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On the product representation of powers. I. (In English)

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The authors study the solvability of the equation (1) $a_1a_2\cdots a_k = z^2$, $a_1, a_2, \ldots, a_k \in A$, $a_1 < a_2 < \cdots < a_k$, $x \in \mathbb{N}$ for fixed k and dense sets A of natural numbers. It is shown that if k is even, $k \geq 4$, and if A is of positive density then the above equation is solvable. In particular, it is proved that if

$$F_k(n) := \max_{A\subseteq[1,n];(1) \text{ is not solvable for } A} |A|, \text{ and}$$

$$L_k(n) := \max_{A\subseteq[1,n];(1) \text{ is not solvable for } A} \sum_{a\in A} \frac{1}{a}$$

then for all $n \in \mathbb{N}$, $F_2(n)$ is equal to the number of square-free integers not exceeding n, that is $F_2(n) \sim 6/\pi^2 \cdot n$. Non-trivial upper and lower bounds for $F_k(n)$ for $k = 3, 4, 4j, 4j+2$ are established and it is shown that $(\log 2 - \varepsilon)n < F_{2k+1} < n - (1-\varepsilon)n(\log n)^{-2}$ (the authors remark that the above lower bound could be improved slightly by a lengthy computation). Moreover, the following results on L_k are established for $n \to \infty$:
$L_{4j} = (1 + o(1)) \log\log n$, $L_{4j+2} = (3/2 + o(1)) \log\log n$, $L_{2j+1} = 1 + (1/2 + o(1)) \log n$.

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