A and B are two non zero 1-forms defined by

$$g(X, \alpha) = A(X), g(X, \beta) = B(X), \tag{1.3}$$

for all vector fields X, ∇ denotes the operator of covariant differentiation with respect to the metric g. The 1-forms A and B are called the associated 1-forms. The name almost pseudo symmetric was chosen because if $A = B$ in (1.2) then the manifold reduces to a pseudo symmetric manifold, introduced by M. C. Chaki [2]. If $A = B = 0$ in (1.2) then the manifold reduces to a symmetric manifold in the sense of E. Cartan. It is to be noted that the notion of pseudo symmetry in the sense of Chaki [2] is different from that of R. Deszcz [7]. It may be mentioned that the almost pseudo symmetric manifold is not a particular case of a weakly symmetric manifold introduced by L. Tamássy and T. Q. Binh [16]. pseudo symmetric manifolds [2, 12], weakly symmetric manifolds [16], almost pseudo concircularly symmetric manifolds [6], etc.

In a paper [4], Chaki and T. Kawaguchi introduced a type of non-flat Riemannian manifold (M, g), whose Ricci tensor S of type $(0, 2)$ satisfies the condition

$$(\nabla_X S)(Y, Z) = [A(X) + B(X)]S(Y, Z) + A(Y)S(X, Z) + A(Z)S(Y, X), \tag{1.4}$$

where A, B and ∇ have the meaning already stated. Such a manifold was called an almost pseudo Ricci symmetric manifold.

A Ricci soliton is a natural generalization of an Einstein metric and is defined on a Riemannian manifold (M, g). A Ricci soliton is a triple (g, V, λ) with g a Riemannian metric, V a vector field, and λ a real scalar such that

$$L_V g + 2S + 2\lambda g = 0, \tag{1.5}$$

where S is Ricci tensor of M and L_V denotes the Lie derivative operator along the vector field V. The Ricci soliton is said to be shrinking, steady, and expanding accordingly as λ is negative, zero, and positive respectively [9].

Motivated by these ideas, in the present paper we have made an attempt to study almost pseudo Bochner symmetric, almost pseudo Bochner Ricci symmetric and Bochner flat almost pseudo Ricci symmetric of generalized complex space forms.

2. Preliminaries

A Kaehler manifold is a complex n-dimensional manifold M, with a complex structure J and a positive-definite metric g which satisfies the following conditions [17]

$$J^2 = -I, \ g(JX, JY) = g(X, Y) \text{ and } \nabla J = 0,$$

where ∇ means covariant derivation according to the Levi-Civita connection.

Using the second Bianchi identity, we infer

$$(\operatorname{div} R)(Y, Z)U = (\nabla_Y S)(Z, U) - (\nabla_Z S)(Y, U), \tag{2.1}$$

and the scalar curvature $r = \Sigma S(e_i, e_i)$ and

$$(\nabla_X S)(e_i, e_i) = \nabla_X r = dr(X). \tag{2.2}$$

Let Q be the Ricci operator defined by $g(QX, Y) = S(X, Y)$,

$$(\nabla_Z S)(X, Y) = g((\nabla_Z Q)(X), Y).$$

Taking $Y = Z = e_i$ and taking summation over i in the above equation, we get

$$(\nabla_{e_i} S)(X, e_i) = g((\nabla_{e_i} Q)(X), e_i).$$

$$\Rightarrow (\operatorname{div} Q)(X) = \operatorname{trace}(Z \to (\nabla_Z Q)(X)) = \sum g((\nabla_{e_i} Q)(X), e_i).$$

$$(\operatorname{div} Q)(X) = \frac{1}{2} dr(X).$$

$$(\nabla_{e_i} S)(X, e_i) = \frac{1}{2} dr(X).$$

For generalized complex space form $M(f_1, f_2)$ we have

$$S(X, Y) = \{(n-1)f_1 + 3f_2\} g(X, Y). \tag{2.3}$$

$$QX = [(n-1)f_1 + 3f_2]X. \tag{2.4}$$

$$r = n[(n-1)f_1 + 3f_2], \tag{2.5}$$

where S is the Ricci tensor, Q is the Ricci operator and r is scalar curvature of the space form $M(f_1, f_2)$. Putting $Y = \alpha$ in (2.3) we get,

$$S(X, \alpha) = \{(n-1)f_1 + 3f_2\} A(X). \tag{2.6}$$

Definition 1. A Kaehler manifold is called an almost pseudo Bochner symmetric manifold if its Bochner curvature tensor D of type (0,4) is not zero and satisfies the condition

