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Erdős, Paul; Faudree, Ralph J.; Rousseau, C.C.; Schelp, R.H.

The size Ramsey number. (In English)

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Let $\mathcal{C}$ be the class of all graphs G which satisfy $G \to (G_1, G_2)$. As a way of measuring minimality for members of $\mathcal{C}$, we define the size Ramsey number $\hat{r}(G_1, G_2)$ by $\hat{r}(G_1, G_2) = \min_{G\in\mathcal{C}} |E(G)|$. As usual, $\hat{r}(G)$ signifies $\hat{r}(G, G)$. For comparison purposes, we let $\hat{R}(G_1, G_2) := \binom{r(G_1,G_2)}{2}$, where $r(G_1, G_2)$ denotes the standard Ramsey number. It is clear that $\hat{r}(G_1, G_2) \leq \hat{R}(G_1, G_2)$ and we note in the paper that for all m, n, $\hat{r}(K_m, K_n) = \hat{R}(K_m, K_n)$. On the other hand, $\hat{r}$ can be much less than $\hat{R}$, a notion made precise by the following definition. An infinite sequence of graphs $\{G_n\}$ is said to be an o-sequence if $\hat{r}(G_n) = o(\hat{R}(G)n))$ $(n \to \infty)$. We prove several theorems related to the o-sequence concept. For example, we prove that if m is fixed, then $\{K_{m,n}\}$ is an o-sequence. In the course of this work, we find some new results for standard Ramsey numbers. For example, letting $K_m{*}\bar{K}_n$ denote the graph obtained from K_m by making one of its vertices adjacent to n additional vertices, we prove that if m is fixed and n is sufficiently large, then $r(K_m{*}\bar{K}_n) = (m-1)(m+n-1)+1$.

P.Erdős

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05C35 Extremal problems (graph theory)