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Ramsey-remainder. (In English)

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The following general question is considered: Given a positive integer k, find the minimum number $rr(k)$ such that any sufficiently large set S belonging to some class $\mathcal{S}$ can be decomposed into "regular" sets of size at least k with a remainder set of size at most $rr(k)$. The number $rr(k)$ is the Ramsey-remainder. It is shown for example, that if $\mathcal{S}$ is the set of all posets, and regularity refers to a poset being a chain or an antichain, then $rr(k)=(k-1)(k-2)=r(k,k-1)$, where $r(k,k-1)$ is the poset Ramsey number. A similar result is proved when $\mathcal{S}$ is the class of all finite r-uniform complete hypergraphs the edges of which are colored by c colors and a regular hypergraph is one that is monochromatic. In this case $rr(k)=r_{c,k}(k)-1$. Other interesting Ramsey-remainder results are investigated, in particular, when $\mathcal{S}$ is the class of finite sets of points in general position in the plane and regularity refers to convexity. In this later case a sharp bound for the corresponding Ramsey-remainder number is obtained if the Erdős-Szekeres conjecture on the Ramsey number for convex sets in the plane is true.

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05C55 Generalized Ramsey theory
05D10 Ramsey theory

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