$$\begin{aligned}(\nabla_X D)(Y, Z, U, V) &= [A(X) + B(X)]D(Y, Z, U, V) + A(Y)D(X, Z, U, V) \\ &\quad + A(Z)D(Y, X, U, V) + A(U)D(Y, Z, X, V) + A(V)D(Y, Z, U, X),\end{aligned} \tag{2.7}$$

where A, B are 1-forms(not simultaneously zero) and D is given by [20],

$$\begin{aligned} D(X, Y, Z, U) &= R(X, Y, Z, U) \\ &- \frac{1}{2n+4}[g(Y, Z)S(X, U) - S(X, Z)g(Y, U) + g(JY, Z)S(JX, U) \\ &- S(JX, Z)g(JY, U) + S(Y, Z)g(X, U) \\ &- g(X, Z)S(Y, U) + S(JY, Z)g(JX, U) \\ &- g(JX, Z)S(JY, U) - 2S(Y, JX)g(JZ, U) \\ &- 2S(JZ, U)g(JX, Y)] \\ &+ \frac{r}{(2n+2)(2n+4)}[g(Y, Z)g(X, U) - g(X, Z)g(Y, U) \end{aligned} \tag{2.8}$$

$$+g(JY,Z)g(JX,U) - g(JX,Z)g(JY,U) - 2g(JX,Y)g(JZ,U)].$$

Definition 2. A Kaehler manifold is called an almost pseudo Bochner Ricci symmetric manifold if its Bochner Ricci tensor K of type (0,2) is not identically zero and satisfies the condition

$$(\nabla_X K)(Y,Z) = [A(X)+B(X)]K(Y,Z)+A(Y)K(X,Z)+A(Z)K(Y,X) \tag{2.9}$$

where A, B are nowhere vanishing 1-forms and K is given by,

$$K(Y,Z) = \frac{n}{2n+4}[S(Y,Z) - \frac{r}{2(n+1)}g(Y,Z)]. \tag{2.10}$$

Suppose (M,g) is an generalized complex space form and (g,V,λ) is a Ricci Soliton in (M,g). If V is Killing vector field, then

$$(L_V g) = 0 \tag{2.11}$$

and if V is conformal Killing vector field, then

$$(L_V g) = \rho g. \tag{2.12}$$

3. Almost pseudo Bochner symmetric generalized complex space form

If the manifold M is an almost pseudo Bochner symmetric Generalized Complex Space Forms, then we can readily write it as

$$D(JY,JZ,U,V) = D(Y,Z,U,V). \tag{3.1}$$

Taking the covariant derivative of (3.1), we get

$$(\nabla_X D)(JY,JZ,U,V) = (\nabla_X D)(Y,Z,U,V). \tag{3.2}$$

Using (2.7) in (3.2), we get

$$\begin{aligned} A(Y)D(X,Z,U,V) + D(Z)R(Y,X,U,V) \\ = A(JY)D(X,JZ,U,V) + A(JZ)D(JY,X,U,V). \end{aligned} \tag{3.3}$$

Putting $Z = U = e_i$ in (3.3) where $\{e_i\}$ is an orthonormal basis of the tangent space at each point of the manifold and taking summation over i $(1 \leq i \leq n)$ we get

$$A(Y)K(X,V) - A(D(Y,X)V) = -A(JY)K(JX,V) - A(D(JY,X)JV). \tag{3.4}$$

Simplify using equations (2.8) and (2.10) in (3.4). Again simplify by putting $X = V = e_i$, $1 \leq i \leq n$. Further using equations (2.6) and (2.5) we get,

$$A(Y)r = 0. \tag{3.5}$$

Thus if $r \neq 0$, then from (3.5) we get $A(Y) = 0$. Using $A(Y) = 0$ in (2.7) we have

$$(\nabla_X D)(Y,Z,U,V) = [B(X)]D(Y,Z,U,V).$$

That is, an almost pseudo Bochner symmetric generalized complex space forms reduces to recurrent one. Therefore we can state the following:

Theorem 3.1. *An almost pseudo Bochner symmetric generalized complex space form with non zero scalar curvature is recurrent.*

Suppose $r = 0$. Putting $Z = U = e_i$ $(1 \le i \le n)$ in (2.7) and on simplification, we get

$$(\nabla_X K)(Y,V) = [A(X)+B(X)]K(Y,V) + A(Y)K(X,V) - A(D(Y,X)V) + A(D(X,V)Y) + A(V)K(Y,X). \tag{3.6}$$

