Zbl 536.10007

Erdős, Paul; Guy, R.K.; Selfridge, J.L.

Another property of 239 and some related questions. (In English)

Numerical mathematics and computing, Proc. 11th Manitoba Conf., Winnipeg/Manit. 1981, Congr. Numerantium 34, 243-257 (1982).

[For the entire collection see Zbl 532.00008.]

Regarding the decomposition $n! = a_1 a_2 ... a_k$ of n! into k factors the authors prove the following three interesting theorems:

Theorem 1. If $n > 239$ there is no factorization with $n < a_1 < a_2 < ... < a_k \leq 2n$.

Theorem 2. For every $n > 13$ there is a factorization with $n < a_1 \leq a_2 \leq ... \leq a_k \leq 2n$.

Theorem 3. Let $f(n)$ denote the smallest integer a_k for which there exists a factorization with $n < a_1 < a_2 < ... < a_k$. Then there are constants $0 < c_1 < c_2$ such that

$$2n + c_1 n/\log n < f(n) < 2n + c_2 n/\log n.$$

Besides they ask many interesting open questions.

K.Ramachandra

Classification:
11A25 Arithmetic functions, etc.
11A41 Elemementary prime number theory
05A10 Combinatorial functions

Keywords:
factors of n factorial