

Zbl 334.04003

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Unsolved and solved problems in set theory. (In English)

Proc. Tarski Symp., internat. Symp. Honor Alfred Tarski, Berkeley 1971, Proc. Symp. Pure Math. 25, 269-287 (1974).

[For the entire collection see Zbl 291.00009.]

The problems in this paper are in the domain of combinatorial set theory. The major part of the paper is a report on solutions, or progress toward solutions, made by themselves and others, on problems in their previous paper [Axiomatic Set Theory, Proc. Sympos. pure Math. 13, Part I, 17-48 (1971; Zbl 228.04001)]. In the last third of the paper, the authors pose and explain nine new problems or groups of problems, of which we list several below (chosen on the basis of brevity).

Problem V (Erdős-Prikry). Let $|S| = \aleph_1$, $[S]^{\aleph_1} = \cup_{\xi < \omega_1} I_\xi$. Does there exist $\xi < \omega_1$ and sets $A, B, C \in I_\xi$ such that $A \cup B = C$?

Problem VI. Assume G.C.H. Let $|S| = \aleph_2$. Does there exist a disjoint partition $[S]^2 = \cup_{\nu < \omega_1} I_\nu$ satisfying the following condition: For all $S' \subset S$, $|S'| = \aleph_2$, there is $Z \subset S$, $|Z| = \aleph_1$ such that all different pairs of Z belong to different I_ν ?

Problem IX. Assume G.C.H. and let $\langle S, I \rangle$ establish $\aleph_{\alpha+1} \nrightarrow ([\aleph_\alpha, \aleph_{\alpha+1}]_2)^2$. (This means that $|S| = \aleph_{\alpha+1}$, $[X]^2 \not\subset I$ for all $x \in [S]^{\aleph_\alpha}$ and $[Y]^2 \cap I \neq \emptyset$ for all $Y \in [S]^{\aleph_{\alpha+1}}$.) For what $\beta \leq \alpha$ is $\langle Z, g \rangle$ isomorphic to a substructure of $\langle S, I \rangle$ for $|Z| \leq \aleph_\beta$?

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Classification:

04A20 Combinatorial set theory

00A07 Problem books