Using equations (2.8) and (2.10) in (3.6) after simplification, again putting $Y = V = e_i$ and taking sum over i, $1 \le i \le n$ we get

$$[n(n+2)]drX = r[(n^2-2n)A(X) + n(n+2)B(X)] + 8(n+4)(n+1)S(X,\alpha). \tag{3.7}$$

Hence $drX = 0$ as $r = 0$, we get $S(X,\alpha) = 0$. It can also be written as

$$S(Z,\alpha) = 0.g(Z,\alpha), \tag{3.8}$$

which, by replacing α by ω, leads to

$$S(Z,\omega) = 0.g(Z,\omega). \tag{3.9}$$

Hence, from equations (3.8) and (3.9), we conclude;

Theorem 3.2. *If M is an almost pseudo Bochner symmetric generalized complex space form then it is Ricci flat and hence α and ω are eigenvector of the Ricci tensor S with respect to the zero eigenvalue.*

Using equation (3.8) in (1.5), we get

$$(L_V g)(Z,\alpha) + 2\lambda g(Z,\alpha) = 0. \tag{3.10}$$

Equation (2.11) in (3.10), we get

$$2\lambda g(Z,\alpha) = 0 \text{ and } \lambda = 0.$$

Then we can state the following

Lemma 1. *Let (g,V,λ) be a Ricci soliton in an almost pseudo Bochner symmetric generalized complex space form. If V is Killing vector field then it is steady.*

Take V as conformal Killing vector field then using equation (2.12) in (3.10), we get

$$(\rho + 2\lambda)g(Z,\alpha) = 0, \ \lambda = -\frac{\rho}{2}.$$

Then we can write the following

Lemma 2. *Let (g,V,λ) be a Ricci soliton in an almost pseudo Bochner symmetric generalized complex space form. If V is conformal Killing vector field then it is shrinking.*

Contracting (2.7) with respect to the pair of arguments Y, V(i.e, taking $Y = V = e_i$ into (2.7) and summing up over i), we have

$$(3.11)\quad (\nabla_X K)(Z,U) = [A(X)+B(X)]K(Z,U) + A(Z)K(X,U) + A(U)K(Z,X) + A(D(X,Z)Y)) - A(D(U,X)Z).$$

Using equations (2.8) and (2.10) in (3.11) and after simplification. Putting $X = U = e_i$ and taking sum over i, $1 \le i \le n$, and further on simplification using equations (2.6) and (2.5), we get

$$(3.12)\quad drZ = \frac{(n+1)}{n^2}[(n+2)rA(Z) + 2nS(Z,\beta) - \frac{nr}{(n+1)}B(Z)].$$

Let us suppose that the manifold under consideration of non zero constant scalar curvature. Then from (3.12) we get,

$$(3.13)\quad S(Z,\beta) = \frac{r}{2(n+1)}B(Z) - \frac{(n+2)r}{2n}A(Z).$$

This shows that $S(Z,\beta)$ cannot be of the form $\kappa B(Z)$, where κ is a scalar. Hence β cannot be an eigenvector corresponding to any eigenvalue κ of S. This leads to the following theorem:

Theorem 3.3. *In an almost pseudo Bochner symmetric generalized complex space form of non zero constant scalar curvature, β cannot be an eigenvector corresponding to any eigenvalue of S.*

If in particular $A = B$, then from (3.13), we have

$$(3.14)\quad S(Z,\beta) = -\frac{(n^2+2n+2)}{2n(n+1)}rB(Z).$$

Corollary 1. *In a pseudo Bochner symmetric generalized complex space form of non zero constant scalar curvature, $-\frac{(n^2+2n+2)}{2n(n+1)}r$ is an eigenvalue corresponding to the eigenvector β*

Using equation (3.14) in (1.5), we get

$$(3.15)\quad (L_V g)(Z,\beta) + 2(-\frac{(n^2+2n+2)}{2n(n+1)}r)g(Z,\beta) + 2\lambda g(Z,\beta) = 0.$$

Suppose V is Killing vector field then using equation (2.11) in (3.15), we get

$$\lambda g(Z,\beta) = \frac{(n^2+2n+2)}{2n(n+1)}rg(Z,\beta), \text{ and } \lambda = \frac{(n^2+2n+2)}{2n(n+1)}r.$$

Then we can state the following

Lemma 3. *Let (g, V, λ) be a Ricci soliton in a pseudo Bochner symmetric generalized complex space form of non zero constant scalar curvature. If V is Killing vector field then it is expanding.*

