
Zbl 718.11041
Balog, A.; Erdős, Paul; Tenenbaum, G.
On arithmetic functions involving consecutive divisors. (In English)
Analytic number theory, Proc. Conf. in Honor of Paul T. Bateman, Urbana/IL (USA) 1989, Prog. Math. 85, 77-90 (1990).
[For the entire collection see Zbl 711.00008.]
The primary focus of this paper is the behavior of the functions

$$H(n) = \sum_{1\le i<\tau(n)} (d_{i+1} - d_i)^{-1}, \quad \kappa(n) = \sum_{d(d+1)|n} 1,$$

where $\tau(n)$ is the divisor function and $1 = d_1 < d_2 < ... < d_{\tau(n)} = n$ denotes the sequence of divisors of n. The function H was introduced by Erdős and has been recently investigated by *P. Erdős* and *G. Tenenbaum* [J. Number Theory 31, 285-311 (1989; Zbl 676.10030)]. In the present paper, the authors establish the upper bound

$$\max_{n\le x} H(n) \le D(x)^{1-c+o(1)},$$

where $D(x) = \max_{n\le x} \tau(n)$ and $c = 5/3 - (\log 3)/(\log 2)$, and conjecture that $H(n) \ll \tau(n)^{1-\delta}$ holds for some absolute constant $\delta > 0$. They also show that for sufficiently large x

$$\max_{n\le x} \kappa(n) > (\log x)^{(\log_3 x)/(9\log_4 x)},$$

where $\log_k x$ denotes the k times iterated logarithm, thereby improving a result of *P. Erdős* and *R. R. Hall* [J. Aust. Math. Soc., Ser. A 25, 479-485 (1978; Zbl 393.10047)]. The proof of the second result depends on an estimate of independent interest, namely the bound

$$\#\{n \le x : \ P(n(n+1)) \le y\} \gg xu^{-u^{7u}}$$

for $\max(2, x^{(8\log_3 x)/(\log_2 x)}) \le y \le x$, where $P(n)$ denotes the largest prime factor of n and $u = \log x/\log y$. This last result represents a quantitative version of a result of the reviewer [Proc. Am. Math. Soc. 95, 517-523 (1985; Zbl 597.10056)].

A.Hildebrand (Urbana)

Classification:
11N25 Distribution of integers with specified multiplicative constraints
11N37 Asymptotic results on arithmetic functions

Keywords:
consecutive divisors; divisor function; largest prime factor

©European Mathematical Society & FIZ Karlruhe & Springer-Verlag