
Zbl 482.28001
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Some additive properties of sets of real numbers. (In English)
Fundam. Math. 113, 187-199 (1981). [0016-2736]

The paper is concerned with some additive properties of subsets of the real line R. The following finite version of *G.G.Lorentz*'s theorem [Proc. Am. Math. Soc. 5, 838-841 (1954; Zbl 056.039)] is proved: There is a positive number c so that for any positive integers n, m, and k, if A is a set of integers, $A \subset [m, m+k]$, with $|A| \geq l$, there is a set B of integers, $B \subset [n, n+2k]$ so that $A + B := \{a+b : a \in A, b \in B\}$ contains all integers in the interval $(n+m+k, n+m+2k]$ with $|B| < c \log l/l$. The following theorems are also obtained: Theorem 4. If S is a subset of R which is concentrated about a countable subset C, Then $\lambda(S+P) = 0$, for every closed set P with Lebesgue measure zero. Theorem 5. There are subsets G_1 and G_2 of R both of which are subspaces of R over the field of rational such that $G_1 \cap G_2 = \{0\}, G_1 + G_2 = R$ and both G_1 and G_2 have Lebesgue measure zero. Theorem 12. Assume $2^{\aleph_0} = \aleph_1$. Then there is a subset X of R such that (1) $|X| = \aleph$, (2) $\forall G \subseteq R[\lambda(G) = 0 \Rightarrow \lambda(G+X) = 0]$, (3) X is concentrated on the rationals. Open questions: Can one prove in ZFC that there is an X satisfying (1) and (2) of theorem 12?

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Classification:
28A05 Classes of sets
03E15 Descriptive set theory (logic)
28A12 Measures and their generalizations

Keywords:
ZFC; Lorentz's theorem