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Erdős, Paul; Spencer, Joel
Evolution of the n-cube. (In English)
Comput. Math. Appl. 5, 33-39 (1979). [0097-4943]

Let C^n denote the graph with vertices $(\varepsilon_1, \dots, \varepsilon_n)$, $\varepsilon_i = 0, 1$ and vertices adjacent if they differ in exactly one coordinate. We call C^n the n-cube. Let $G = G_{n,p}$ denote the random subgraph of C^n defined by letting $\text{Prob}(\{i,j\} \in G) = p$ for all $i, j \in C^n$ and letting these probabilities be mutually independent. We wish to understand the "evolution" of G as a function of p. Section 1 consists of speculations, without proofs, involving this evolution. Set

$$f_n(p) = \text{Prof}(G_{n,p} \text{ is connected}).$$

We show in Section 2: Theorem

$$\lim_n f_n(p) = \begin{cases} 0 & \text{if } p < 0.5 \\ e^{-1} & \text{if } p = 0.5 \\ 1 & \text{if } p > 0.5. \end{cases}.$$

The first and last part were shown by *Yu.Burtin*. For completeness, we show all three parts.

Classification:
05C99 Graph theory
05C40 Connectivity
60C05 Combinatorial probability
60D05 Geometric probability

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