Zbl 435.10028
Erdős, Paul; Ruzsa, I.Z.
On the small sieve. I. Sifting by primes. (In English)
J. Number Theory 12, 385-394 (1980). [0022-314X]

The authors continue some investigations on what the reviewer has called the Erdős-Szemerédi sieve. Let A be a set of natural numbers not containing 1. Let $F(x,A)$ denote the number of natural numbers $n \leq x$, not divisible by any element of A. Let $K > 0$ be any constant. Let P run over all possible sets of primes the sum of whose reciprocals do not exceed K. Put $G(x,K) = \min F(x,P)$. The authors prove that

$$G(x,K) \geq x(\exp\exp(cK))^{-1},$$

where $x \geq 2$ and c is an absolute positive constant. (The proof involves a curious induction procedure which they call real type induction). They have also other results. For example if P is contained in $[2,x^{1-\delta}]$ then $G(x,K) \geq c$, $\delta e^{-K}x$, where $\delta > 0$ is arbitrary and c_1 is an absolute positive constant. They also study $\min F(x,A)$ where A ranges over more general sets of integers.

K.Ramachandra

Classification:
11N35 Sieves
11N05 Distribution of primes

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small sieve; sifting by primes; coprime sifting set