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Remarks on some problems in number theory. (In English)
Math. Balk. 4, 197-202 (1974). [0350-2007]

The paper, presented at the fifth Balkan Congress, held in Beograd in 1974, consists of three parts in each of which a different subject is treated. Part I considers the problem of finding an estimate from above for $S(x)$, the number of integers $n \leq x$ for which there is a non-cyclic simple group of order n. Let V denote the sequence of integers $v_1 < v_2 \ldots$ with the property that for every prime p/v_i, v_i has a divisor $d_i \equiv 1(\mod p)$, $d_i > 1$. Further U denotes the sequence of integers (u_i) for which this property holds at least for the largest prime dividing u_i. If $V(x)$ and $U(x)$ denote the number of integers not exceeding x in the respective sequences, then $S(x) \leq V(x) \leq U(x)$. It is then proved that

$$U(x) < x \exp((-\frac{1}{2} + 0(1))(\log x \log\log x)^{1/2}),$$

which improves an earlier result of Dornhoff and Spitznagel for $S(x)$. The main tool is de Bruijn's well-known result on the number of integers not exceeding x, all whose prime factors are not greater than y. It is conjectured that

$$V(x) = x \exp(-(1 + 0(1))c_5(\log x)^{1/2} \log\log x).$$

Part II discusses a problem due to Hadwiger: Let $D(n)$ denote the set of integers with the property that if $k \in D(n)$ then the n dimensional unit cube can be decomposed into k homothetic n dimensional cubes. Let $c(n)$ be the smallest integer such that $k \geq c(n)$ implies $k \in D(n)$. It is proved that

$$c(n) \leq (2^n - 2)((n+1)^n - 2) - 1$$

and a number of related number theoretical problems are discussed.
Part III is devoted to the functions σ and φ. Several previous results of the author are mentioned and, as goes without saying for a paper of Erdős's, various unsolved problems are stated. Finally, this part of the paper contains some new results with hints of their proofs. We mention one:

$$\nu(\sigma(n)) = (\frac{1}{2} + 0(1))(\log\log n)^2,$$

with ν the function that counts the different prime factors of n.

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Classification:
11N37 Asymptotic results on arithmetic functions
11A25 Arithmetic functions, etc.
11-02 Research monographs (number theory)
20D05 Classification of simple and nonsolvable finite groups
11H99 Geometry of numbers
00A07 Problem books