4. Almost Pseudo Ricci Symmetric Generalized Complex Space Form

If the manifold M is an almost pseudo Bochner Ricci symmetric generalized complex space form, then we can be written as

$$K(JY, JZ) = K(Y, Z). \tag{4.1}$$

Taking the covariant derivative of (4.1), we get

$$(\nabla_X K)(JY, JZ) = (\nabla_X K)(Y, Z). \tag{4.2}$$

Using (2.9) in (4.2), we get

$$A(JY)K(X, JZ) + A(JZ)K(JY, X) = A(Y)K(X, Z) + A(Z)K(Y, X). \tag{4.3}$$

Using equation (2.10) in (4.2) and on simplification. Put $X = Z = e_i$ and taking sum over i, $1 \leq i \leq n$, on further simplification using equations (2.6) and (2.5) we get,

$$A(Y)r = 0. \tag{4.4}$$

Thus if $r \neq 0$, then from (4.4) we get $A(Y) = 0$. Using $A(Y) = 0$ in (2.9) we have

$$(\nabla_X K)(Y, Z) = B(X)K(Y, Z). \tag{4.5}$$

Theorem 4.1. *An almost pseudo Bochner Ricci symmetric generalized complex space form with non zero scalar curvature reduces to Ricci recurrent one.*

Also from (4.5) we get

$$(\nabla_X K)(Y, Z) - (\nabla_Y K)(X, Z) = B(X)K(Y, Z) - B(Y)K(X, Z). \tag{4.6}$$

Contracting (4.6) over Y and Z and using (2.10), (2.3) and (2.5), we get

$$dr(X) = r(n+1)B(X) - \frac{n}{2n+4}S(X, \beta). \tag{4.7}$$

If the scalar curvature r is constant, then

$$dr(X) = 0. \tag{4.8}$$

By virtue of (4.7) and (4.8) yields

$$S(X, \beta) = \frac{(2n+4)(n+1)}{n}rB(X). \tag{4.9}$$

In the other way, we assume that the Bochner curvature tensor of this space form is Codazzi type [8]. Then we have

$$(\nabla_X K)(Y, Z) - (\nabla_Y K)(X, Z) = 0. \tag{4.10}$$

Using (4.10) in (4.6), we get

$$B(X)K(Y, Z) - B(Y)K(X, Z) = 0. \tag{4.11}$$

Using equations (2.10) and (2.3) again putting $Y = Z = e_i$ and taking sum over i, $1 \leq i \leq n$ in (4.11), later simplification using equation (2.5) we get

$$S(X, \beta) = \frac{(2n+4)(n+1)}{n} rB(X). \tag{4.12}$$

This leads the following theorem.

Theorem 4.2. *In an almost pseudo Bochner Ricci symmetric generalized complex space form, if*

- *non zero scalar curvature or*
- *Bochner curvature tensor is Codazzi type*

then $\frac{(2n+4)(n+1)}{n} r$ *is an eigenvalue of the Ricci tensor* S *corresponding to the eigenvector* β.

Using equation (4.9) or (4.12) in (1.5), we get

$$(L_V g)(X, \beta) + 2(\frac{(2n+4)(n+1)}{n} r) g(X, \beta) + 2\lambda g(X, \beta) = 0. \tag{4.13}$$

By virtue of equation (2.12) in (4.13), we get

$$\lambda = -\frac{n\rho + 2(2n+4)(n+1)r}{2n}.$$

Then we can state the following

Lemma 4. *Let* (g, V, λ) *be a Ricci soliton in an almost pseudo Bochner Ricci symmetric generalized complex space form of non zero scalar curvature or Bochner curvature tensor is Codazzi type. If* V *is conformal Killing vector field then it is shrinking.*

5. Bochner flat almost pseudo Ricci symmetric generalized complex space form

From (1.4) we get

$$(\nabla_X S)(Y, Z) - (\nabla_Y S)(X, Z) = B(X)S(Y, Z) - B(Y)S(X, Z). \tag{5.1}$$

Setting $Y = Z = e_i$ in (5.1) and taking summation over i, $1 \leq i \leq n$, we obtain

$$dr(X) = 2rB(X) - 2S(X, \beta). \tag{5.2}$$

Putting $X = JX$ and $Y = JY$ in (5.1), we get

$$\begin{aligned}(\nabla_{JX} S)(JY, Z) - (\nabla_{JY} S)(JX, Z) \\ = B(JX)S(JY, Z) - B(JY)S(JX, Z).\end{aligned} \tag{5.3}$$

