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On some diophantine problems involving powers and factorials. (In English)
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An old conjecture states that any solution in positive integers n and x of the equation $1 + n! = x^2$ is given by $n = 4, 5, 7$. In this paper it is shown that for every $r \in \mathbb{N}$ there is an $n_0 = n_0(r)$ such that none of the integers $\sum_{i=1}^{r} n_i!$ $(n_0 < n_1 < \cdots < n_r)$ is powerful, i.e. not all exponents of the primes occurring in the prime factorization of such an integer are larger than 1. Unfortunately, the $n_0(r)$ can not be given explicitly. The authors also show that there is an effectively computable upper bound for the solution in positive integers a, k, p ($p > 2$ and prime) of the equation $(p-1)! + a^{p-1} = p^k$. The proof depends on deep results on linear forms in logarithms. Finally the same method is applied to obtain an effectively computable upper bound for k in a solution in positive integers x, k, p ($p > 1$) of the Ramanujan-Nagell equation $x^2 + D = p^k$ ($D \neq 0$, $D \in \mathbb{Z}$) which is close to being best possible in D.

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Classification:
11D61 Exponential diophantine equations

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