Zbl 669.10011
Erdős, Paul; Lacampagne, C.B.; Selfridge, J.L.
Prime factors of binomial coefficients and related problems. (In English)
Acta Arith. 49, No.5, 507-523 (1988). [0065-1036]

Consider a sequence of k positive integers $\{a_i\}_{i=1}^k$ with the following properties: (i) $a_i \leq k$ for $i = 1, ..., k$; (ii) there is an n such that a_i is the quotient when $n + i$ is divided by all its prime factors greater than k (in other words: for each prime $p \leq k$, the pattern of that prime and its powers in the sequence $a_1, ..., a_k$ is the same as the pattern of that prime and its powers in some sequence of k consecutive integers).
Example: the sequence $1, 4, 3, 2, 5$ satisfies (i) and (ii) since $91, 92, 93, 94, 95$ factor into $1 \cdot 7 \cdot 13$, $4 \cdot 23$, $3 \cdot 31$, $2 \cdot 47$, $5 \cdot 19$.
This long but captivating paper starts with a proof that any sequence which satisfies (i) and (ii) is a permutation of $1, 2, ..., k$. However, only very few of the $k!$ possible permutations can actually occur in such sequences. Six different types of solutions are characterized, and it is proved - the main result of the paper - that these indeed are the only possible patterns which can occur in sequences which obey (i) and (ii). From this, all the solutions for $k \leq 27$ are derived and listed.
Next, for given k, the number of possible sequences satisfying (i) and (ii) is studied. It is proved that the logarithmic density of those values of k with exactly two solutions is positive. Finally, these results are applied to the problem of estimating the least prime factor of binomial coefficients.

H.J.J.te Riele

Classification:
11A41 Elemementary prime number theory
11B39 Special numbers, etc.
05A10 Combinatorial functions
11A05 Multiplicative structure of the integers

Keywords:
binomial coefficients; consecutive integers; logarithmic density; least prime factor of binomial coefficients