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On the Schnirelmann and asymptotic densities of sets of non-multiples. (In English)

Combinatorics, graph theory and computing, Proc. 16th Southeast. Conf., Boca Raton/Fla. 1985, Congr. Numerantium 48, 67-79 (1985).

[For the entire collection see Zbl 619.00006.]

Let $\delta(S)$, $\sigma(S)$ denote the asymptotic density (when it exists), the Schnirelmann density, respectively, of the set of natural numbers not divisible by any element of a set S of natural numbers, and let $D(S) = \delta(S) - \sigma(S) \geq 0$. When S is a finite set or a subset of the set $\mathcal{P}$ of all primes, the authors prove some interesting results concerning $D(S)$; for example: (1) $\sup\{D(S) : S \quad finite\} = 1$. (2) If $S \subset \mathcal{P}$, there exists S' with $S \subset S' \subset \mathcal{P}$ such that $\sigma(S') = \sigma(S)$ and $D(S') = 0$.

They also derive upper and lower bounds for $\sup\{D(S) : S \subset \mathcal{P}\}$. The paper concludes with a stimulating discussion describing related unsolved problems, their setting and implications.

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Classification:
11B05 Topology etc. of sets of numbers
11N37 Asymptotic results on arithmetic functions
11A25 Arithmetic functions, etc.

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sets of non-multiples; asymptotic density; Schnirelmann density; finite set; upper and lower bounds; unsolved problems