Again setting $Y = Z = e_i$ in (5.3) where $e_i, i = 1, 2, 3, \ldots, n$, is an orthonormal basis of the tangent space at each point of the manifold and taking summation over i, $1 \leq i \leq n$, we get

$$dr(JX) = -2S(X, \beta). \tag{5.4}$$

Let us consider a Bochner flat almost pseudo Ricci symmetric. Then we have

$$(\operatorname{div} D)(X,Y)Z = 0. \tag{5.5}$$

Differentiating (2.8) covariantly and contracting we obtain

$$\begin{aligned}(\operatorname{div} D)(X,Y)Z &= (\operatorname{div} R)(X,Y)Z \\ &-\frac{1}{2n+4}[g(Y,Z)dr(X) - (\nabla_Y S)(X,Z) + g(JY,Z)dr(JX) \\ &-(\nabla_{JY}S)(JX,Z) + (\nabla_X S)(Y,Z) - g(X,Z)dr(Y) + (\nabla_{JX}S)(JY,Z) \\ &-g(JX,Z)dr(JY) - 2(\nabla_{JZ}S)(Y,JX) - 2g(JX,Y)dr(JZ)] \\ &+\frac{1}{(2n+2)(2n+4)}[g(Y,Z)dr(X) - g(X,Z)dr(Y) + g(JY,Z)dr(JX) \\ &-g(JX,Z)dr(JY) - 2g(JX,Y)dr(JZ)].\end{aligned} \tag{5.6}$$

Using equations (5.5) and (2.1) in (5.6), we get

$$\begin{aligned}&\frac{2n+3}{2n+4}[(\nabla_X S)(Y,Z) - (\nabla_Y S)(X,Z)] \\ &-\frac{1}{2n+4}[(\nabla_{JX}S)(JY,Z) - (\nabla_{JY}S)(JX,Z) - 2(\nabla_{JZ}S)(Y,JX)] \\ &= \frac{2n+1}{(2n+2)(2n+4)}[g(Y,Z)dr(X) - g(X,Z)dr(Y) \\ &+g(JY,Z)dr(JX) - g(JX,Z)dr(JY) - 2g(JX,Y)dr(JZ)].\end{aligned} \tag{5.7}$$

By virtue of (5.1) and (5.3), it follows from (5.7) that

$$\begin{aligned}&\frac{2n+3}{2n+4}[B(X)S(Y,Z) - B(Y)S(X,Z)] \\ &-\frac{1}{2n+4}[B(JX)S(JY,Z) - B(JY)S(JX,Z) \\ &-2(\nabla_{JZ}S)(Y,JX)] \\ &= \frac{2n+1}{(2n+2)(2n+4)}[g(Y,Z)dr(X) - g(X,Z)dr(Y) \\ &+g(JY,Z)dr(JX) - g(JX,Z)dr(JY) - 2g(JX,Y)dr(JZ)].\end{aligned} \tag{5.8}$$

Using equation (2.3) in (5.8), we get

$$\begin{aligned}&\frac{2n+3}{2n+4}\{(n-1)f_1 + 3f_2\}[(n-1)B(X)] \\ &-\frac{1}{2n+4}[B(JX)S(JY,Z) - B(JY)S(JX,Z) \\ &-2(\nabla_{JZ}S)(Y,JX)] \\ &= \frac{2n+1}{(2n+2)(2n+4)}[g(Y,Z)dr(X) - g(X,Z)dr(Y) \\ &+g(JY,Z)dr(JX) - g(JX,Z)dr(JY) - 2g(JX,Y)dr(JZ)].\end{aligned} \tag{5.9}$$

Putting $Y = Z = e_i$ and taking sum over i, $1 \leq i \leq n$, on further simplification using equations (5.2), (5.4) and (2.5) we get

$$S(X,\beta) = \frac{4n+3}{-2n^2+4n+4}(n-1)\{(n-1)f_1 + 3f_2\}B(X)$$

and

$$S(X,\beta) = \phi g(X,\beta) \tag{5.10}$$

where $\phi = \frac{4n+3}{-2n^2+4n+4}(n-1)\{(n-1)f_1+3f_2\}$. Then we can state the following theorem.

Theorem 5.1. *In a Bochner flat almost pseudo Ricci symmetric generalized complex space form, λ is an eigenvalue corresponding to the eigenvector β.*

Using equation (5.10) in (1.5), we get

$$(L_V g)(X,\beta) + 2\phi g(X,\beta) + 2\lambda g(X,\beta) = 0. \tag{5.11}$$

Equation (2.12) in (5.11), we get

$$\lambda = -\frac{\rho + 2\phi}{2}.$$

Then we can write the following

Lemma 5. *Let (g,V,λ) be a Ricci soliton in a Bochner flat almost pseudo Ricci symmetric generalized complex space form. If V is conformal Killing vector field then it is shrinking.*

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