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Some remarks on infinite series. (In English)
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The authors prove four theorems.
(1) Suppose $a_n > 0$, $a_n \geq a_{n+1}$, $\sum_1^\infty a_n = \infty$. Then for every $c > 0$, $\sum a_n^2$, $\sum_{k=1}^\infty a_{n_k(c)}$ are equiconvergent, where $n_{k(c)}(c)$ is the minimal m such that kc$\leq \sum_{j=1}^m a_j$.
(2) Suppose $a_n > 0$, $\sum a_n = \infty$. (i) If (a_n) has a majorant $(b_n) \in \ell_2$ with $b_n \geq b_{n+1}$ for $n \geq 1$, then there exists a sequence of natural numbers $N_0 = 0$, $N_i \nearrow \infty$, such that (*) $\sum_{j=N_i+1}^{N_{i+1}} a_j \geq \sum_{j=N_{i+1}+1}^{N_{i+2}} a_j$ $(i = 0, 1, 2, ...)$; (ii) If $a_n \geq a_{n+1}$ for $n \geq 1$ then there exists a series $\sum b_n$ having no decomposition (*) and $1/3 < a_n/b_n < 3$.
(3) Suppose $a_n > 0$, $\sum a_n = \infty$. If $\sum a_n^2 < \infty$ then $X =^{def} \{c : \sum_{k=1}^\infty a_{n_k(c)} = \infty\}$ is of measure zero, and if $\sum a_n^2 = \infty$, then $Y =^{def} \{c : \sum_{k=1}^\infty a_{n_k(c)} < \infty\}$ is meagre (i.e. of first category).
(4) X can be residual, and Y can be of cardinality continuum.

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Classification:
40A05 Convergence of series and sequences

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decomposition of series